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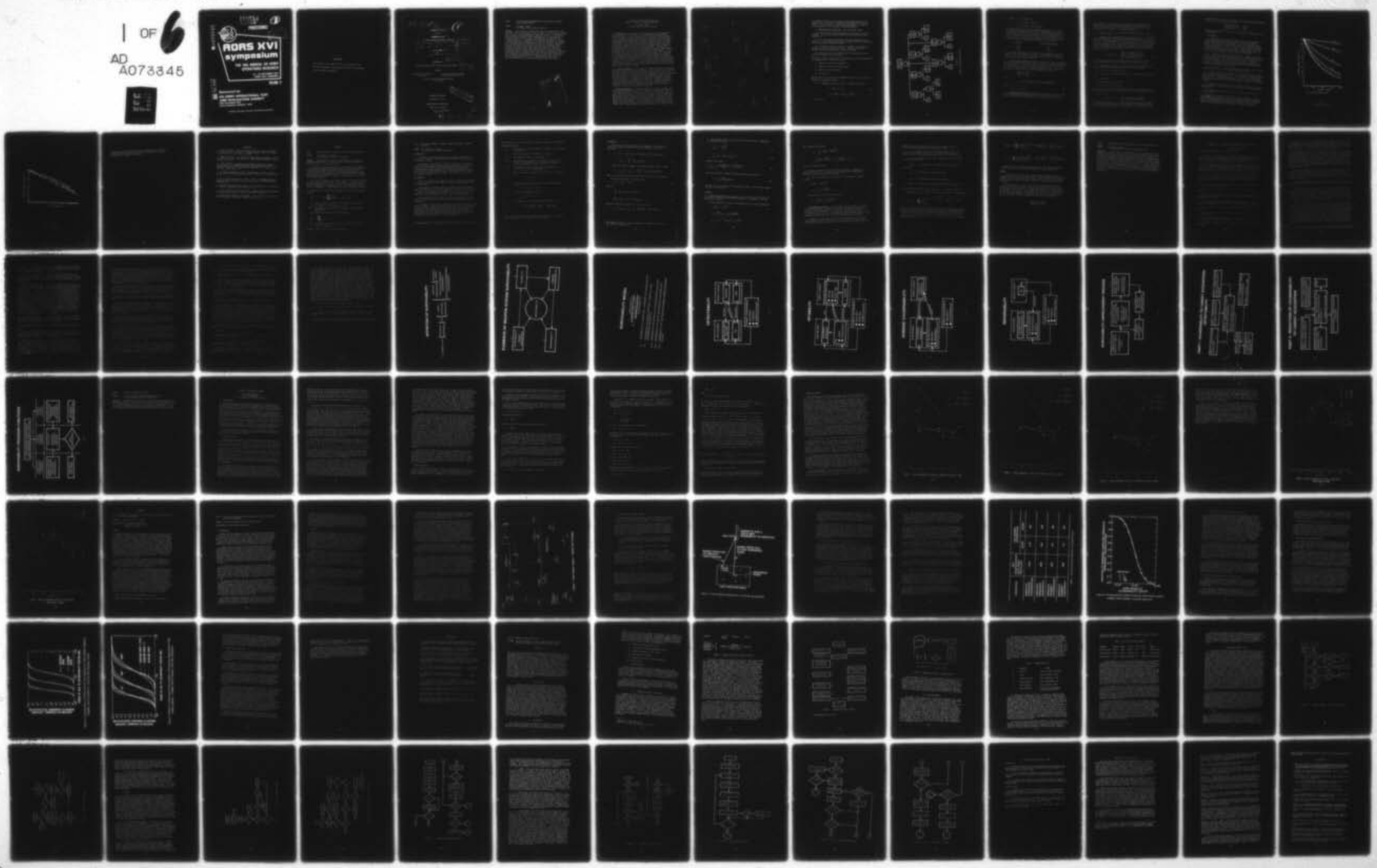

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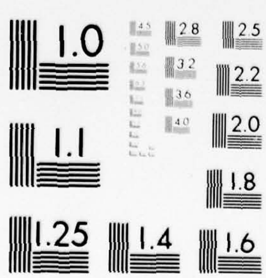

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TITLE: A Survivability Methodology for the Analysis of Theater Nuclear-Capable Units

AUTHOR: LTC John L. Hesse
USA TRADOC Systems Analysis Activity

ABSTRACT: Most studies of land warfare rely on the belief that units lose their combat effectiveness, and are unable to perform their assigned mission, when a fixed percentage of their men are lost. This belief is widely held by experienced combat commanders, but is not supported by historical data. Military units are assemblages of equipment and personnel which act in accordance with rehearsed procedures to achieve a combat capability. Accordingly, it seems more meaningful to analyze units microscopically to determine the effect of personnel and materiel losses on the subfunctions the unit must perform in combat, then to aggregate from this base as required by the study being performed. In this paper a method is developed, and exemplified for a 155mm self-propelled howitzer battery, to relate the firing, ammunition supply, command and control, communications, and supply and maintenance functions to the personnel and equipment responsible for functional performance. The mathematical model developed is used to examine the results of hostile counterbattery fire so as to determine the "weak links" in unit survivability, and to test the fixed percentage loss theory.

A SURVIVABILITY METHODOLOGY FOR THE ANALYSIS OF THEATER NUCLEAR-CAPABLE UNITS

LTC JOHN L. HESSE
USA TRADOC Systems Analysis Activity

1. Introduction: As a result of the recent Warsaw Pact conventional force buildup in Central Europe, and an essentially unchanged NATO defense posture, a greater implicit reliance on tactical nuclear forces (TNF) has emerged as de facto NATO policy.[1] The TNF role is twofold: they reinforce the credibility of the NATO deterrent, and they provide a supplement to the conventional defense of Western Europe.[2] In order for the TNF to fill these roles successfully, they must be perceived by potential adversaries as able to survive whatever conflict environment precedes political release of nuclear weapons. Survivability, the degree to which a unit can remain combat capable while withstanding hostile actions, is a relatively new measure of combat results. It can be thought of in terms of a SEE-HIT-KILL-REPAIR cycle, which says that if a unit can be seen by enemy target acquisition means, it can be hit by counterbattery fires. Given sufficient fire volume, unit elements can be killed, thus degrading the unit's combat capability. Restoration of full capability requires repair. The extent to which combat capability survives is a vital data point in the conduct of force-on-force analyses. The purpose of this paper is to suggest a methodology for relating vulnerability data to combat capability degradation, and hence to surviving unit combat power following an attack.

2. Existing Methodology: Most studies of land warfare rely on the belief that units lose their combat effectiveness, and are unable to perform their assigned mission, when a fixed percentage of their men are lost. This belief is widely held by experienced combat commanders, but is not supported by historical data.[3] Military units are assemblages of equipment and personnel which act in accordance with rehearsed procedures to achieve a combat capability. Accordingly, it seems more meaningful to analyze units microscopically to determine the effect of personnel and materiel losses on the subfunctions the unit must perform in combat, then to aggregate from this base as required by the study being performed. This forms the thrust of the approach described here.

3. Proposed Methodology: From the viewpoint of a hostile planner, four target configurations are of analysis interest in planning an attack on a nuclear-capable unit. These are the garrison, or peacetime, posture; the dispersal area posture; the road march posture; and the combat deployment posture. Each requires allocation of different delivery means and amounts of ordnance to produce the desired effect. Doctrinally, Soviet forces employ two damage criteria: neutralization, which implies 20-25 percent fractional damages to men and materiel, and annihilation, which implies 50-60 percent losses to men and materiel.[4] The discussion which follows uses a combat-deployed 155mm howitzer battery, as shown in Figure 1,

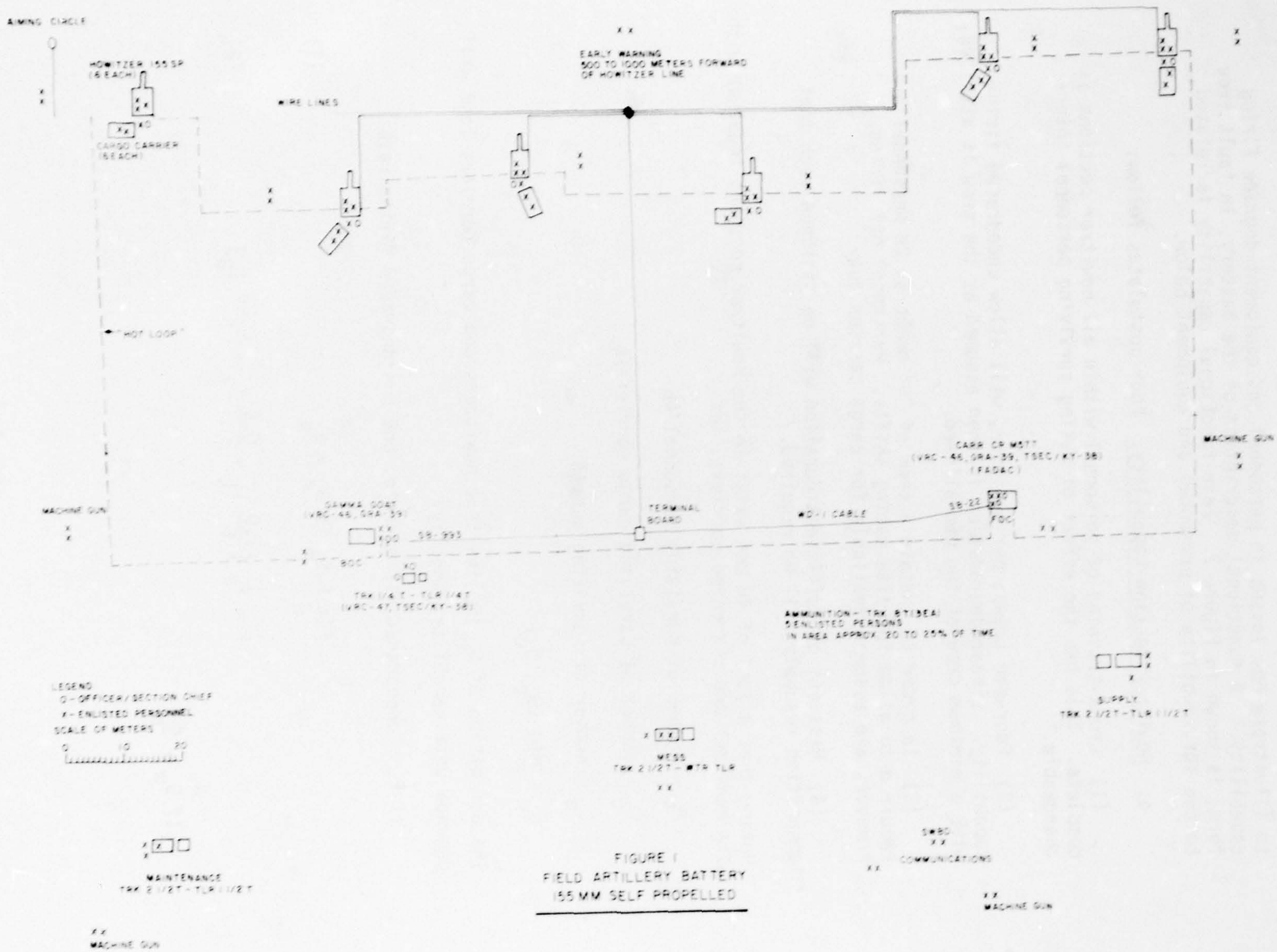

FIGURE 1
FIELD ARTILLERY BATTERY
155 MM SELF PROPELLED

to illustrate how losses in personnel and equipment degrade firing capability.[5] A functional description of the battery, in fault tree form, is shown in Figure 2. Each functional capability is related to the survivability of personnel and equipment below.

a. Howitzer Section Capability. Four postulates follow:

(1) Cross-training of personnel within all howitzer sections is complete. This has the effect of making surviving personnel interchangeable.[6]

(2) Four-man crews, per howitzer, will allow undegraded firing capability. Linear degradation is then assumed as the crew is attrited, with a minimum crew of two men allowed.

(3) In order to sustain a crew of "n" members, 2n personnel are required to allow for alternating shifts. Personnel not firing, however, are assumed available for cargo carrier duty.

(4) Missions of sufficient duration will be assigned such that ammunition resupply will be required.

Noting that a pool of 48 men exist in the howitzer sections together with six howitzer and six cargo carriers, let:

S_p = number of surviving personnel/48

S_c = number of surviving cargo carriers/6

S_g = number of surviving guns/6

$S_m = \min(S_g, S_c)$

The definition of S_m implies that howitzers and cargo carriers lost will degrade unit capability equally.

If $\dot{F}_o$ = undegraded firing rate, and $\dot{F}$ = degraded firing rate:

$$\dot{F} = \dot{F}_o S_m \text{ if } S_m \leq S_p \tag{1}$$

$$\dot{F} = \dot{F}_o \left[\frac{1.0}{6} I_1 + \frac{0.5}{6} I_2 + \frac{0.25}{6} I_3\right] \tag{2}$$

If $S_m > S_p$

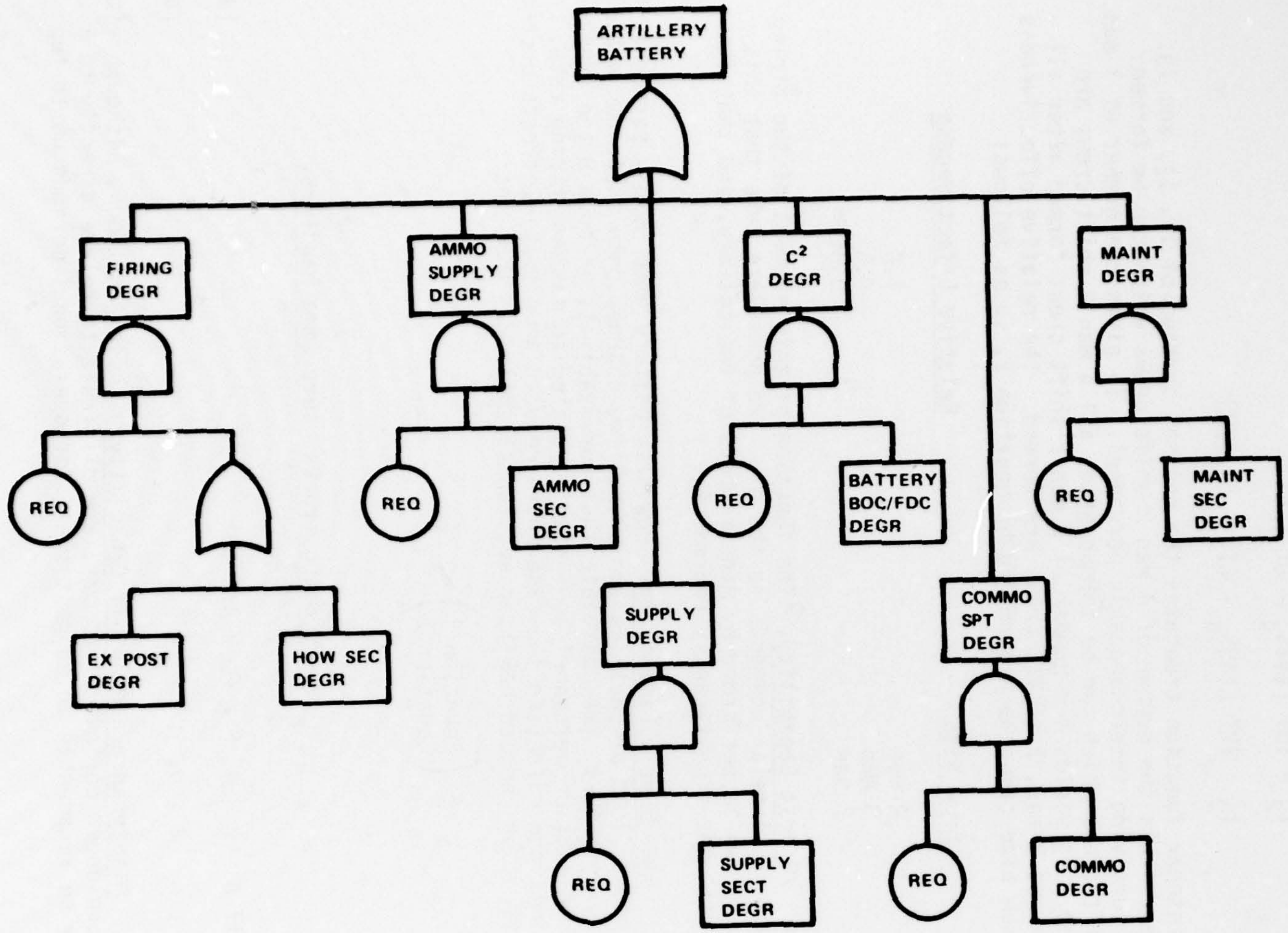

FIGURE 2

BATTERY FUNCTIONAL DESCRIPTION

where: $I_1 = \text{INT}\ [(48S_p)/8]$

$I_2 = \text{INT}\ [(48S_p - 8I_1)/6]$

$I_3 = \text{INT}\ [(48S_p - 8I_1 - 6I_2)/4]$

The integer function truncates the decimal portion of I_1, I_2, and I_3. I_1 represents the number of 4 man 2 shift crews which can be formed from surviving interchangeable personnel. I_2 gives the number of 3 man 2 shift crews which can be formed after all 4 man 2 shift crews are formed. I_3 yields the number of 2 man 2 shift crews formed after all 4 man and 3 man, 2 shift, crews are formed. The relative effectiveness of each size crew, as determined by equation 2, is as follows:

Crew Size	Relative Effectiveness
4 man	1.0
3 man	0.5
2 man	0.25

b. XO Post Capability. The function of this element of the firing battery is to relay commands to the gun. We postulate here that this function can be performed by many members of the battery, and that its loss will not significantly degrade $\dot{F}_o$.

c. Ammo Supply Capability. Howitzer battery ammo supply personnel are required to load projectiles and propelling charges onto their own vehicles as ASPs and SASPs. We postulate that survivability of both 8 ton trucks and ammo section personnel is linearly related to maximum firing rate, but limit the effect to the case where reduced ammo supply effects exceed the effect of reduced howitzer section capability. Thus let:

$$A = \left(\frac{\text{Equation 1}}{\text{Equation 2}}\right) \div \dot{F}_o$$

and

$$B = \left[\min\left(\frac{\text{surv 8 ton trucks}}{3}, \frac{\text{surv ammo handlers}}{5}\right)\right]$$

Then if $A - B \leq 0,\ \dot{F} = \dot{F}_o \times A$ (3)

$A - B > 0,\ \dot{F} = \dot{F}_o \times B$ (4)

d. Maintenance and Supply Capability. These functions, although vital over prolonged periods of combat, are difficult to relate directly to $\dot{F}_o$. It may be reasonable to suggest that, for operational periods up to two

weeks in duration, a maximum of 20% degradation in $\dot{F}_o$ could occur as a linear function of surviving personnel assumed interchangeable. Loss of vehicular capability could probably be made up for by using other unit vehicles for resupply. $\dot{F}$ is then represented by:

$$\dot{F} = [\text{Equation } (\tfrac{3}{4})] \times [.2\ (\frac{\text{maintenance + supply survivors}}{9}) + 0.8] \qquad (5)$$

e. <u>Communications, Command and Control Degradation (C^3)</u>. C^3 functions are concentrated in the BOC, FDC, and communications sections. Internal wire nets are believed replaceable by voice or messenger, and will thus not affect $\dot{F}_o$. In the BOC/FDC, it is postulated that 9 of the 12 total members are interchangeable in terms of performing C^3 functions. Of these, 3 at a time are required for full capability. Degradation will be nonlinear based on discussions with experienced battery officers, and is postulated to occur as follows:

$$\dot{F} = [\text{Equation 5}] \quad \times \quad \begin{bmatrix} \text{1.0, 3 FDC/BOC survivors} \\ \text{0.85, 2 FDC/BOC survivors} \\ \text{0.35, 1 FDC/BOC survivors} \\ \text{0, 0 FDC/BOC survivors} \end{bmatrix} \qquad (6)$$

This inherently ignores the residual capability to respond to forward observer-directed missions, and considers only computed missions.

f. <u>Communications Section</u>. Communications with the supported unit, higher, and adjacent units depends on survival of the following:

(1) One of two AN/VRC-46/TSEC/KY-38.

(2) A source of power (M577, Gamma Goat, or either battery generator set).

(3) An operator.

(4) Code changer key KYK-28/TSEC.

(5) One of two RC-292 antennas.

Operator skills are sufficiently widespread within the battery that $\dot{F}_o$ should not be affected. The code changer key will be considered survivable.

$\dot{F}$ then becomes

$$\dot{F} = [\text{Equation 6}] \quad \times \quad \begin{bmatrix} 1,\ \text{radio} \cap \text{power} \cap \text{antenna} \\ 0,\ \text{any other combination} \end{bmatrix} \qquad (7)$$

4. <u>Inherent Availability</u>: The preceding paragraphs treat combat damage. In force-on-force calculations, the true value of firing rate will also be affected by availability of battery equipment. Typical values for the

European theater, derived from records at the USA Maintenance Management Center, are as follows for major items:[7]

Howitzer, M109	0.67
Cargo Carrier, M548	0.41
Truck, 8 ton	0.93

These values would enter equations 1, 2, 3, and 4 as multipliers of the equipment terms.

5. An Example:

a. The 155mm howitzer battery shown in Figure 1 was targeted, in counterbattery mode, by a 122mm multiple rocket launcher battery. Target acquisition error was parametrically varied from zero to 800 meters. The resultant residual combat capability, computed using the foregoing methodology, is shown in Figure 3 as a function of the number of 122mm rockets fired at it. Given that a battery volley equals 240 rounds, and that the 122mm multiple rocket launcher battery has a 10 minute reload time between volleys,[8] we see the following:

(1) In the worst case situation where hostile target acquisition is perfect, a 155mm howitzer battery under attack by a 122mm multiple rocket launcher battery can preserve 65 percent of its combat capability by moving to a new area if the old area can be cleared in 10 minutes. It is critical, however, that the type of arriving hostile ordnance be recognized, since other types may produce different results and, hence, different required actions for survival.

(2) The ability of a 155mm howitzer battery to defeat hostile target acquisition is critical to a surviving combat capability.

b. Another analysis was conducted to compare the foregoing methodology with the fixed-percentage-loss theory. A large number of 122mm multiple rocket launcher counterbattery attacks on a 155mm howitzer battery were conducted, and the residual combat capability as a function of percentage of personnel casualties was determined as shown in Figure 4. We note the following:

(1) Personnel shortages/losses have a stronger than linear impact on combat capability. A priority effort should be made to improve personnel survivability.

(2) The fixed-percentage-loss theory overestimates survivability for the case studied.

6. Validation: The foregoing methodology assumes a number of linear or quasi-linear relationships. Clearly, any supportable analytic representation of the degradation of each functional capability can be substituted. Data to allow refinement and validation could readily be collected during unit field training and evaluations. Further, the concept of

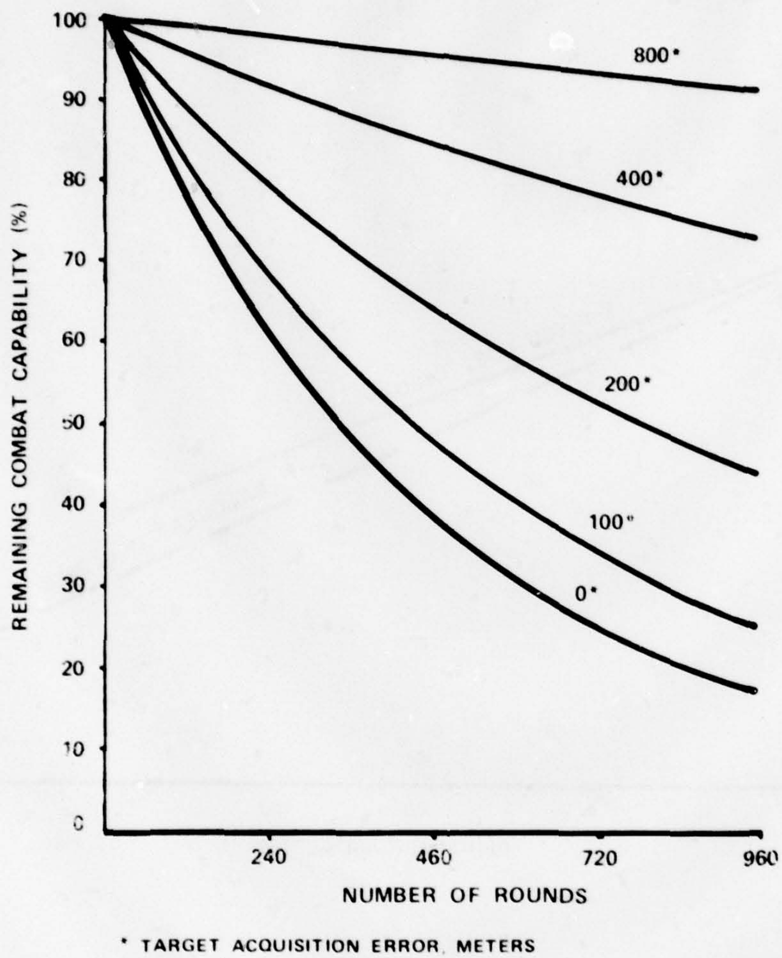

FIGURE 3

COUNTERBATTERY ATTACK

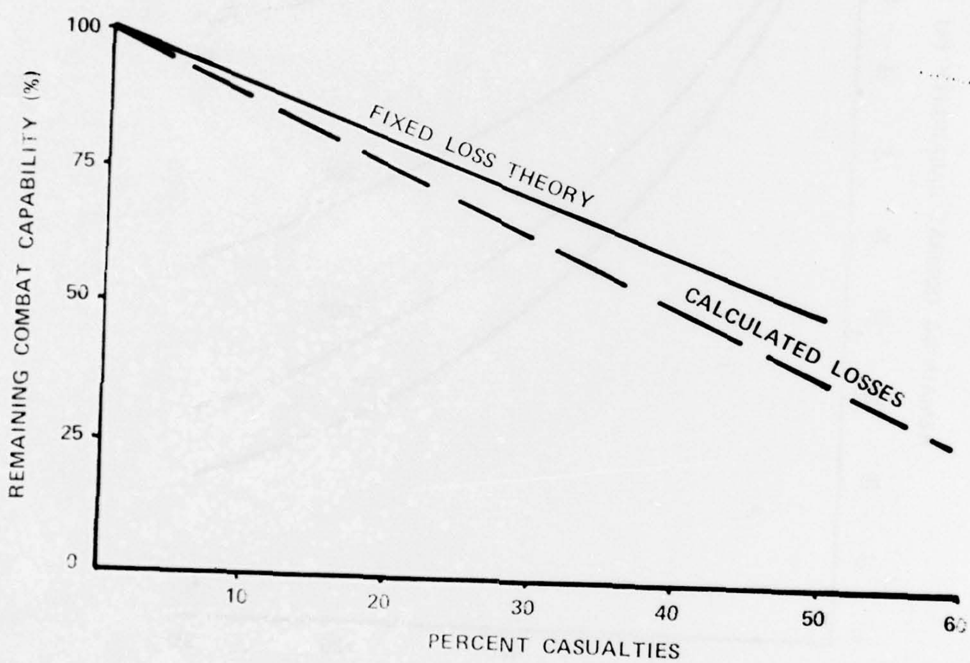

FIGURE 4

METHODOLOGY COMPARISON

relating personnel and materiel losses to degradation of functional capabilities has equal application to any military unit for which survivability in combat is of concern.

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ABSTRACT

TITLE: Expected Number of Attempts to Achieve a Binary Objective

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US Army Ballistic Research Laboratory

ABSTRACT: An equation (A) is derived which predicts the number of attempts one would expect to make to achieve an objective which has only two states, i.e. success, failure.

The equation is particularly useful in the analysis of firepower systems which can change method and/or type of munition delivered. It can be used to determine the expected depletion rate per success associated with a given strategy or to determine the optimum strategy resulting in the lowest depletion rate. Since time to kill is a monotonic function of the number of attempts, the equation is also useful in assessing lethality.

A strategy is defined as an ordered sequence of attempts predicated on failure of preceeding attempts. Each sequence is partitioned such that the probability of success associated with an attempt in a subsequence is independent of the probabilities associated with attempts in all other subsequences. Interdependence within a subsequence can exist.

The derivation is described and examples given for the following equation:

$$\text{(A)} \cdots\cdots \qquad E_n = \sum_{K=1}^{T} \frac{\Pi_{K-1}}{(1 - \Pi_T)} * (N_K - \psi_K)$$

E_n = Expected number of attempts to first achieve the objective

T = Total number of subsequences in a cycle

Π_{K-1} = The probability of failing all attempts through the "k^{th}" subsequence

N_K = Number of attempts in the "K" subsequence

$$\psi_K = \sum_{i=1}^{N_{K-1}} P_K(i) \;, \quad \psi_0 = 0$$

$P_K(i)$ = Probability of at least one success by the "i^{th}" attempt in the "k^{th}" subsequence

CATEGORY: Special Session: Lethality/Resources

TITLE: THE EXPECTED NUMBER OF ATTEMPTS REQUIRED TO ACHIEVE A BINARY OBJECTIVE

AUTHOR: *Mr. Lawrence D. Johnson*
US Army Ballistic Research Laboratory

INTRODUCTION:

An equation is derived which predicts the number of attempts one would expect to make to achieve an objective which has only two states, i.e. success, failure.

The equation is particularly useful in the analysis of firepower systems which can change method and/or type of munition delivered. It can be used to determine the expected depletion rate per success associated with a given strategy or to determine the optimum strategy resulting in the lowest depletion rate. Since time to kill is a monotonic function of the number of attempts, the equation is also useful in assessing lethality.

THE PROBLEM

This is an expected value problem. We wish to determine the number of attempts one might expect to make to achieve a binary objective* for a given strategy.

By strategy, we mean the sequence of attempts which are planned to *achieve the objective.* It will be convenient to represent each strategy as a sequence of subsequences of attempts, i.e., $\{\S_1\}$, $\{\S_2\}$, $\{\S_3\}$..... $\{\S_t\}$ where $\{\S_i\}$ represents the sequence of the first N_1 attempts, $\{\S_2\}$ represents the next N_2 attempts, $\{\S_3\}$ the next N_3 attempts and so on.

This sectioning of the sequence is done to take advantage of intra-sequence similarities which will greatly reduce the number of required calculations.

For example, assume a man is attempting to break a bottle with a rifle having ten shots. Assume also that one wishes to analyze the strategy of firing two rounds to register and eight for effect. By sectioning this strategy into two subsequences of two attempts and eight attempts respectively, it can be shown that only four terms need be calculated. If done otherwise, over ten terms must be individually calculated. The advantages will become clearer as the note progresses.

* A binary objective is one which has only two states, success or failure.

Before proceeding to the derivations, an index of terms will be introduced.

DEFINITION OF TERMS

E_n $\equiv$ The expected value of the number of attempts to first achieve the objective

$\{\S_i\}$ $\equiv$ The ith subsequence of attempts in a strategy

N_i $\equiv$ The number of attempts in subsequence $\{\S_i\}$

$P_i(j)$ $\equiv$ The probability of achieving at least one success in "j" attempts of subsequence $\{\S_i\}$ given that all attempts in previous subsequences failed* (Note: $P_o(j) = P_j(o) = o, \forall j$)

$\tilde{P}_i(j)$ $\equiv$ The probability that the jth attempt in subsequence $\{\S_i\}$ is the first success in subsequence $\{\S_i\}$

S_j $\equiv$ The total number of attempts prior to the "j+1" subsequence, i.e.,

$$S_j = \sum_{i=1}^{j} N_i$$

t $\equiv$ The total number of subsequences before strategy repeats itself,

$$\text{i.e., } P_{(i+t)}(j) = P_i(j) \forall i$$

Ψ_k $\equiv$ An artificial variable defined as

$$\Psi_k = \sum_{i=1}^{N_k - 1} P_k(i), \quad \Psi_o = o$$

Π_k $\equiv$ The probability of failures through S_k attempts, i.e.,

$$\Pi_k = \left[1-P_1(N_1)\right] * \left[1-P_2(N_2)\right] * \;\ldots * \left[1-P_k(N_k)\right]$$

* If one chooses to define the subsequences such that they are independent of each other, the conditional aspects disappear.

DERIVATION

By definition, the expected value of a quantity is the sum of all values of the quantity multipled by their probability of occurence. This means,

$$E_n \equiv E\{i\} = \sum_{i=1}^{N_1} i*\tilde{P}_1(i) + [1-P_1(N_1)] * \sum_{i=1}^{N_2} (i+N_1)*\tilde{P}_2(i)+\ldots$$

$$+ \Pi_{k-1} * \sum_{i=1}^{N_k} (i+S_{k-1}) *\tilde{P}_k(i)+\ldots \qquad (1)$$

Assume that after S_t attempts, the strategy repeats itself.* Then:

$$E_n = \sum_{j=o}^{\infty} [\Pi_t[^j * \sum_{k=1}^{t} \Pi_{k-1} *[\sum_{j=1}^{N_k} (i+S_{k-1}+j*S_t)*S_t)*\tilde{P}_k(i)] \qquad (2)$$

where Π_t is the probability that success does not occur in "S_t" attempts.

With little difficulty one can show that

$$\tilde{P}_i(i) = P_k(i) - P_k(i-1) \qquad (3)$$

and that

$$\sum_{i=1}^{N_k} [P_k(i)-P_k(i-1)] = P_k(N_k) \qquad (4)$$

$$\sum_{i=1}^{N_k} i*[P_k(i)-P_k(i-1)] = N_k*P_k(N_k) - \Psi_k \qquad (5)$$

With these identities, equation (2) can be reduced to

$$E_n = \sum_{j=0}^{\infty} [\Pi_t]^j * \sum_{k=1}^{T} \Pi_{k-1} *[S_k*P_k(N_k) - \Psi_k+j*S_t*P_k(N_k)] \qquad (6)$$

*For example, a tank has a finite complement of ammunition after which it must reload and engage again.

The infinite series can be put in closed form solution by remembering the simple relationships.

$$\sum_{i=0}^{T} x^i = \frac{1-x^{T+1}}{1-x} \tag{7}$$

$$\sum_{i=0}^{T} i*x^i = x \frac{\partial}{\partial x} \left[\sum_{i=0}^{T} x^i \right] \tag{8}$$

Equation (6) becomes

$$E_n = \sum_{k=1}^{T} \Pi_{k-1}*\left[\frac{S_k*P_k(N_k)-\Psi_k}{1-\Pi_t} + \frac{S_t*P_k(N_k)*\Pi_t}{} \right] \tag{9}$$

With more work than is probably warranted, equation (9) reduces to

$$E_n = \sum_{k=1}^{T} \frac{\Pi_{k-1}*}{(1-\Pi_t)} [N_k-\Psi_k] \tag{10}$$

Equation (10) is the solution to the question at hand. The following examples may show its application.

EXAMPLES

E-1. Constant probabilities of success. Every so often, the sun shines and the problem at hand is one where each attempt has an independent, constant probability of success.

If the probability of success is designated as "p" for each attempt, then

$$P_k(N_k) = 1-(1-p)^{N_k}$$

$$\Psi_k = \sum_{i=1}^{N_k-1} P_k(i) = N_k-\left[\frac{1-(1-p)^{N_k}}{p}\right]$$

$$\Pi_{k-1} = (1-p)^{N_1}*(1-p)^{N_2}\ldots(1-p)^{N_k-1}$$

Thus, equation (10) becomes

$$E_n = \sum_{k=1}^{T} \frac{\Pi_{k-1}}{1-\Pi_t} \left[\frac{1-(1-p)^{N_k}}{p}\right]$$

$$= \sum_{k=1}^{T} \frac{\Pi_{k-1}}{1-\Pi_t} * \left[\frac{P_k(N_k)}{p}\right] = \sum_{k=1}^{T} \left[\frac{\Pi_{k-1}-\Pi_k}{1-\Pi_t}\right] * \frac{1}{p} = \frac{1}{p}$$

not a very stunning result!

E-2. Constant probabilities which change with sequence. Sometimes the probability of success is constant for each attempt within subsequences, but has a different value for each sequence.

Let "p_k" be the probability of success for each attempt in sequence k, then

$$P_k(N_k) = 1-(1-p_k)^{N_k}$$

$$\Psi_k = N_k - \left[\frac{1-(1-p_k)^{N_k}}{p_k}\right]$$

Substitution into equation (10) results in

$$E_n = \sum_{k=1}^{T} \frac{\Pi_{k-1}}{(1-\Pi_t)} * \left[\frac{1-(1-p_k)^{N_k}}{p_k}\right]$$

Intradependent attempts. In this example, the probability of success of each attempt within a sequence is implicitly dependent upon the success or failure of previous attempts. This is the case with most firepower systems which, unfortunately, are plaqued with random and bias errors. It is the latter which causes the intradependence whereas changing munition types, choosing a new aim point, etc. causes the interdependence of attempts.

Assume one is analyzing a firepower system which has an error budget table such that random, variable bias, and fixed bias error distributions are known for the range in question. Also, assume that the conditional kill

probability can be reasonably estimated for the targets of interest. Using the following definitions, we set up the solution. Let:

$(x, y) \equiv$ spatial variables indicating the impact point of the projectile in the plane of the target. The plane may be defined as vertical or horizontal depending on weapon type, e.g., tank, artillery.

$\eta_x, \eta_y \equiv$ variable describing the system bias consisting of both a fixed and variable bias.

$\sigma^2_{R_x}, \sigma^2_{R_y} \equiv$ the variance of the random errors, in the x and y direction.

$\sigma^2_{\eta_x}\ \sigma^2_{\eta_y} \equiv$ the variance of the variable biases.

$\mu_x, \mu_y \equiv$ the fixed bias in the x and y direction

$P_{c_k}(x,y) \equiv$ the conditional probability that the target will be defeated if the projectile impacts at point x, y in the "kth" sequence.

Assume that all error sources have normal distributions. Further define

$$\rho_k(\eta_x,\eta_y) = \iint_{-\infty}^{\infty} \frac{P_{c_k}(x,y)}{2\Pi\sigma_{R_x R_y}} \left[e^{-\frac{1}{2}\left[\frac{(x-\eta_x)^2}{\sigma_{\eta_x}^{\ 2}} + \frac{(y-\eta_y)^2}{\sigma_{R_t}^{\ 2}}\right]}\right] dxdy$$

which is the probability of defeating a target with an attempt given specific values for η_x, η_y. Note that for many munitions against hard targets, such as tanks, the bounds of this integration are limited to the projection of the target on the target plane. This occurs because $P_{c_k}(x,y) = o$ when the projectile misses these target types. The probability of killing the target in "n" attempts is:

$$P_k(N_k) = 1 - \iint_{-\infty}^{\infty} \frac{[1-\rho k(\eta_x,\eta_y)]^{N_k}}{2\pi\sigma_{\eta_x}\sigma_{\eta_y}} * \exp\left\{-\frac{(\eta_x-\mu_x)^2}{2\sigma_{\eta_x}^{2}} - \frac{(\eta_y-\mu_y)^2}{2\sigma_{\eta_y}^{2}}\right\} d\eta_x d\eta_y$$

$$\Psi_k = N_k - \iint_{-\infty}^{\infty} \frac{[1-\{1-\rho k(\eta_x,\eta_y)\}^{N_k}]}{\rho_k(\eta_x,\eta_y)2\pi\sigma_{\eta_x}\sigma_{\eta_y}} * \exp\left\{-\frac{(\eta_x-\mu_x)^2}{2\sigma_{\eta_x}^{2}} - \frac{(\eta_y-\mu_y)^2}{2\sigma_{\eta_y}^{2}}\right\} d\eta_x d\eta_y$$

When these are substituted into equation (10), the expected number of rounds used and thus time to defeat the target is easily calculated.

SUMMARY

The equation for predicting the expected number attempts required to achieve a binary objective has been derived. These predictions are a necessary ingredient in most systems analyses. They are also essential in determining optimum strategies when the objective-function is dependent upon the number of attempts.

Before concluding, it must be pointed out that the qualification of "binary objectives" is not minor. Many times, although a previous attempt has failed to meet an objective, it alters the probability of future success, and therefore is not binary. For example, when a boxer hits an opponent with the objective of a knockout, although the blow fails to meet the objective, it can sufficiently condition the opponent so that a lesser subsequent blow will be successful. With this homey disclosure, the note concludes.

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TITLE: Implementation of Survivability Program for Weapon Systems

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ABSTRACT: In present times of expensive and sophisticated weapons, high combat attrition rates, often limited stockpiles, and numerical superiority of military personnel and materiel demonstrated by potential enemies, survivability is becoming an element of weapon effectiveness of primary importance to our armed forces. This paper defines the type and extent of effort required for an effective implementation of a survivability program for Army weapon systems. It reveals and highlights all essential factors related to the weapon and its combat environment that could affect survival of Army weapons on a battlefield. Further, this paper outlines a program procedure for a systematic evaluation of these factors in the context of a total weapon system and in a manner that could assure cost-effective weapon survivability improvements.

IMPLEMENTATION OF SURVIVABILITY PROGRAM FOR WEAPON SYSTEMS

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In present times of expensive and sophisticated weapons, high combat attrition rates, often limited stockpiles and increased vulnerability to detection and suppression by enemy action, weapon survivability is becoming a military discipline of primary importance to our armed forces.

Today, whether designing a new weapon or improving the existing one, we must ask ourselves not only what can we do to make these weapons perform better, but also, with equal importance, what can we do to improve their chance of survival on a battlefield. In other words, our objective in the weapon development process must equally stress performance and survival.

This emphasis on survivability has been reflected in the DARCOM policy of recent years. Accordingly, the AMSAA has been designated as a Lead Activity for survivability, and survivability program requirements have been set for DARCOM elements. These requirements can be paraphrased as follows:

- Survivability considerations and investigations will be applied to the total life cycle of a weapon system.

- Survivability requirements will be included into materiel planning documents.

- DARCOM elements will establish their own survivability implementation programs and will develop needed capability for conducting survivability improvement investigations.

The purpose of this paper is to offer a set of guidelines that could assist in the implementation of DARCOM survivability program for Army assets such as weapon systems. These guidelines could be summed-up as follows:

- Define the scope of your program by first defining the weapon survivability.

- Identify factors that could affect the survival of your weapon system on a battlefield.

- Establish a mechanism for evaluation of these factors and consequent identification of survivability improvements with payoffs.

- Establish a workable procedure for implementation of survivability improvements.

To define the range of our program effort, we must, first of all, define survivability as it pertains to our program needs. Figure 1 shows that the definition could vary in scope, depending on what we want to include or not into our program. In situations where some survivability activities already exist, the extent of our program must be adjusted accordingly.

Having defined the scope of our program, next, we must attempt to identify those elements and factors that could affect survival of our weapons on a battlefield. Figure 2 shows four basic elements of weapon system survivability: detectability (or target acquisition), hitability, vulnerability to damage and repairability. In other words, the survival of our weapons on a battlefield will depend upon how vulnerable our weapons are to detection and location by enemy's target acquisition equipment, how vulnerable they are to being hit if detected, and how vulnerable they are to damage by enemy munitions if hit. Their survival will depend also on how easily they could be repaired when damaged.

A mathematical relationship of the weapon survivability to these four elements is shown in Figure 3. This is a basic survivability model. It can be seen that our survivability improvement goals should be pursued through the reduction of the vulnerability of our weapons to being detected, to being hit, to being damaged when hit, and by making our weapons easier to repair when damaged.

The four survivability elements defined above must be related to various inter-actions between our weapons, the enemy threat and the combat environment.

Let us consider the first element that can affect the survivability of our weapons - the detectability (figure 4). Thus, the vulnerability of our weapons to detection and location by enemy forces will depend on the signatures that our weapons present, the enemy ability to exploit them for location of our weapons, our ability to countermeasure such enemy efforts and, in turn, his ability to oppose our countermeasures. Also, these factors of our weapon and the enemy threat effectiveness will be influenced by a combat environment - weather, terrain, and combat situation.

Likewise, the probability of being hit by enemy fire (figure 5) will depend, to a great measure, on the performance of enemy weapons with respect to their range, hit accuracy and response time and the type of tactics employed. But the chances of being hit will also depend upon our capability to countermeasure this enemy effort through the effectiveness of our weapons that could help us to either avoid the enemy attacks, or to counteract them with our suppressive fire. Thus, improvements in our weapon performance such as mobility, hit accuracy, lethability, extended range of engagements, rapid response; or improvements in weapon operation such as training, tactics and logistic support could reduce the probability of being hit and hence must be included in our consideration of factors affecting survivability. Here again, the effect of combat environment must be taken into account.

The damage sustained by our weapons and personnel (figure 6) will depend, in the first place, on the lethality of enemy munitions or the severity

of the effects of ordinary or extraordinary environments. But it will depend also on the type of protection that we can provide against such threats. This element of vulnerability has already been extensively studied and hence does not require more elaboration at this time.

Finally, the element of repairability (figure 7) will depend on the easily-to-repair characteristics inherent in the design of our weapons, the level of logistic support provided for their repair and replacement, the enemy ability to disrupt this support effort, and also on the effect of combat environment.

Merely identification of factors affecting survivability will be of little value, unless their significance to the weapon survival is understood and assessed, and the resultant promising improvements are incorporated into a weapon system. For this purpose we need an effective program mechanism structured around the framework of the Survivability Improvement Process (figure 8). This process, normally, will consist of five different but interrelated stages involving managerial, analystical and engineering efforts. In the first stage, the vulnerability of the weapon system (that is, the threat posed to the weapon) is assessed and, thus, its existing survivability level is determined. During the second stage, various vulnerability reduction approaches are identified and explored. These efforts should yield various survivability improvement potentials. In the third stage these improvement potentials are subjected to trade-off analysis. Improvements with high payoffs are selected and recommended for implementation. In the next stage of the survivability process, selected improvements are implemented into the weapon system--weapon design, training, operation and/or tactics. Finally, after the implementation, the survivability is re-assessed and a need for any further improvements is determined.

During the early phases of the weapon development, these survivability functions will be less demanding and rather general in nature; but later in the development, more sophisticated analytical approach requiring mathematical modeling and field demonstration will be needed.

For our program to succeed, this Survivability Improvement Process must be incorporated into and become an integral part of the weapon system life cycle.

The Survivability Improvement Process contains two major and basic program functions--assessment of weapon vulnerability and evaluation of vulnerability reduction potentials. To perform these two functions, appropriate analytical procedures must be developed and applied.

As an example, shown here in figure 9 is an outline of the first part of a procedure to be used for the vulnerability assessment of our weapon system to enemy target acquisition threat. The analytical steps show that the assessment of enemy target acquisition potentials (with respect to range, accuracy and time) must be based on consideration of the effects of our weapon signatures, enemy sensor capabilities and battlefied environment. This assessment must then be compared with target acquisition needs dictated by a particular combat situation in order to yield vulnerability (i.e. detectability) assessment.

Vulnerability assessments provide us with direction for further action they reveal to us the areas where vulnerability reduction needs are urgent. This brings us to the second part of our analytical procedure: The Reduction of Vulnerability to Target Acquisition (figure 10). Here, various potentials for the reduction of signatures and for the degradation of the enemy target acquisition capabilities are investigated and assessed with respect to their impact on cost and possible degradation of weapon effectiveness. Vulnerability reduction measures with high payoffs are, in effect, our survivability improvements, selected for implementation.

Such analytical procedures must be developed for the assessment of other elements and other factors of the survivability. This is a subsystem and component approach to vulnerability and vulnerability reduction assessments. A more sophisticated, system approach, will be to consider all factors affecting survivability simultaneously in context of a total weapon system.

Because of its extensive range and complexity, the weapon survivability program should be established and implemented gradually, preferably in three phases.

The first, the preparatory phase, should encompass activities that are related to the structuring and the establishment of the program base. Thus, guidelines for survivability considerations during a weapon life cycle with survivability milestones must be established. The associated program survivability functions and tasks must be defined and their funding needs estimated. Further, these functions and tasks must be assigned to various elements of the command.

During this phase groundwork must be prepared for analytical efforts planned for later phases of survivability activities. Thus, various interrelated factors and parameters that govern the weapon survivability outcome must be determined, and plans and analytical procedures for investigation of their effects on survivability must be outlined.

At this time any on-going DOD projects on/or related to survivability must be identified and reviewed and their application to weapon survivability determined. Likewise, your organization's portion of DA projects should be reviewed with the purpose of determining if and how survivability needs have been considered so far. Projects lacking or with inadequate survivability improvement plans should be identified, and program requests to fill existing gaps should be initiated. During this phase, sources of data and expertise in survivability related disciplines must be identified with contacts established and their support assured.

The second phase of the program should be devoted to the exploration of various survivability potentials. Activities during this phase are characterized by a subsystem or component approach. Both qualitative and quantitative studies leading to survivability improvement proposals are conducted. If the risk of implementation of these proposed improvements is low, they could be recommended and implemented into a weapon system at this time.

The second phase must be initiated with two major prerequisite activities:

·Acquisition and evaluation of existing data related to weapon vulnerability and to reduction of this vulnerability, and

·Acquisition and evaluation of available analytical and simulation modeling techniques.

Based on data and evaluation techniques so acquired, weapon vulnerabilities are assessed at a subsystem/component level. These assessments must be related to various elements and factors affecting survivability such as signatures of our weapons, capability of enemy target acquisition equipment, or reaction time of enemy weapons.

Next, based on such vulnerability assessments, various vulnerability reduction or survivability improvement measures are identified and assessed. These efforts again must relate survivability improvement values to treat reduction measures such as signature reduction, ECM, camouflage, mobility or armor portection.

Finally, the most promising, low risk survivability improvements are implemented into the weapon system.

The third phase of the program should be concerned with optimization of weapon survivability. The activities during this phase are more complex and also more expensive. Here, numerous elements and factors affecting survivability improvement must be quantified whenever possible. Further, the burden associated with the implementation of survivability improvement such as incurred costs, delayed program schedules, and any degradation in weapon performance/effectiveness must be weighed against the benefits (both tangible and intangible) derived from the survivability improvements such as reduced rate of combat losses in equipment and personnel, reduced demands on logistics, prolonged equipment life, improved combat posture. These benefits must outweigh the burdens in order to yield realistic payoffs.

During the survivability optimization phase, we must rely heavily on analytical techniques and mathematical system modeling. The major activities during this phase should include:

·Development of system and subsystem models

·Collection of new vulnerability and vulnerability reduction data through field and laboratory tests.

·Assessment of survivability improvements through tradeoffs.

·Selection of cost-effective survivability improvements and their recommendation for implementation into a weapon system.

Survivability improvements must be implemented through command channels. This could be an engineering effort if it pertains to equipment modification, or an operation effort if it involves changes in deployment, training or tactics

It is necessary to emphasize that this three-phase program effort that I have described should not be carried alone by system analysis groups that exist at various commands and centers, but must be extended to include scientific, engineering, logistic, user, and management elements responsible for the development and operation of the weapon system. Figure 11 depicts how these various organizational elements must be involved in the survivability improvement efforts. Generally, it should be the responsibility of system or project management elements to develop and establish a program base for the weapon survivability, namely to plan, organize, direct and coordinate numerous interactions among the other concerned functional elements. The task of scientific and engineering elements is to identify, study and quantify, whenever possible, various design factors affecting survivability. In likewise manner, logistic and user elements must be concerned with operational and environmental factors. In turn, the system analysis elements must provide a meaningful interpretation of these factors in a context of a total weapon system and with a cost-effective approach as a goal. Also, it is up to system analysis elements to come up with survivability improvement proposals convincing enough to justify their implementation into a weapon system. Finally, engineering logistic and user elements must carry the burden of implementing survivability improvement proposals.

In conclusion, here are basic program recommendations:

Set weapon survivability as an objective on equal footing with performance.

Assure that survivability requirements are properly reflected in your planning documents: Incorporate survivability needs into the weapon life cycle.

Develop and establish a workable program mechanism for identification, study and implementation of survivability improvements with potential payoffs.

DEFINITION OF SURVIVABILITY

CAPABILITY OF
- MATERIEL
- PERSONNEL

TO
- AVOID
- WITHSTAND
- COUNTERACT

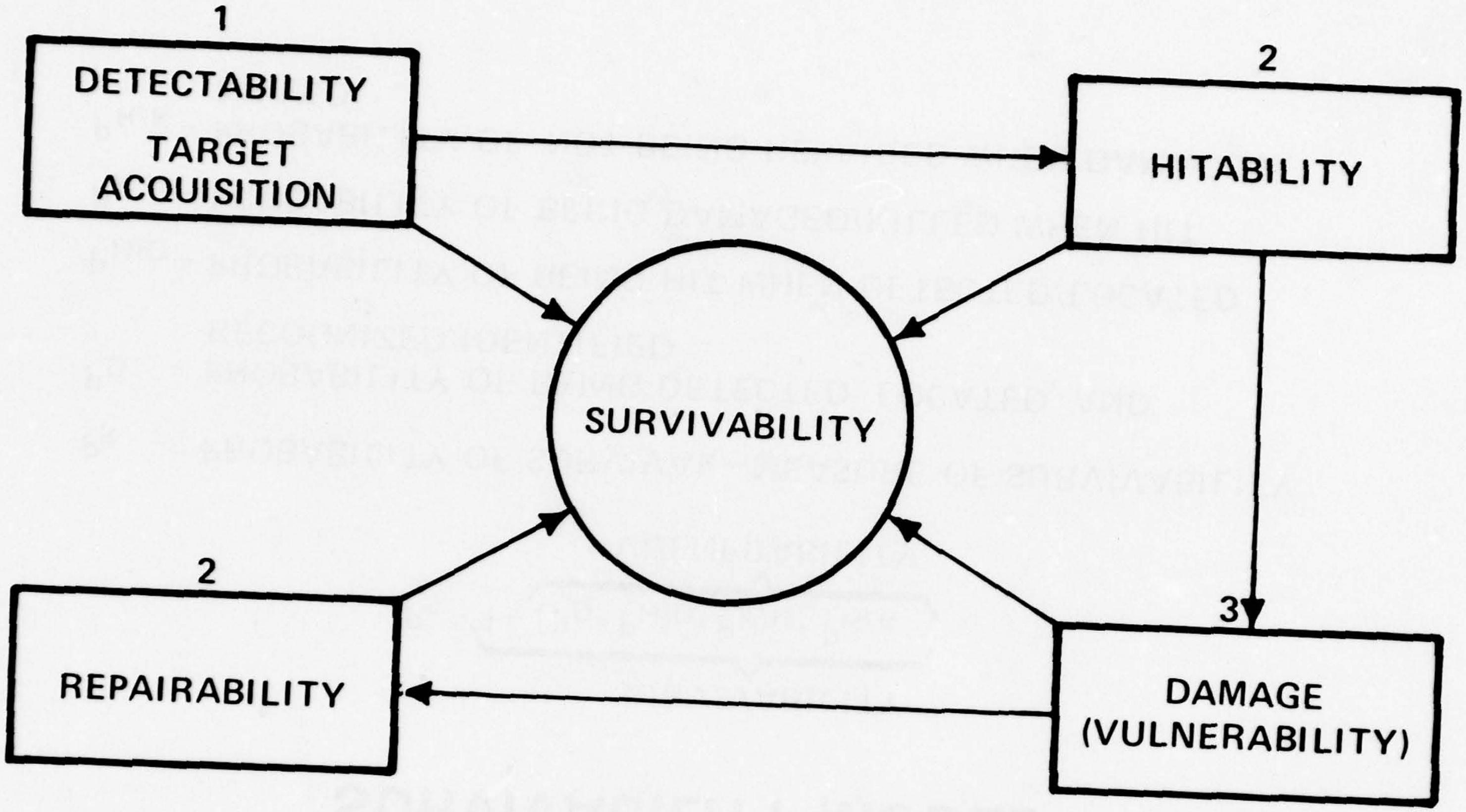
- NUCLEAR
- NONNUCLEAR

MILITARY THREAT

- EFFECTS OF ORDINARY ENVIRONMENT
- EFFECTS OF EXTRAORDINARY ENVIRONMENTS

Fig. 1

ELEMENTS OF WEAPON SYSTEM SURVIVABILITY



SURVIVABILITY MODEL

$$P_S = \overbrace{1 - \underbrace{[P_D \cdot P_{H/D} \cdot P_{K/H} \cdot P_{R/K}}_{\text{VULNERABILITY}}}^{\text{SURVIVABILITY}}$$

P_S = PROBABILITY OF SURVIVAL—MEASURE OF SURVIVABILITY

P_D = PROBABILITY OF BEING DETECTED, LOCATED, AND RECOGNIZED/IDENTIFIED

$P_{H/D}$ = PROBABILITY OF BEING HIT WHEN DETECTED/LOCATED

$P_{K/H}$ = PROBABILITY OF BEING DAMAGED/KILLED WHEN HIT

$P_{R/K}$ = PROBABILITY OF NOT BEING REPAIRED WHEN DAMAGED/KILLED

Fig. 3

DETECTABILITY

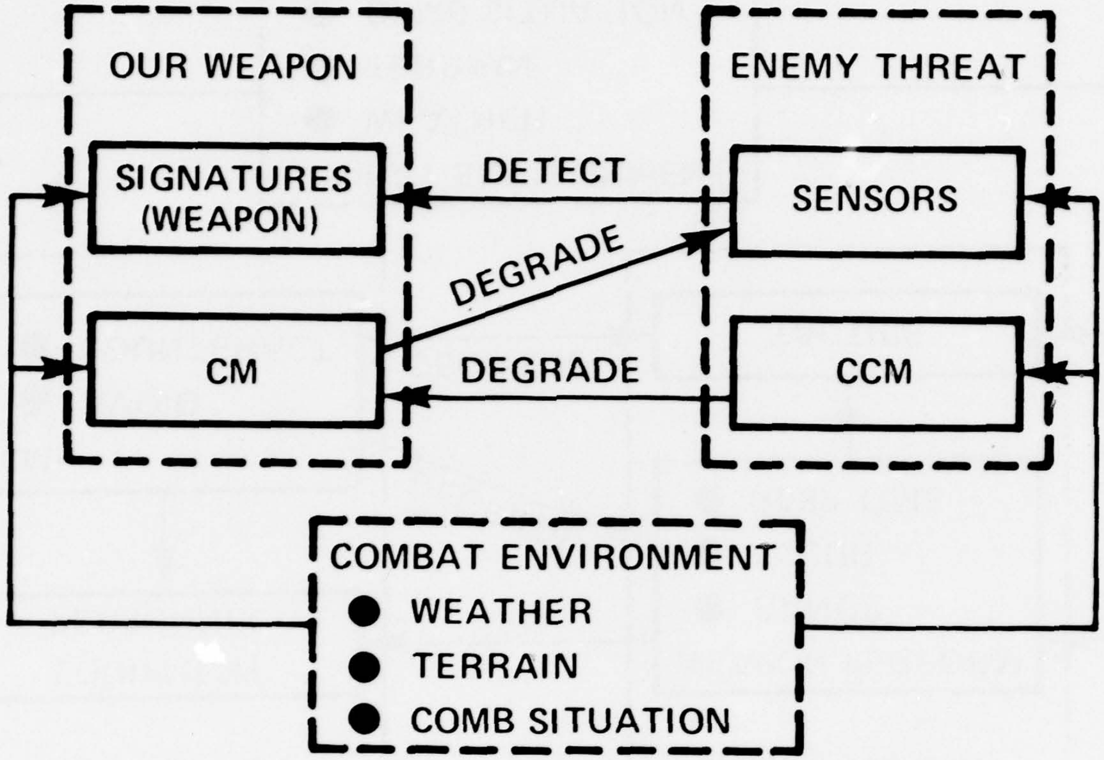


Fig. 4

HITABILITY

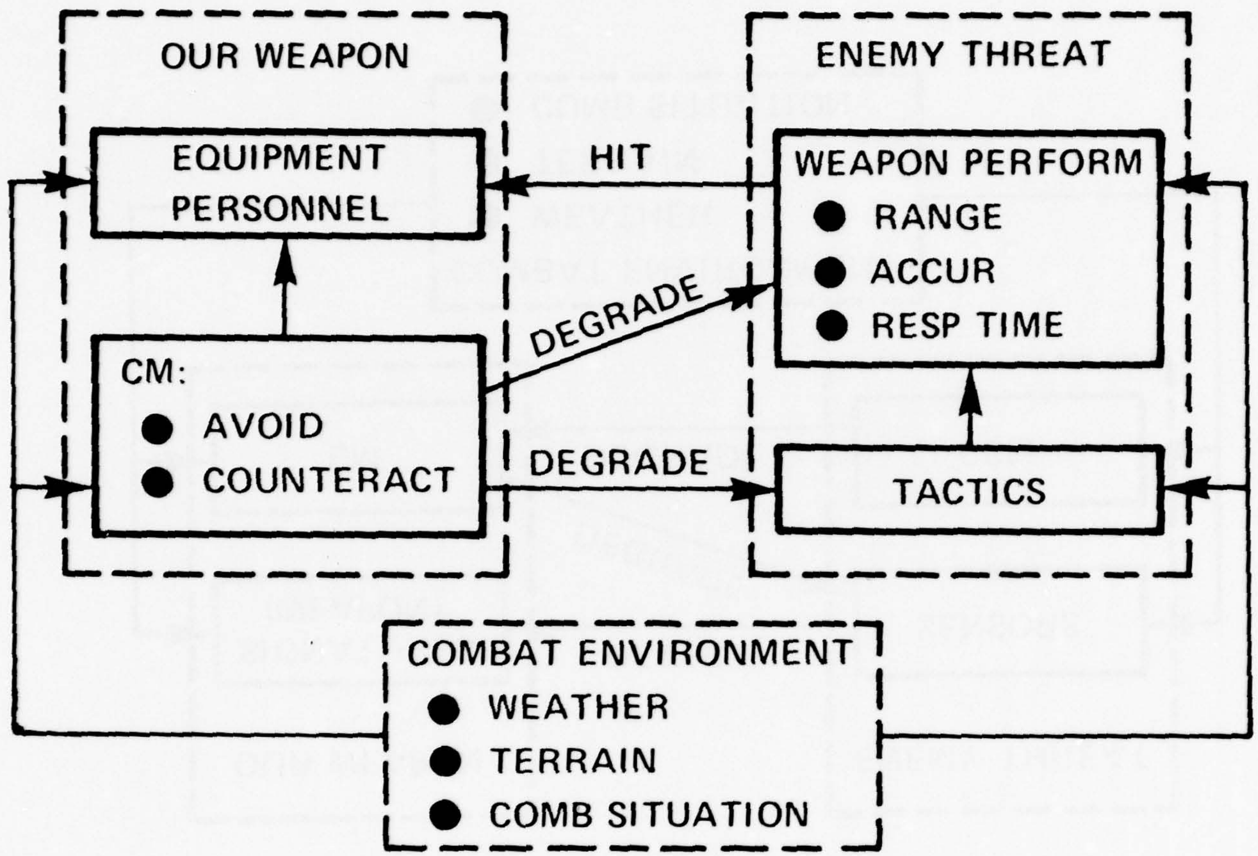


Fig. 5

DAMAGE (VULNERABILITY)

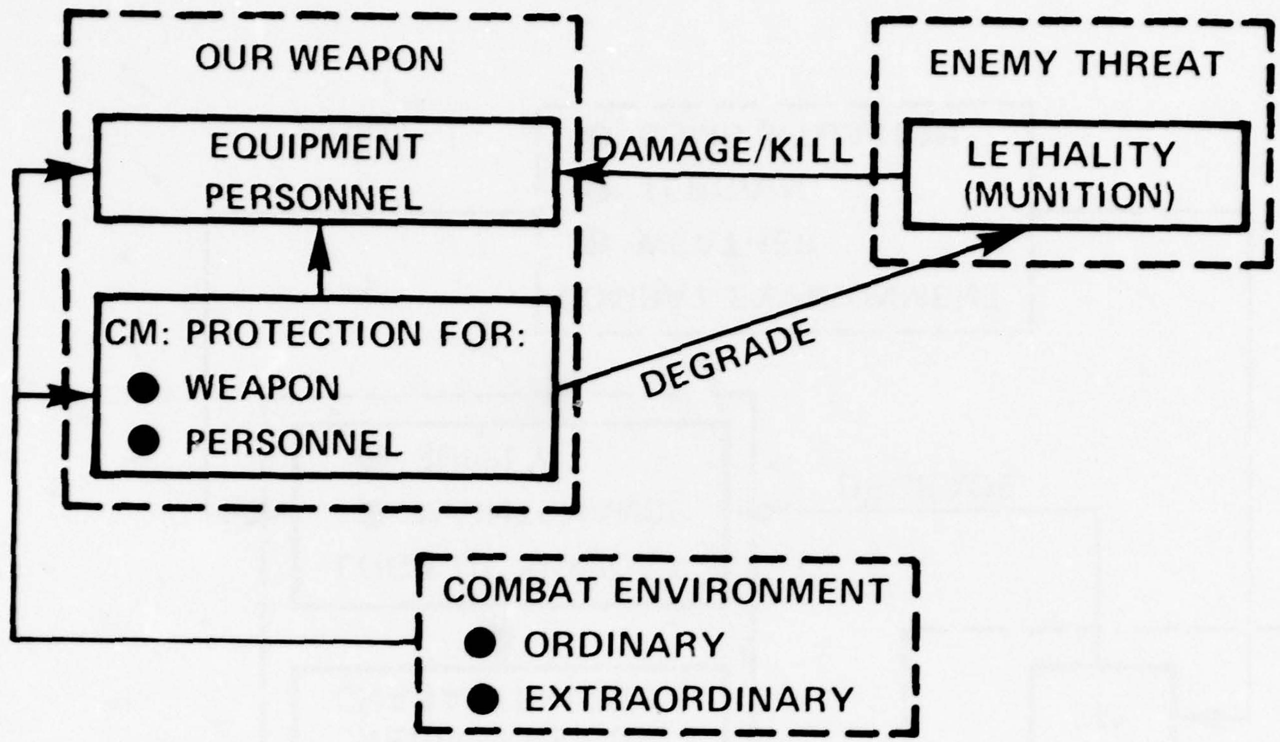


Fig. 6

REPAIRABILITY

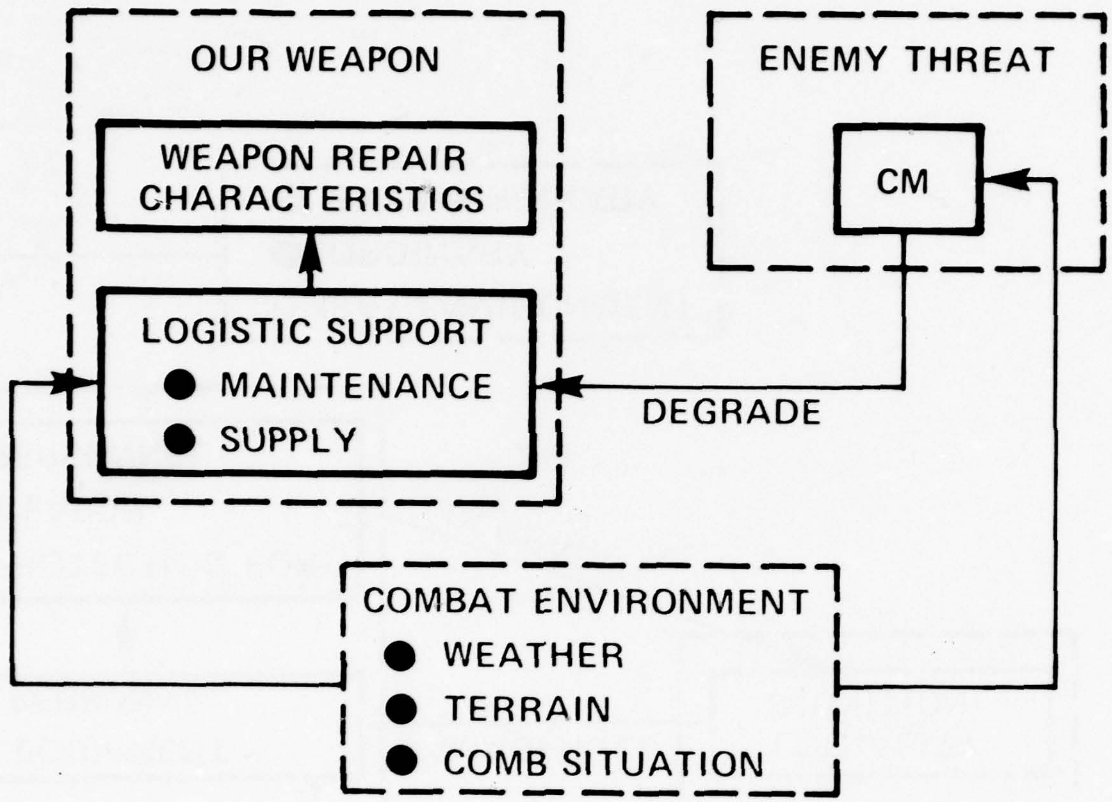


Fig. 7

SURVIVABILITY IMPROVEMENT PROCESS

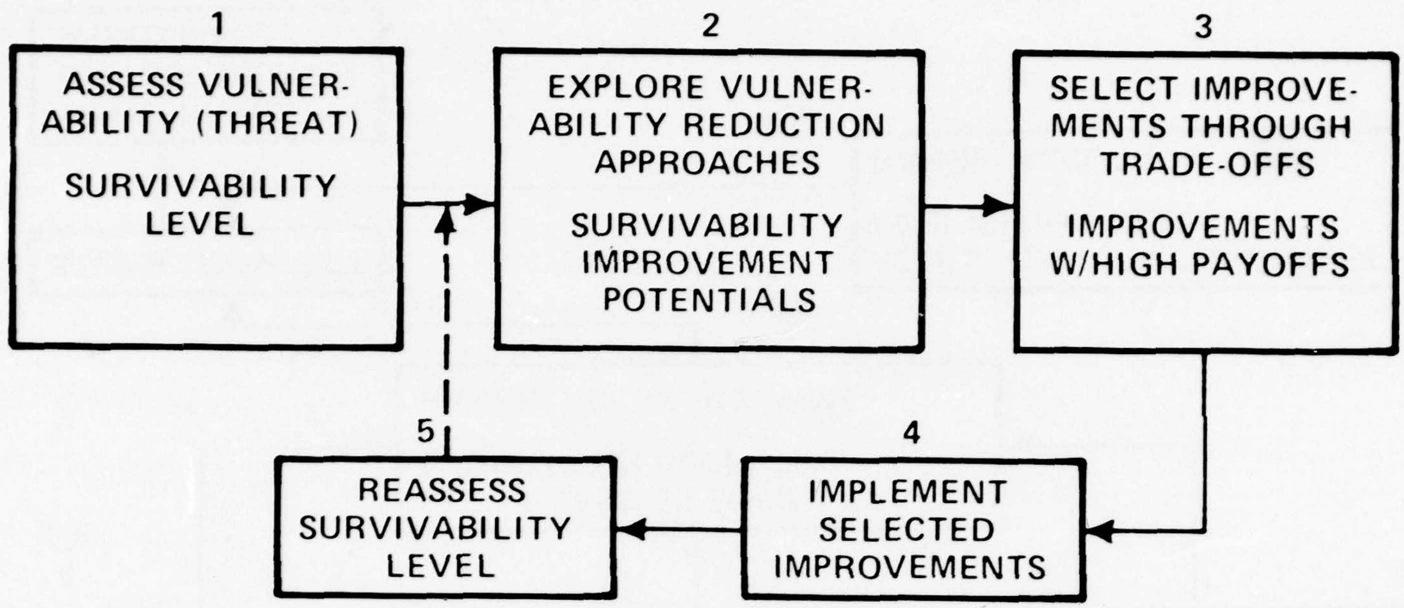


Fig. 8

PART I: VULNERABILITY TO TARGET ACQUISITION: DETECTABILITY

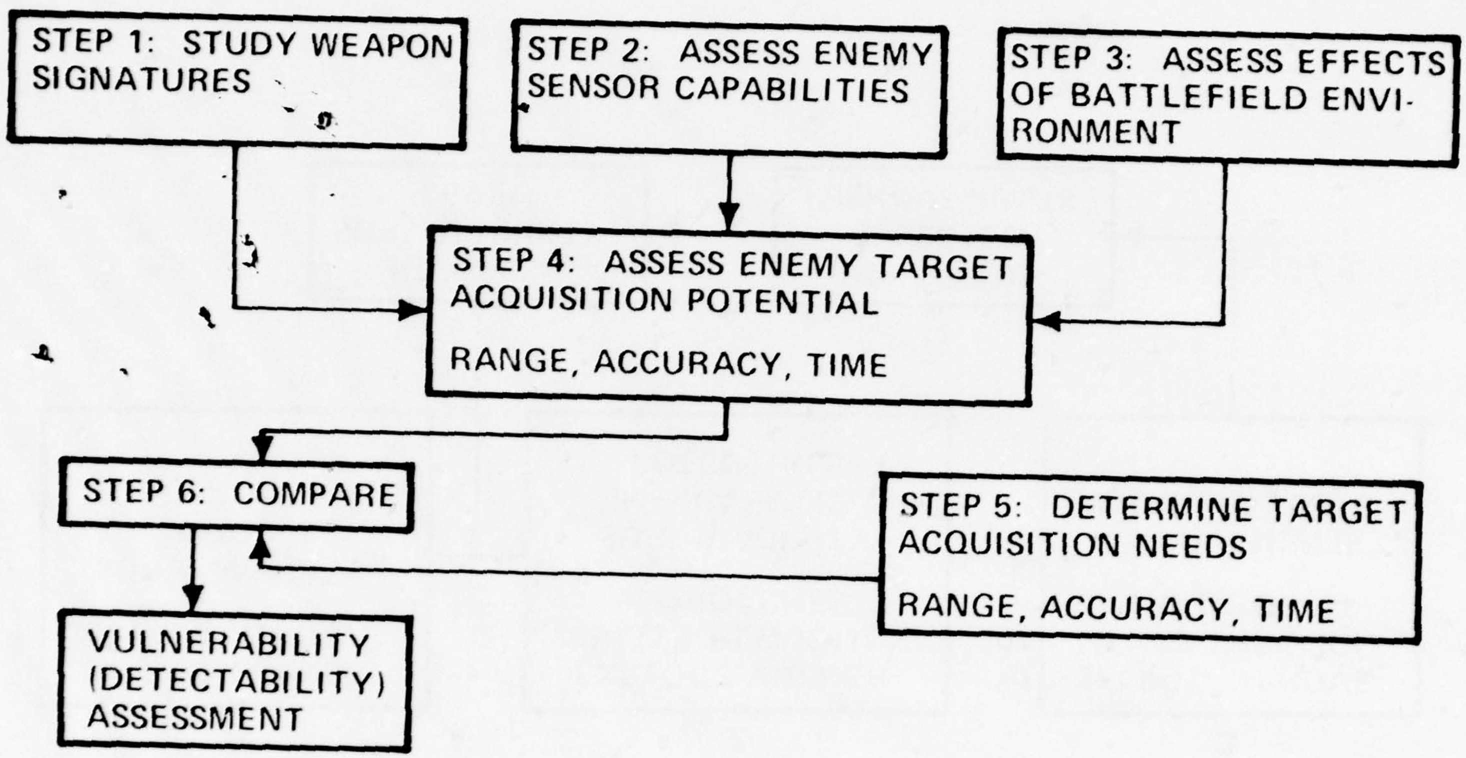


Fig. 9

PART II: REDUCTION OF VULNERABILITY TO TARGET ACQUISITION

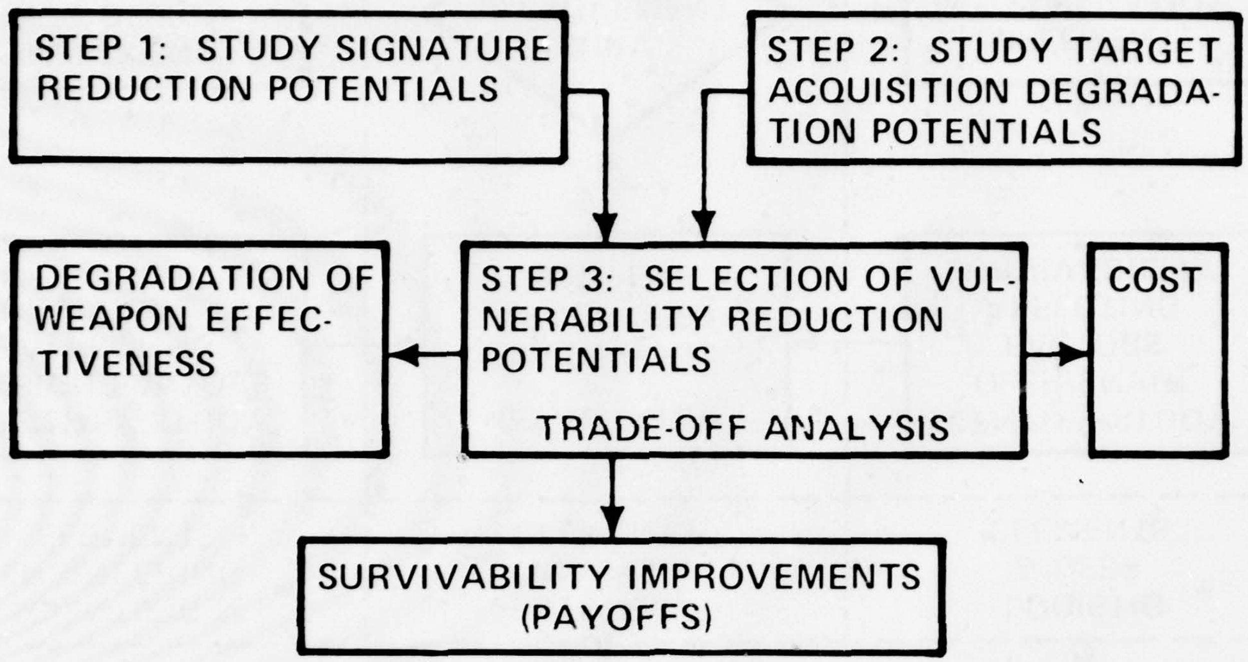


Fig. 10

SURVIVABILITY PROGRAM FUNCTIONS

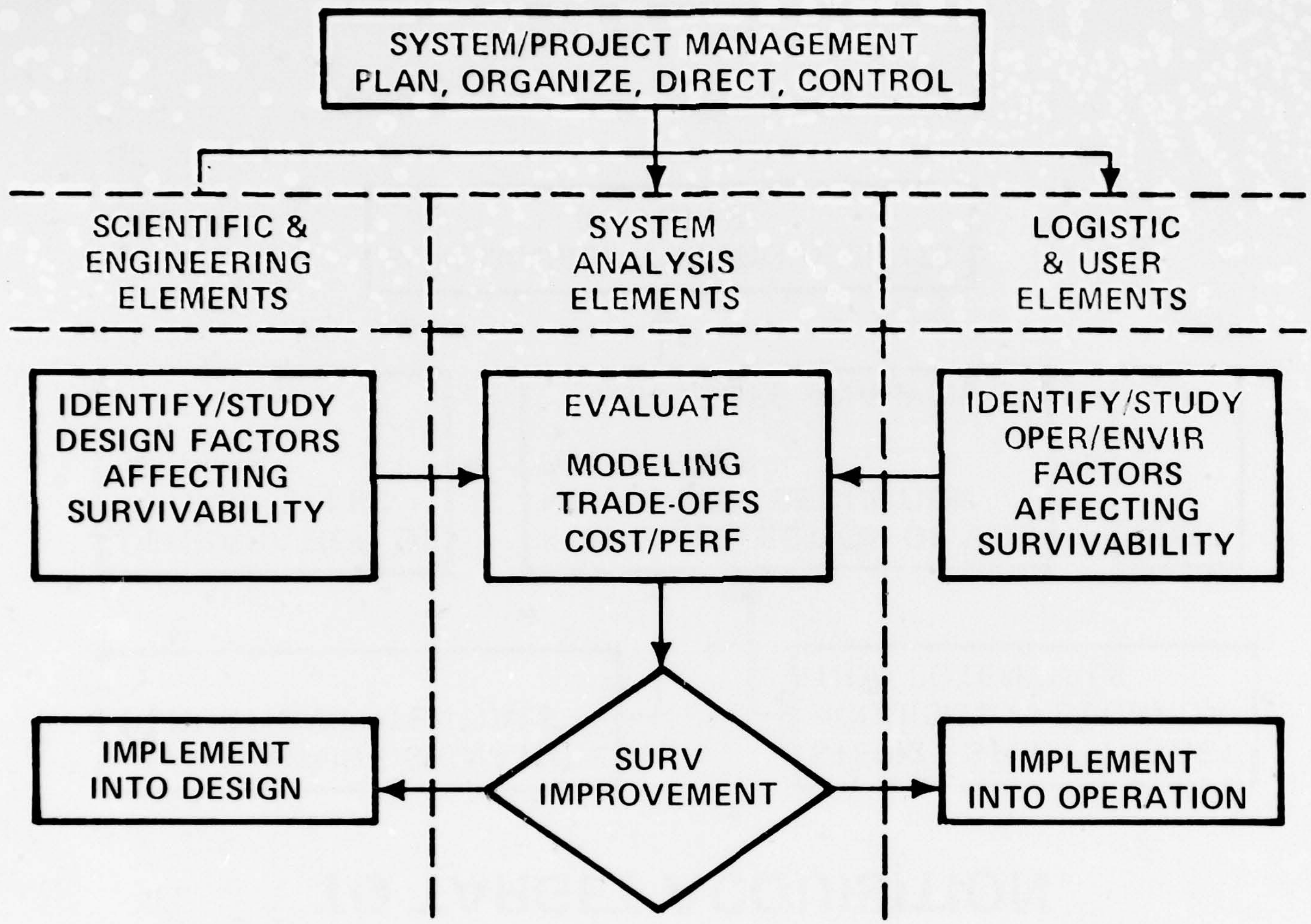


Fig. 11

TITLE: Parametric Vulnerability Model

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ABSTRACT: Available vulnerability data together with the assumption that kill probability by a kinetic energy round is an antitonic function of vehicle weight and range are used to derive a parametric vulnerability model that defines conditional mobility or firepower kill probability as a function of weight, range, aspect angle, and distribution of the impact point.

PARAMETRIC VULNERABILITY MODEL

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1. Introduction

Trade-off criterion between various combat vehicle characteristics depends on their quantification and on assignment of a value and a cost to each characteristic. Our armor and mobility trade-off study considers, among others, the probability p_s of surviving a shot. Thus, the value of this characteristic is its effect on the probability of surviving a cross-country run while under fire and it can be readily computed provided the hit probability is expressed in terms of a vehicle's mobility and other characteristics. The cost of an increment of p_s is expressed in terms of the associated increase of the gross weight of the vehicle. The relation between a cost chosen in this way and the probability of surviving a shot (p_s) is the topic of the present paper. In principle we are only seeking a set of interpolation and extrapolation formulas of vulnerability data obtained by a complex numerical transformation of firing and damage test results.

We present here the derivation of these interpolation formulas to illustrate our approach to smoothing the vulnerability data and handling a large number of independent variables.

2. Vulnerability Data

Vulnerability data supplied by the Vulnerability/Lethality Division, Ballistic Research Laboratory provides kill probability, given a hit, $p_{K/H}$. Thus, we obtain the required relation between gross weight and probability of surviving a shot ($p_s = 1 - p_{K/H}p_H$) by deriving such a relation for both $p_{K/H}$ and p_H. We describe below how this relation is derived from the vulnerability data corresponding to 115mm APDS round and full exposure of the target. The two types of kill probabilities $p_{K/H}$ available are catastrophic and mobility or firepower kill probabilities, i.e., $p_{K/H}$ may be either the probability of total loss of a combat vehicle or only of its mobility or firepower. We discuss here the interpolation and extrapolation procedures for the latter. The analogous method has also been applied to catastrophic kill probability.

The conditional kill probability depends on range, aspect angle (the firing direction relative to the longitudinal axis of the target vehicle), and the probability distribution of the impact point. Available vulnerability data contain kill probability for uniform distribution of the impact point and also for bivariate normal distributions with certain distribution parameters. The data are available for six armored vehicles, namely, M60A1 (54.7 tons), T10 (50 tons), T62 (39.9 tons), T54 (36 tons), M551 (16.8 tons),

and PT76 (14 tons). For each vehicle the kill probability with uniform distribution of the impact point depends only on range R and θ. Bivariate normal distribution of an impact point is specified by the means μ_h and μ_v of horizontal and vertical displacement from the aiming point and by the corresponding standard deviations σ_h and σ_v. Thus, in this case the vulnerability data of each vehicle depends on six variables: R, θ, μ_h, μ_v, σ_h, and σ_v.

Vulnerability data with normal distribution are available for $\mu_h = \mu_v = 0$ and $\sigma_h = \sigma_v = 30.5$ k cm, k = 1,2,..., 10. These data were computed for three APDS 115mm rounds, one with a steel penetrator, another with tungsten alloy penetrator, and the third an experimental round with a steel penetrator but with a slightly lower muzzle velocity than the first two. According to the authors of these data the first two penetrators produce almost identical damage and the third becomes comparable to the first two if the kill probability by this round at range R is compared with the corresponding kill probability by the other rounds at range R + 500 m. Vulnerability data for the round with a steel penetrator are available for T62 and M551. The tungsten alloy penetrator was evaluated against an M60 tank, while the data for the experimental round are available for T10, T54, and PT76.

The data contain probabilities of kill for seven values of the aspect angle ranging from 0 to 180° in equal steps of 30° for seven values of range from 500 to 3500 m in steps of 500 m. Data for the experimental round are available for ranges from 0 to 3000 m except for PT76 data which are only at 500 and 3000 m. We call these data the first set.

The second set of vulnerability data contains kill probability for six different normal distributions of the impact point with distribution parameters (μ_{hi}, μ_{vj}, σ_h, σ_v), i = 1,2,3, j = 1,2,3 and $\mu_{h1} = -300$ cm, $\mu_{h2} = 250$ cm, $\mu_{h3} = 500$ cm, $\mu_{v1} = -35$ cm, $\mu_{v2} = 35$ cm, $\mu_{v3} = 70$ cm, $\sigma_h = 100$ cm, and $\sigma_v = 110$ cm. These values were selected as representative of simulated firing results during the ARSV field tests The second set contains kill probabilities corresponding to each of thenine distributions for the same seven aspect angles as the first set and for ranges of 500, 1000, 2000, and 3000 m. These data are the kill probabilities for M60A1, T10, and T62 by the standard 115mm round with a steel penetrator.

The kill probability corresponding to a fixed distribution, aspect angle, and range depends on the amount of armor as well as other variables such as number, size, location, and sensitivity of various components, including crew, ammunition, etc. The six vehicles for which vulnerability data are available differ quite considerably with respect to these characteristics. We note that, in general, armor protection is proportional to the armor weight and that total armor weight of fielded combat vehicles constitutes nearly a fixed percentage of their gross weight. Thus, other things being equal, kill probability should be an antitonic function of gross weight. This is not the case with the available vulnerability data because of many

other differences besides armor protection. We note, that for a particular pair of vehicles differing considerably in amount of armor the kill probability of a less protected vehicle is smaller at aspect angles of 60° and 180° while at other angles of attack the lighter vehicle is more vulnerable as should be expected. The situation is even more complicated since at a 60° aspect angle the heavier vehicle is tougher at 3000 and 3500 m ranges while the lighter is tougher at shorter ranges. In the case of another pair of vehicles with a still larger difference in gross weight the kill probability of the lighter vehicle is lower than that for the heavier one at a 90° angle of attack from a range of 2500 and 3000 m while for all other ranges at this angle of attack the heavier vehicle is tougher than the lighter vehicle. At an angle of 60° the kill probability of the lighter vehicle in this pair is lower than that of the heavier one for all ranges.

We compare kill probabilities with uniform distribution of the impact point for all 15 pairs of the six vehicles. The first set yields 560 pairs of probabilities with the elements of each pair corresponding to the same aspect angle and range. Of these 136 pairs show that the conditional kill probability of a lighter vehicle is smaller than the corresponding kill probability of a heavier one. We further observe that for certain values of range and aspect angle the correlation between kill probability and gross weight is positive, i.e., in general, the vulnerability increases as we pass toward heavier vehicles. This happens at a range of 1500 m and an aspect angle of 150° as well as at a range of 2000 m and an aspect angle of 60°. The kill probability computed with normal distribution of the impact point shows equally poor correlation with armor weight. For instance, three cases specified by impact point distribution parameters seven aspect angles and four ranges provide 56 sets of kill probabilities for three vehicles. If these vehicles differed only in armor protection each of the 56 probability sets ordered by the corresponding vehicle weight would consist of three monotonically decreasing values. However we observe that in two cases out of 56, the kill probability decreases with weight, in four cases it is an isotonic function of weight, while in the remaining 50 cases it is not monotone.

Consequently the vulnerability data must be supplemented by certain assumptions in order to derive suitable formulas for extrapolation and interpolation as required for comparing relative benefit of armor protection and mobility and by a study of other trade-off relations. Simple monotonicity is the basic assumption of our vulnerability model. Besides, we assume that existing vehicles because of many other variable parameters constitute a random sample from a family of vehicles that satisfy monotonicity conditions.

3. Model Assumptions

Let w and w' be the weights of vehicles A and B, respectively. We say that B is an increment of A if $w' > w$ and B is obtained from A by increasing armor protection at various locations of A and by modifying its structure, tracks, suspension, etc., to accommodate the increased armor weight. We

also say that vehicles of the family V are of compatible design if for every pair A and B in V either A is an increment of B or B is an increment of A.

We note that the increment relation is transitive and a family of compatible design can be obtained by starting with an optimal design of a combat vehicle and constructing the other vehicles by successive increases of the amount of armor.

Let p be kill probability with a fixed distribution of the impact point computed for vehicles of compatible design. Then p is an antitonic function of the gross weight w for every range R and attack angle θ. We assume that it is an analytic function of w and R:

(1) $p = p\ (w,\ r,\ \theta).$

The antitonicity implies

(2) $\frac{\partial p}{\partial w} \leqslant 0.$

Since we consider a kinetic energy round we also have

(3) $\frac{\partial p}{\partial R} \leqslant 0.$

We assume that armored combat vehicles, for which vulnerability data are available, which we call sample vehicles, are random samples from a family V' that contains the designs from a compatible family V each modified by a moderate but random change of sensitive components, their location, and armor distribution. This assumption implies that p (w, R, θ) is the mean of the family V'. Therefore it can be estimated by a least squares fit of the vulnerability data for the sample vehicles.

Since the hull of M551 is aluminum while the hulls of all other sample vehicles are made of steel we do not include this vehicle in the sample from the family V'. The vehicle PT76 is also not included since its crew consists of only three men while heavier vehicles have a crew of four. However we compare the data of M551 and PT76 with approximations obtained from our model.

We express kill probability $p_u = p\ (w,\ R,\ \theta)$ with uniform distribution of the impact point in terms of a truncated power series in w and R and use the data for a fixed θ in a stepwise multiple regression fit. This computation shows that the significant terms are linear in R and quadratic in w, i.e., we get

(4) $p_u = a_1 + a_2 R + (b_1+b_2R)\ w + (c_1+c_2R)\ w^2.$

A direct least squares fit, such as stepwise multiple regression, produces an estimate of p (w, R, θ) that does not satisfy (2) for every θ and R or (3) for every θ and w. Therefore the conditions (2) and (3) must be included as constraints in the least squares fit of the data.

Equation (4) is a parabola in the (w, p_u) plane. Thus, condition (2) can be satisfied either by a convex or concave arc of a parabola. A visual inspection of the data plots suggests a concave curve. Therefore we choose a parabola with a positive maximum at a non-positive value of w, i.e., we require that

(5) $a_1 + a_2 R > 0$,

(6) $c_1 + c_2 R < 0$,

and

(7) $-\dfrac{b_1 + b_2 R}{2(c_1 + c_2 R)} \leq 0$.

In view of (6), condition (7) is equivalent to

(8) $b_1 + b_2 R \leq 0$.

Conditions (5), (6), and (8) must be satisfied for every R of interest. We choose the values of R in the interval between 0 and 4 km. Therefore (5) is equivalent to

(9) $a_1 > 0$ and

(10) $a_1 + 4 a_2 > 0$.

Condition (6) is satisfied, provided

(11) $c_1 < 0$ and

(12) $c_1 + 4 c_2 < 0$.

Similarly, from (8) we have:

(13) $b_1 \leq 0$ and

(14) $b_1 + 4 b_2 \leq 0$.

Condition (3) implies that $a_2 + b_2 w + c_2 w^2 \leq 0$ for every w in the interval of interest, say, in $[0, w_m]$. This last condition is satisfied if

(15) $a_2 \leq 0$,

(16) $c_2 > 0$,

and

(17) $a_2 + b_2 w_m + c_2 w_m^2 \leq 0$.

We select w_m as the right endpoint of the interval in which $0 \leq p_u$, i.e., w_m is defined by $a_1 + 4 a_2 + (b_1+4 b_2) w_m + (c_1+4 c_2) w_m^2 = 0$. Since (4) is antitonic in w and R the condition $p_u \leq 1$ is equivalent to

(18) $a_1 \leq 1$.

Thus, for each θ we obtain a least squares fit of (4) with the constraints (9)-(18) with $w_m = (-\beta - \sqrt{\beta^2 - 4\alpha\gamma})/2\gamma$, where $\alpha = a_1 + 4 a_2$, $\beta = b_1 + 4 b_2$ and $\gamma = c_1 + 4 c_2$. This defines mobility or firepower kill probability (with uniform distribution of the impact point) as a function of weight w and range R with coefficients as tabular functions of θ. The vulnerability data for normal distribution also deviates from monotonicity quite considerably. Therefore we obtain a least squares fit of form (4) with constraints (9)-(18) for each θ and each distribution of the impact point. These fits yield an approximation to each value of kill probability in the second set of data. The approximation values obtained this way are smoothed values that possess the required monotonicity. These values depend on w, R, θ, and the impact point distribution parameters, μ_h, μ_v, σ_h, and σ_v, i.e., kill probability is a function of seven variables. Examination of the data shows that an interpolation formula dependent on four variables only can be constructed by considering as intermediate variables the kill probability p_u for uniform distribution of impact point which is a function of w, R, and θ, and hit probability p_H which is a function of μ_h, μ_v, σ_v,

and σ_v, the quantity $r = \dfrac{w \sqrt{\mu_h^2 + \mu_v^2}}{1 + \sqrt{\mu_h^2 + \mu_v^2}}$, and R. A test for significance of various combinations of these variables leads to the following approximation for mobility or firepower kill probability p with normal distribution of the impact point:

(19) $p_{K/H} = p_u (1 + A p_H + Br + CRr)$,

where p_u is given by (4). Thus (19) together with (4) constitute our parametric vulnerability model or an interpolation-extrapolation procedure in the seven-dimenstional space of variables: w, R, θ, μ_h, μ_v, σ_h, and σ_v.

4. Numerical Results

The procedure described in the preceding section yields extrapolation-interpolation formulas for computing smoothed values p_u of mobility or firepower kill probability with uniform distribution of the impact point. Because of the imposed monotonicity on the approximation by equation (4) which is not present in the data, the discrepancy between smoothed values and the vulnerability data is appreciable in some cases. The root mean square of this discrepancy varies between 0.02 and 0.06 depending on the aspect angle θ. The largest discrepancy slightly exceeding 0.1 is at $\theta = 0°$, R = 3.5 km and w = 50 tons. Out of 196 differences between original data and smoothed values six exceed 0.1. According to our formula mobility or firepower kill probability tends to 0.89 at range R = 0, $\theta = 0$ and uniform distribution of the impact point as w goes to zero. At $\theta = 150°$ kill probability tends to 0.58. The other limiting values of kill probability are between 0.61 and 0.77. We note that mobility or firepower kill probability of the light vehicle M551 is greatest for $\theta = 0°$ and smallest for $\theta = 150°$ which is consistent with our model, although the vulnerability data for M551 were not used in deriving the approximation for p_u. The values of p_u corresponding to w = 0 suggest that a hit on a vehicle with no armor produces damage with the greatest probability when $\theta = 0°$, i.e., armor protection is most important for this direction, even if we do not consider that a head-on attack is more likely than from other directions. Our result shows that on the average the projected area of sensitive components comprises the least percentage of the total projected area at aspect angles of 60° and 150° (59% and 58%, respectively).

The approximations for p_u obtained from (4) are compared with the corresponding vulnerability data in Figures 1-3. Here the vulnerability data are represented by various symbols with line segments joining the values of p_u computed by (4). Both the data and the approximations are rescaled and plotted as function of w for various values of θ as shown in the figures. Figure 2 presents the results for R = 1 km. The other two graphs compare the data and the model for R = 2 and 3 km., respectively. Because of the rescaling of the data no direct comparison of these graphs can be made.

We applied (4) with the accompanying constraints to obtain smoothed values of vulnerability data with normal distributions of the impact point. The RMSE of the corresponding least squares fits ranges from 0.003 to 0.162 depending on the aspect angle and the impact point distribution. Fortunately, the relatively large discrepancies correspond to extreme mean values of the impact point displacement and, consequently, small probability of hit. Therefore these large discrepancies have little effect on the probability of survival $p_s = 1 - p_H\, p_{K/H}$ which is the ultimate goal of our modeling.

The smoothed values of kill probabilities corresponding to normal distribution of the impact point together with the values of p_u determined by (4) are used to compute the coefficients of (19) by the least squares method.

FIGURE 1. SCALED VULNERABILITY DATA AS A FUNCTION OF WEIGHT vs MODEL

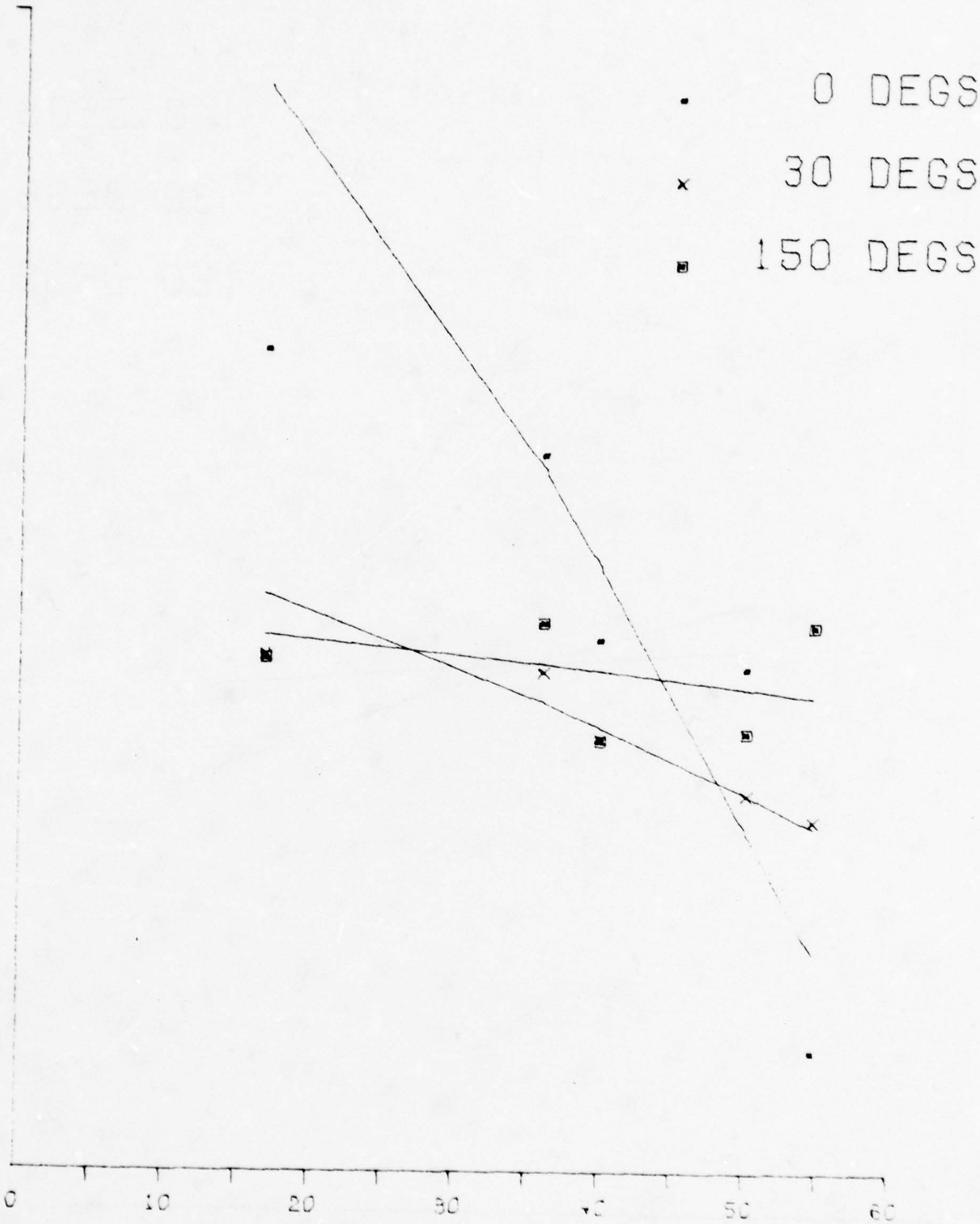

FIGURE 2. SCALED VULNERABILITY DATA AS A FUNCTION OF WEIGHT vs MODEL

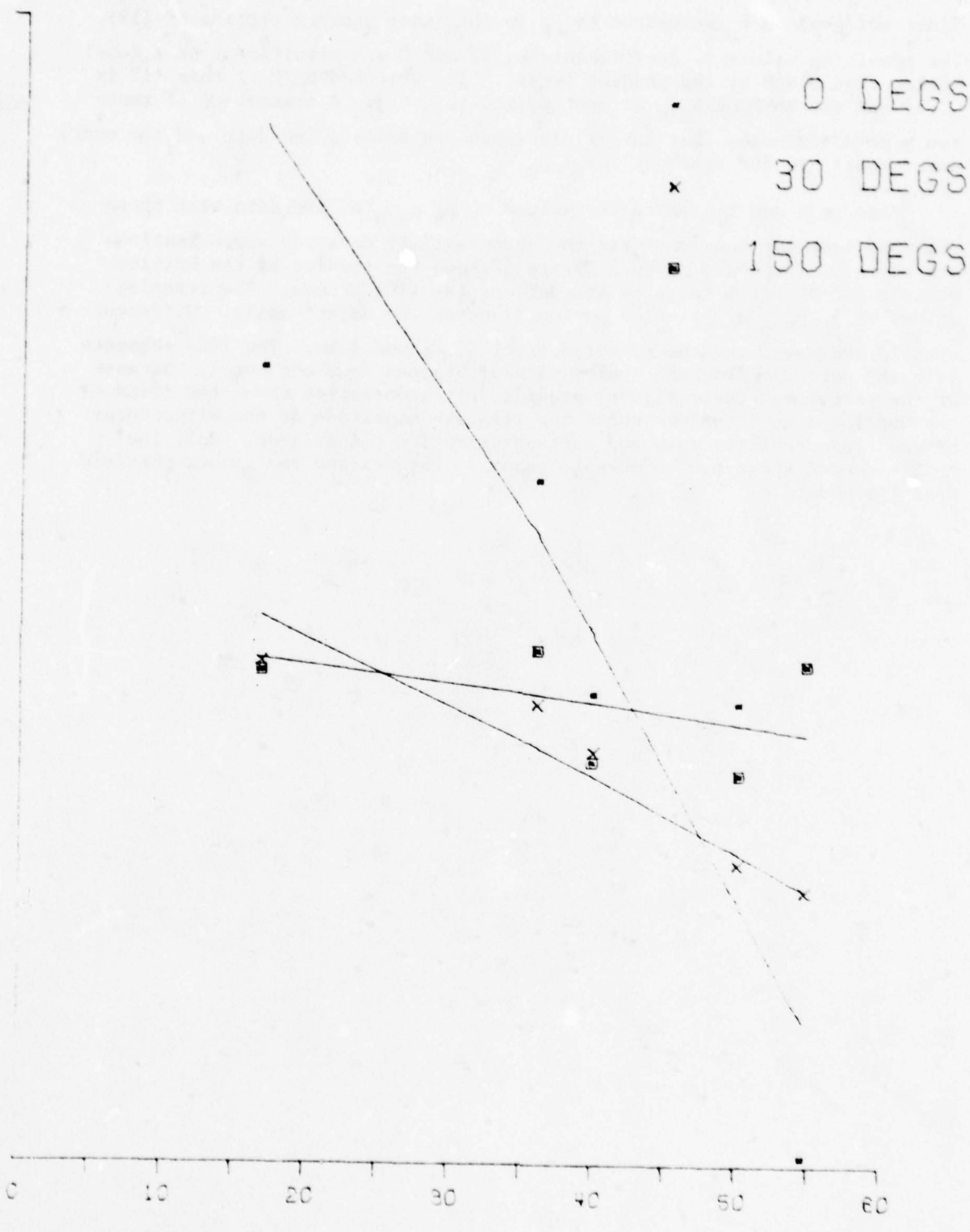

FIGURE 3. SCALED VULNERABILITY DATA AS A FUNCTION OF WEIGHT vs MODEL

Since our goal is p_S we weight by p_H in the least squares fitting of (19). The resulting values of coefficients A, B, and C are significant at a level better than 0.999 by the Student t-test. The weighted RMSE of this fit is 0.019 and the RMSE of $p_{K/H}$ without weight is 0.072. A comparison of these two errors indicates that larger discrepancies between the data and the model occur mostly in the cases of small p_H.

Figures 4 and 5 compare the values of $p_H\, p_{K/H}$ of the data with those obtained from our model. Again the vulnerability data and approximations are rescaled for these plots. Figure 5 shows the results of the heavier vehicle and Figure 6 compares the data of the lighter one. The rescaled values of $p_H\, p_{K/H}$ are plotted as functions of the aspect angle. Different symbols represent the data for ranges of 1, 2, and 3 km. The line segments join the correspondingly rescaled values obtained from our model. Because of the rescaling, these figures provide only information about the trend of the model and no inference about the relative magnitude of the differences between vulnerability data and our approximation can be made. Only the RMSE's quoted above can be used to compare the data and the values obtained from the model.

FIGURE 4. SCALED VULNERABILITY DATA AS A FUNCTION OF ASPECT ANGLE vs MODEL

FIGURE 5. SCALED VULNERABILITY DATA AS A FUNCTION OF ASPECT ANGLE vs MODEL

ABSTRACT

TITLE: A Method for Assessing the Survivability of Small Tactical Units in a Nuclear Engagement

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ABSTRACT:

The Theater Nuclear Force Survivability (TNF/S) R&D Program has been established to investigate the survivability of theater nuclear forces. The objective of this program is to improve the survivability of theater nuclear forces through changes in operations, doctrine, tactics, organization, and materiel. In support of this effort, the Tactical Nuclear Warfare Branch of the Harry Diamond Laboratories is developing a methodology that allows existing vulnerability/survivability information to be presented in a small unit vulnerability assessment format. This approach addresses a need expressed by wargamers for simpler numbers as input to their games, e.g., the integration of small unit vulnerability data into larger assessment routines. This paper describes the methods developed to date and gives, as an example, selected results of a personnel survivability assessment for the Field Artillery Battery, 155 mm, Self-Propelled, in a nuclear engagement.

We have found that cumulative logarithmic normal distributions of properly-selected environmental parameters describe the probability of survival of a tactically deployed unit. Azimuthal variations in vulnerability of the unit can be made negligible by proper selection of a set of unit deployment area reference coordinates.

Initial work has consisted of determining mathematical descriptors of nuclear vulnerability for the 155 mm Field Artillery Firing Battery deployed in tactical, staging, and road march configurations. Initial results of this effort indicate that only two constants, the mean of a logarithmic normal distribution and the standard deviation, are required to describe the vulnerability of the units in tactical and staging areas for each nuclear environment. In effect, the unit has been collapsed into a point target. The road march configuration, which is in effect a one dimensional array, requires the specification of vulnerability descriptors at 3 points along the array. Future work will consist of determining mathematical descriptors of nuclear vulnerability for additional units of the theater nuclear force. This information will facilitate the task of investigating the survivability of theater nuclear forces during conduct of tactical nuclear warfare studies.

ESTIMATED TIME REQUIRED FOR PRESFNTATION - 30 minutes

CATEGORY APPROPRIATE FOR THIS PAPER - Lathality and Vulnerability

TITLE - A Method for Assessing the Survivability of Small Tactical Units in a Nuclear Engagement

AUTHORS - Mr. Chris Spyropoulos and Mr. John Wicklund

ORGANIZATION - Harry Diamond Laboratories

1. Introduction

Wargames that are aggregated at higher levels (division and above) tend to wash out the significant factors concerning force posture that are apparent at lower levels. It is at these lower levels that improvements in tactics and doctrine become evident. Most often this failure of lower level assessments to impact significantly on the outcome of theater level analyses results from inadequacies in the model rather than from the lack of importance of the identified vulnerabilities.

In support of the Department of Army (DA) Theater Nuclear Force Survivability (TNF/S) program, we have developed a method that integrates existing data on personnel and materiel survivability into assessments of the survivability of small tactical units. This approach greatly simplifies the data input into higher level wargames and significantly reduces computer running times for large tactical assessment programs. This paper describes the method and gives, as an example, results of a personnel survivability assessment for the Field Artillery Battery 155 mm, Self-Propelled in a nuclear weapons effects (NWE) environment.

The survivability of a small tactical unit is highly dependent on parameters such as deployment configuration, unit posture (protected versus unprotected), and materiel hardness (unhardened versus hardened). Since these factors, particularly for small tactical units, can vary over a wide range , the NWE environment level required to render a unit ineffective can vary over a wide range. It was a goal of this effort to assess the unit's survival probability taking these factors into account.

The method described in this paper is probabilistic and expresses the survivability of a small tactical unit as a fraction. We have found that cumulative logarithmic normal (CLN) functions of properly selected environmental parameters describe the small unit survival probability for units in tactical deployment, in staging areas, and on road march. For the units tactically deployed and in staging areas, the log normal function describes the probability of survival so well that the unit can be considered a point target at a proper selection of reference coordinates within the unit area.

The method, although probabilistic in approach, differs from both "cookie-cutter" approaches and other probabilistic approaches which average over the area of each affected unit. Further, the method is applicable for determining personnel, electronic and communication

equipment, wheeled and tracked vehicles, and other survivabilities. Section 2 of this paper outlines the steps required for an assessment. Section 3 discusses the rationales for selecting baseline unit position areas used for the assessments. Section 4 details the methodology and identifies limitations where significant. Sections 5, 6, 7, and 8 discuss the analyses for the various configurations and units investigated in this study. Finally, section 9 gives study results and discusses significant conclusions.

2. Approach

The first step in the conduct of a small unit survivability assessment is the selection of an appropriate unit for study. The second step is to identify a standard deployment layout of the unit, including mission-critical materiel and personnel. Next, nuclear vulnerability data are obtained for the materiel and personnel so that the sensitivities to deployment specifics (e.g., location of personnel and weapons) can be determined. Finally, the equipment and personnel vulnerabilities are analyzed on a small unit basis to (1) identify the critical vulnerabilities, (2) specify vulnerability reduction measures, and (3) provide small unit inputs to tactical nuclear wargames.

Azimuthal variations in vulnerability of the tactical unit for each environment can be made negligible by proper selection of a set of reference coordinates, called the unit "vulnerability center" (VC) for that nuclear effect. Thus two constants, the mean of a CLN distribution and its standard deviation, are required to describe the vulnerability of a unit for each environment. In effect, the unit has been collapsed into a single point target. For the road march, which is a linear array, we have found that the unit may be collapsed into three point targets.

In addition to wargaming, the small-unit survivability assessment methodology can be used to perform sensitivity studies. Such studies were performed on the 155 mm battery to compare unit survivabilities of the tactical deployment versus the staging area. We were able to quantify the significances of shielding and dispersion for unit survivability on the nuclear battlefield for the 155 mm battery tactically deployed.

3. Establishment of Baseline Unit Position Area

Vulnerability assessment of area type targets forms a part of many current nuclear damage assessment methods that are employed in Department of Defense (DOD) warfare-related codes. In a survey of current codes (ref. 1), the author cites methods employed currently in characterization of extended targets as well as other specific areas such as the state of the art in target and weapon deployment, nuclear-induced environments, proper use of target vulnerability data, specific damage calculation techniques, and output capabilities. In the codes assessed, extended targets are generally represented as a fixed mathematical distribution of point targets within the area, which is assumed to have

a given geometric shape, usually elliptical or rectangular. Commonly used descriptions include a uniform distribution throughout the area, point targets at the corners and at the center of the area, and a normal distribution of targets about a point.

The actual distribution of personnel and equipment in tactical area targets is such that few (if any) of these targets can be characterized as containing personnel and equipment distribution which can be expressed by a fixed mathematical expression. Tactical targets are mainly nonuniformly distributed point targets of varying vulnerability and target value. Therefore, to maximize the accuracy of the survivability assessment of an area target, damage calculations are performed on a point-by-point basis. For any representative battlefield survivability assessment, such a procedure would be extremely tedious and time consuming. Those problems prompted our effort to represent a typical tactical coherent unit as an effective point target, the vulnerability of the unit being determined from the vulnerabilities of each person and each piece of equipment in the unit.

To perform a nuclear vulnerability assessment of a specific tactical unit, a representative configuration of equipment and personnel configurations must first be defined. Since the Nuclear Kill Subprogram of TNF/S was addressing the vulnerability of the 155 mm battery as its initial effort, a representative basic configuration for this unit was obtained.

Reference data relative to organization, equipment, deployment, and tactics of a 155 mm battery (ref. 2 and 3) are obtained from the U.S. Army Field Artillery School at Ft. Sill, Oklahoma. More specific information relative to the position area was obtained by verbal communication with informed specialists in this area at Ft. Sill and the U.S. Army Materiel Systems Analysis Agency (AMSAA). This information was used to generate Fig. 1, which shows the location of equipment and personnel in a typical tactical position area. In Fig. 1, the distance between flank howitzers is 200m. Deployment considerations may increase this distance up to 250m or decrease it to 150m. The placement of the supporting sections with respect to the howitzers is reduced or increased proportionately.

The typical position area in the battery specifies type and location of each vehicle, radio, and person. Personnel are specified as either enlisted men who perform specific operational functions or officers/section chiefs who are competent in all areas critical to performance of the battery tactical mission. The battery consists of six howitzers and associated ammunition carriers, a fire direction control center (FDC), a battery operations center (BOC), a maintenance section (MAINT), a mess section, a communications section, and a supply section. The battery ammunition section is in the battery position area approximately 20 to 25 percent of the time during a tactical deployment. Personnel in the howitzers and the Command Carrier M577 are assumed to be afforded the shielding of those vehicles. All other personnel are considered to be exposed. The position area was assumed to be in flat terrain for this analysis.

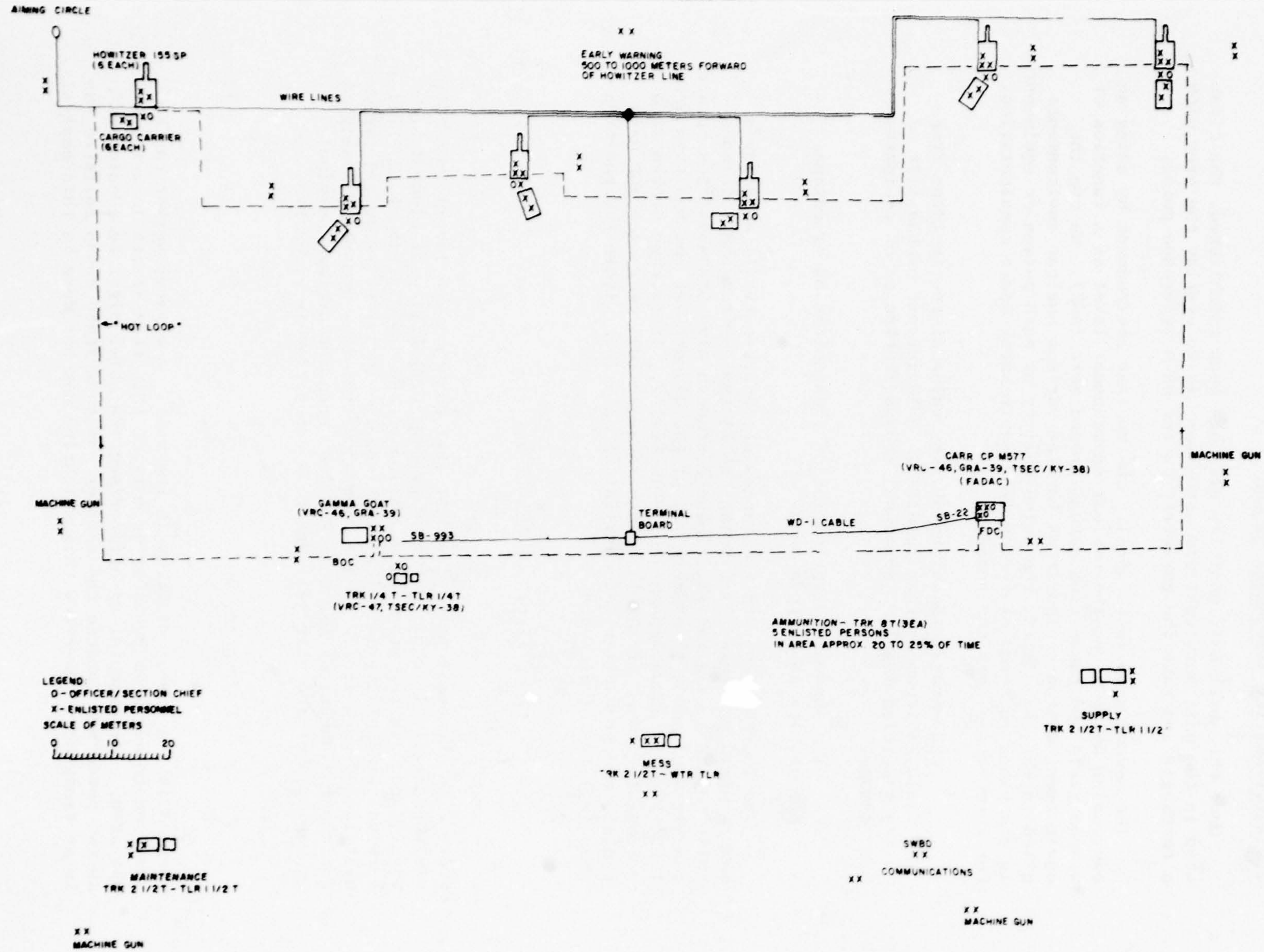

Figure 1. Fiela Artillery Battery 155MM Self Propelled.

4. Survivability Assessment Method

Once the small-unit position area has been established, the first step in the unit survivability assessment is to overlay the area with a rectangle and take the geometric center as a reference point.

The second step is to specify the nuclear environment by using an expression which gives the nuclear environment level as a function of weapon yield and range from actual ground zero (AGZ). We use the environment-distance algorithms for the various nuclear environments given in ref. 4. Third, the vulnerability of each person or equipment is described in terms of the nuclear environment under consideration. Our formulation uses two numbers:

> The natural logarithm of that value of the incident free-field environment that results in a 50 percent probability of a specified level of personnel incapacitation or of equipment damage.
>
> The standard distribution of incapacitation or damage probability about this mean.

The fourth step sets the ranges of interest by (1) assuming a weapon yield and then (2) finding the distances from the reference point to AGZ's on a radial. These distances give probabilities between roughly 0.05 and 0.95 of survival of the elements of the position area. This range is shown schematically in Fig. 2. It is then broken into a reasonable number of AGZ locations (usually 10). For each AGZ on a radial, the probability of survival of each item within the position area (P_i) is calculated by

$$P_i = \frac{1}{2}\left(1 + \mathrm{erf}\ \frac{\ln E - M}{\sigma\sqrt{2}}\right) , \qquad (1)$$

where E is the environment level at that point, M the logarithm of the environment level required for 50 percent probability of element survival, and σ is the standard deviation of the CLN distribution. Unit survival probability is calculated from the individual survival probabilities of each item. If all items have the same importance (weight) for accomplishment of the unit mission, then the average survival probability of the unit (P_s) due to a detonation is given by

$$P_s = \frac{1}{N} \Sigma P_i . \qquad (2)$$

where N is the number of items in the unit. A greater weight can be assigned to any item by assuming more of this item at its location within the area. For example, an officer/section chief might be considered as two personnel because the officer can substitute as an enlisted man in an emergency. However, this weighting was not done in this analysis.

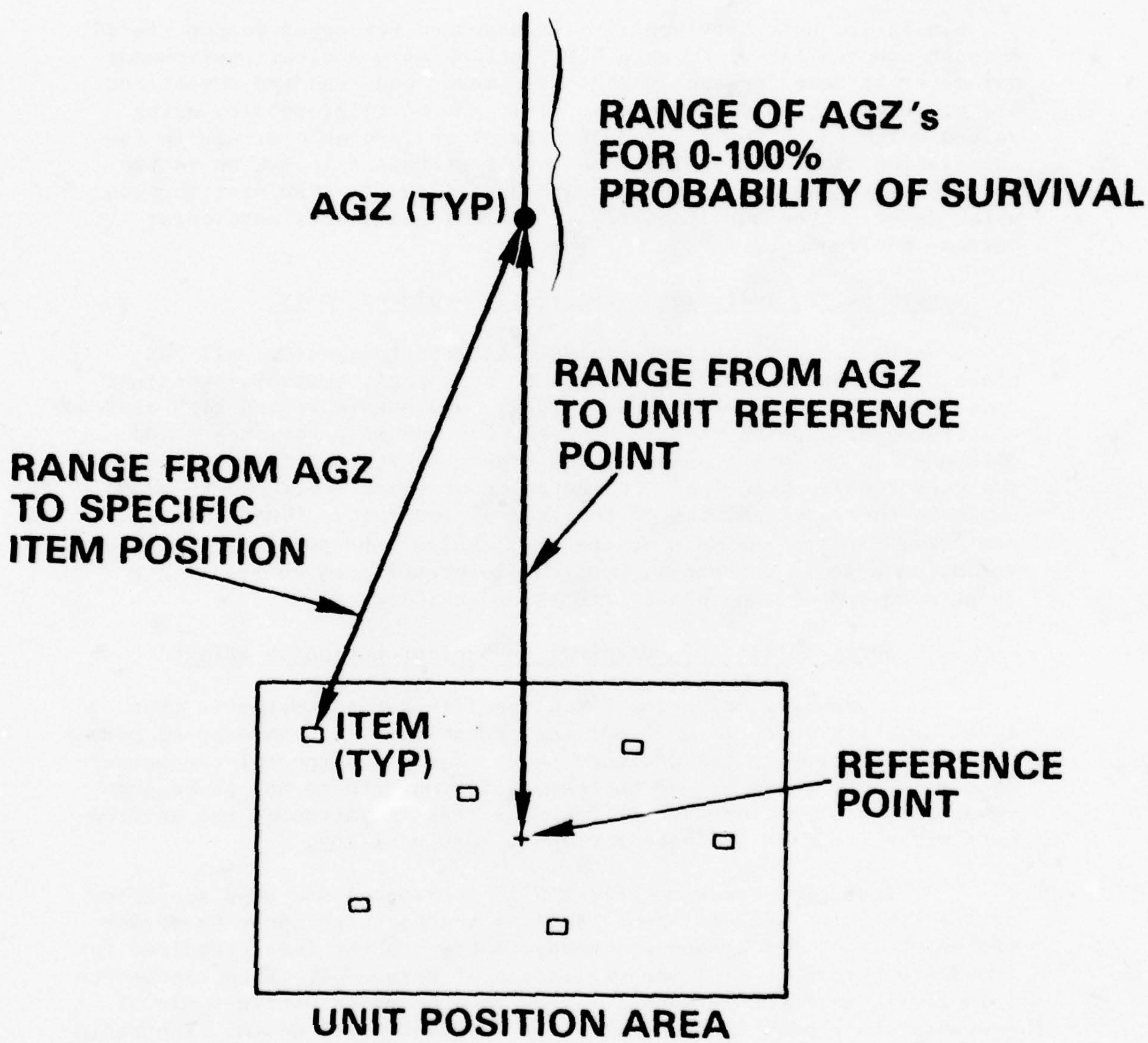

Figure 2. Survivability Assessment of a Small Tactical Unit.

Unit survival probabilities are obtained similarly for all AGZ locations along this radial and for additional radials to envelope the unit position area. The VC, assumed to be the geometric center in the first iteration, is then relocated, and the process is repeated until a point is found about which there is azimuthal symmetry of unit survival probability.

Similarly, unit survivability is assessed for other weapon yields. A least squares fit of P_s as a CLN function of a nuclear environment parameter is done for each yield. The means and standard deviations are close enough in value so that yield can be eliminated by using values weighted by the inverse squares of the probable errors in the calculations for each yield. The result of this calculation is two constants, the mean and the standard deviation of a CLN distribution, which describe the survivability of a small unit for a particular nuclear environment.

5. Survivability Analysis of Tactically Deployed Units

Generally, survivability analyses attempt to consider all NWE: blast, total nuclear radiation dose to personnel, thermal, transient radiation effects on electronics (TREE), and both low- and high-altitude electromagnetic pulse (EMP). However, for specific equipments and personnel in the position area, the order and degree of detail of the analyses require principal consideration of those environments which dominate the survivability of the item of interest. Thus, the NWE considered in the analysis of the small units considered were nuclear radiation dose to personnel, neutron fluence effects on radios and related equipment, and blast effects on vehicles.

5.1 Survivability of Personnel to Nuclear Radiation Effects

A casualty criterion for a specific NWE parameter is that level at which there is a 50 percent probability that an exposed person is unable to perform his assigned task. Battery personnel are vulnerable to blast, thermal, and nuclear radiation effects and their survivability differs for each effect. Nuclear radiation is the environment which seems to dominate personnel survivability.

Radiation casualty criteria for personnel has been specified by the U.S. Army Nuclear Agency (ref. 5 and 6). The upper bound for radiation level for personnel incapacitation is the level required for immediate permanent (IP) incapacitation of personnel. When exposed to this level, half the personnel become incapacitated within 5 min of exposure and remain incapacitated for any task until death. The radiation level causing immediate incapacitation of 50 percent of the personnel performing physically demanding (IPD) tasks is given as the next highest casualty-producing dose level.

The references also cite radiation levels required for less severe levels of incapacitation. Immediate transient (IT) incapacitation refers to the state of incapacitation in which half the personnel become incapacitated within 5 min of exposure and remain so for 30 to 45 min. Personnel recover, but are functionally impaired until death (which occurs in 4 to 6 days). Dose levels are given for undemanding (ITU) as well as demanding (ITD) tasks.

The least severe level of incapacitation is termed "latent lethality." This level of incapacitation is characterized by personnel becoming functionally impaired within 2 hr of exposure, the majority of the exposed personnel remaining functionally impaired until death in several weeks. Unit survivability was assessed for both IP and IT incapacitations. Latent lethality was not considered, since personnel experiencing this type of casualty can continue to perform their combat tasks long enough to conduct a tactical nuclear encounter.

The means and standard deviations for the two types of personnel incapacitation considered are given in Table 1. These values are obtained from a least squares CLN fit to data on radiation-induced incapacitation (ref. 6). The radiation parameter is free-in-air dose in rads.

All personnel except those in the howitzer and the command carrier are considered to be fully exposed. The transmission factor for the shielded personnel is taken as 0.8. All personnel except those in the BOC and FDC are considered to be performing physically demanding tasks.

Unit survivability was assessed for both IP and IT incapacitation. Fig. 3 plots the fraction of personnel surviving IP incapacitation as a function of dose at the VC. The figure shows the value obtained for three tactical yields and the curve which uses their weighted values.

5.2 Survivability of Radios and Related Equipment to TREE Effects

Military electronic and communication equipment is susceptible to the effects of prompt nuclear radiation. For most tactical situations, neutron fluence has been found to be the principal nuclear environmental parameter which causes permanent damage to electronic equipment (ref. 7). The logarithm of the fluence for which there is a 50 percent probability of circuit/system failure defines M in Eq. 1 for permanent damage to electronic and communication systems.

All radios and associated electronic equipment assigned to the battery were considered in the assessment of communications equipment survivability. The mean and sigma for the radios and associated equipment of the battery were determined as described in section 4 employing vulnerability data given in ref. 8.

DESCRIPTOR	50% INCAPACITATION LEVEL (RADS, FREE-IN-AIR)	LOG NORMAL DISTRIBUTION	
		MEAN	SIGMA
IMMEDIATE PERMANENT (UNDEMANDING)	17700	9.78	.581
IMMEDIATE PERMANENT (DEMANDING)	8490	9.05	.638
IMMEDIATE TRANSIENT (UNDEMANDING)	2630	7.87	.308
IMMEDIATE TRANSIENT (DEMANDING)	2260	7.72	.353

Table 1. Personnel Nuclear Radiation Vulnerability Data.

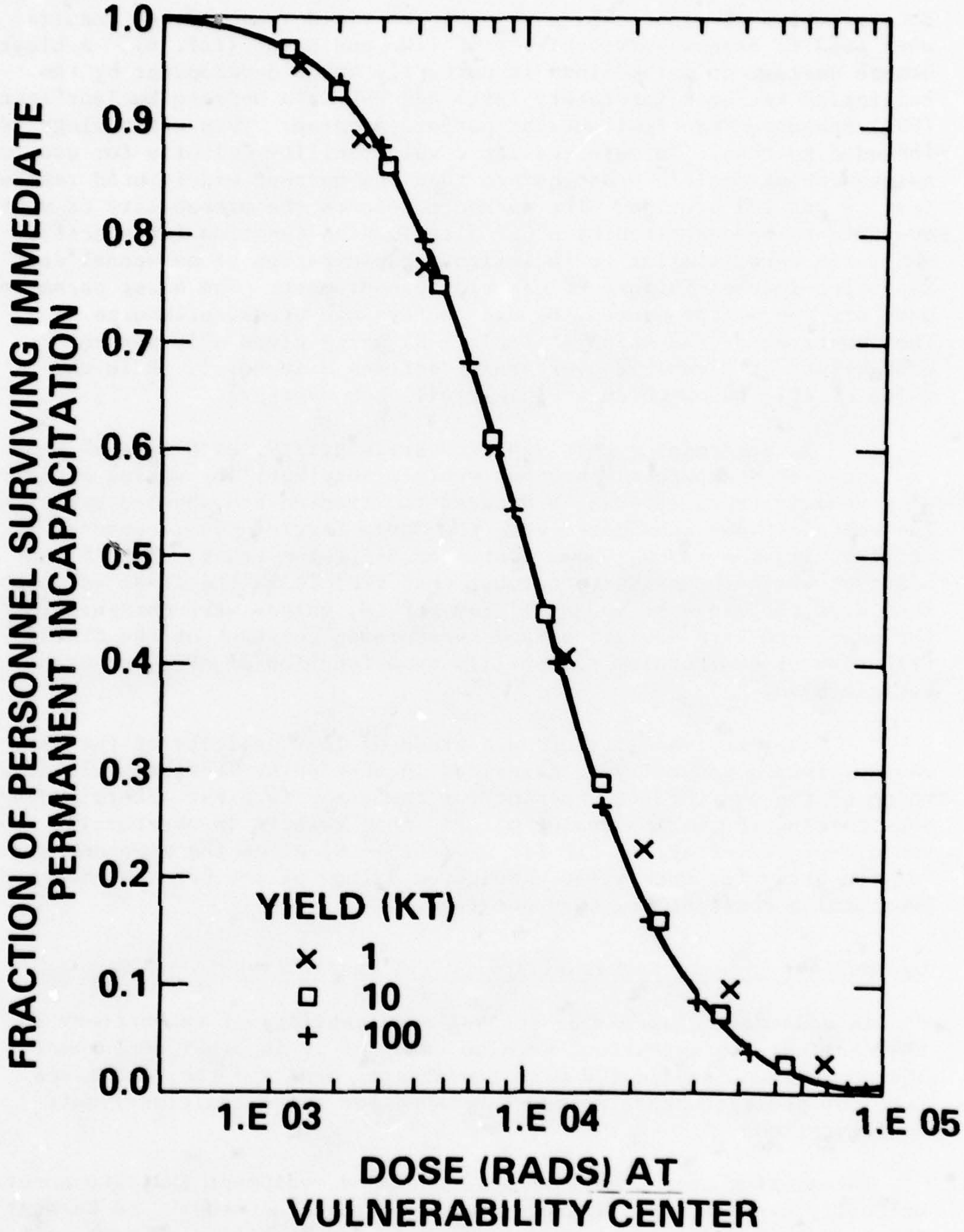

Figure 3. Personnel Survivability Profile for Field Artillery Battery, 155MM, Self Propelled, Tactically Deployed.

5.3 Survivability of Vehicles to Blast Effects

Standard references which specify vulnerability criteria for determination of blast damage to vehicles yield inconsistent results when used to assess survivability of like equipment (ref. 8). A blast damage assessment methodology is currently under development by the Ballistics Research Laboratory (BRL) and HDL in a Defense Nuclear Agency (DNA)-sponsored tactical nuclear warfare program. This methodology is intended to result in more realistic vulnerability criteria for use in assessment of vehicle blast damage than the current widely used references (ref. 9 and 10) provide. The method considers the probability of vehicle overturn to be described by a CLN distribution function for specific NWE parameters, similar to radiation incapacitation of personnel and radiation-induced failure of electronic equipment. The blast parameters used are the overpressure (ΔP) and the dynamic pressure impulse (I_q). The logarithm of the value of (ΔPI_q - K) which gives a 50 percent probability of a vehicle overturning defines M in Eq. 1. K is the value of ΔPI_q below which a vehicle will not overturn.

In determining unit vehicle survivability, each type of vehicle was considered separately because vehicle survivability varies greatly with vehicle type, especially between the tracked and wheeled types. The vehicle types considered were the Cargo Carrier M548, command carrier, 1/4 ton truck, Gamma Goat, and 2-1/2 ton truck. The 155 mm howitzer was not considered because this vehicle is the least vulnerable item over the range of yields. From ref. 8, values were obtained for the mean, standard deviation, and overturning constant of the CLN distribution of overturning probability as a function of ΔPI_q for each vehicle type.

The survivability of each group of like vehicles of the battery was determined generally as described in section 4. For each yield, a value of the overturning constant for the array is first determined by ascertaining the minimum value of ΔPI_q that results in overturning any vehicle of the array. A CLN fit of (ΔPI_q - K) gives the mean and sigma for the array for each yield. Weighted values of the mean, sigma, and overturning constant are then obtained.

6. Nuclear Effects Survivability of 155 mm Battery

In addition to assessing the NWE survivability of the battery in the tactical configuration, we also assessed it in other deployments. The deployments considered were the staging area and the road march. Baseline position areas were established for these additional unit configurations.

The staging area comprises personnel and equipment that are generally uniformly distributed. Therefore, one would expect the VC to be near the unit geometrical center, and it was. Survivability of each unit in a staging area was assessed in the same manner as for the tactical areas.

The road march layouts are unique in that they are one-dimensional arrays spread out over a considerable distance. A modification of the method of assessing survivability was devised to deal with this situation. It was discovered that three vulnerability centers are required to describe a road march.

7. Unit Vulnerability Due to Multiple Bursts

The method of section 4 is applicable for multiple bursts for personnel incapacitation due to prompt radiation and damage to electronic equipment due to neutron fluence because these effects are cumulative. Therefore, to assess the unit survivability, one totals the dose or fluence level at the VC for each effect. Then one determines the survival probability for that level from the appropriate expression for survival probability as a function of the environment at the VC.

8. Tactical Unit Sensitivity Studies

This method was used to conduct sensitivity studies on the 155 mm battery. Unit survivabilities for the different deployments were compared, as well as doctrinal approaches (shielding and dispersion) for increasing unit survivability. The shielding postures were personnel and equipment in foxholes, in a forest, and in foxholes in a forest.

The comparison of unit survivability of the battery in the tactical and staging area configurations showed that personnel are slightly more vulnerable to the effects of initial nuclear radiation in the staging area. This survivability comparison is shown in Fig. 4 for the two configurations for the fraction of personnel surviving IP incapacitation versus the range of AGZ to VC for the 155 mm battery. These configurations differ in survivability because fewer people are shielded in the staging area. Comparison of survivability of radios and vehicles for tactical versus staging area deployments showed no significant differences

The baseline tactical position area for the 155 mm battery was increased to assess the effect of unit dispersion on vulnerability. Fig. 5 shows this effect for personnel survivability to IP incapacitation. For all values of probability of survival greater than about 50 percent, the unit survivability actually decreases as the unit area increases. This apparently paradoxical result arises from the wide range of lethality of a nuclear device. Dispersing a unit increases the probability that someone will be injured, even though it decreases the probability that the entire unit will be destroyed. For values of probability of survival of less than approximately 50 percent, the unit survivability increases as the unit area increases.

The effect of shielding on unit survivability was investigated. The postures considered were personnel and equipment in foxholes, in a forest, and in foxholes in a forest. For the shielding cases

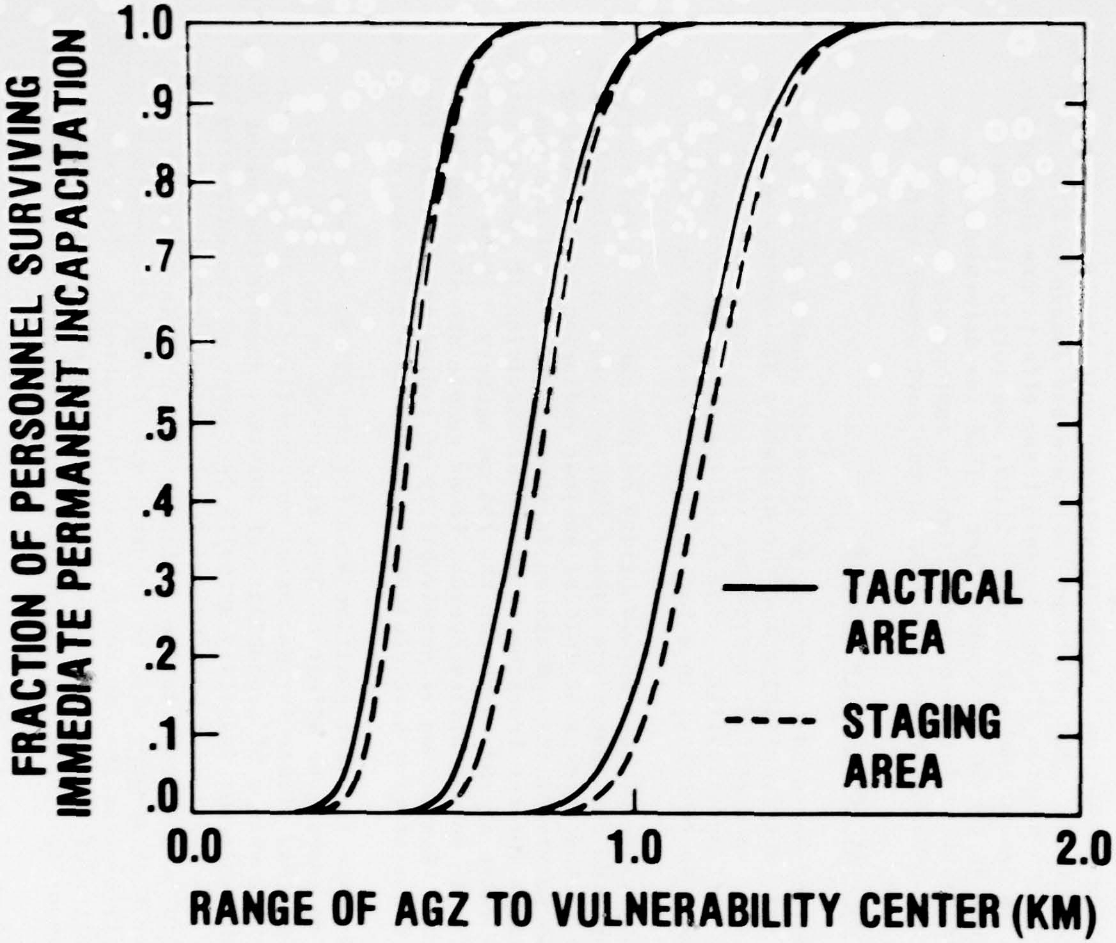

Figure 4. Comparison of Personnel IP Survivability for Field Artillery Battery, 155MM, Self Propelled in Tactical and Staging Areas.

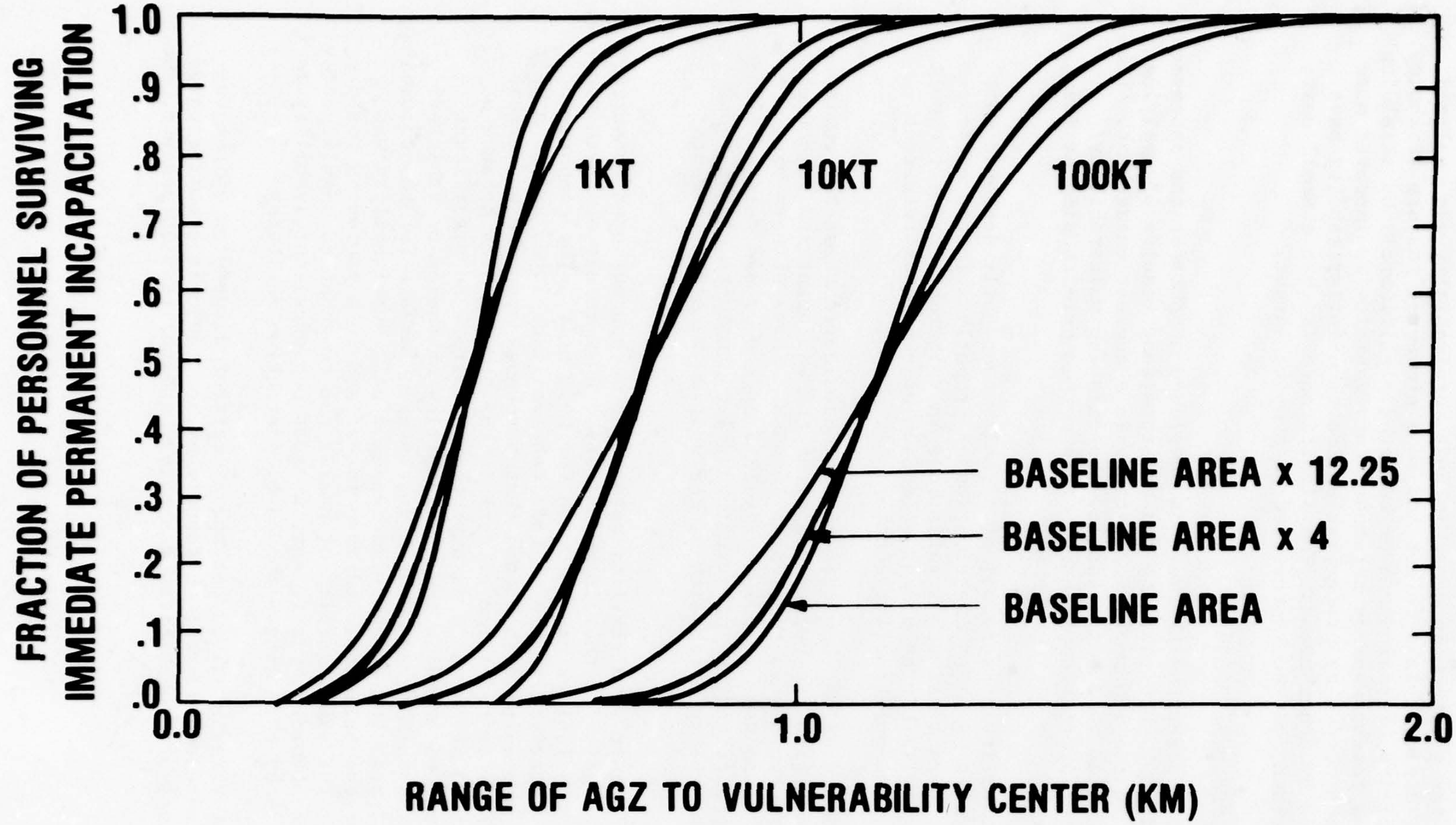

Figure 5. Effect on Personnel IP Survivability of Personnel Dispersal for Field Artillery Battery, 155MM, Self Propelled, Tactically Deployed.

considered, the greatest increase in unit survivability was realized for the unit deployed in a forest and in a protected posture (foxholes). The survivability of radios can be enhanced significantly by shielding afforded by a forest whereas personnel survivability is improved more by shielding afforded by protective shelters. Similar results have been obtained by other means. The significance is that a small unit, however shielded, can be represented as a point target.

9. Summary and Conclusions

This paper describes the methods developed at HDL for the assessment of small unit NWE survivability and presents some results of applying this method to the 155 mm firing battery in a nuclear engagement. The unit is described as a point target, and existing vulnerability/survivability information is used. This method permits simpler input to wargames.

The probability of survival of a small, tactically deployed unit can be described by a CLN distribution of properly selected environmental parameters. Azimuthal variations in vulnerability of the unit can be made negligible by proper selection of a pair of reference coordinates, the vulnerability center.

The analysis of the howitzer battery considered those NWE environments that dominate the vulnerability of the equipment and personnel positioned in typical tactical area layouts. These effects were nuclear radiation dose to personnel, TREE on radios and related electronic equipment, and vehicle overturn. Two constants can be found that describe the vulnerability of the unit to each environmental parameter.

The small-unit survivability assessment method was used to perform sensitivity studies on the 155 mm battery. As a result of these studies, significant conclusions can be drawn for this unit. The comparison between unit survivability of a unit tactically deployed and of a unit deployed in a staging area showed that personnel are more vulnerable to initial nuclear radiation effects in the staging area because a higher fraction of the unit is unprotected there. No significant difference was observed in the survivability of radios or vehicles for a deployed unit versus a unit in a staging area. For the shielding cases considered, the greatest increase in unit survivability was observed for the unit deployed in a forest and in a protected posture (foxholes). The survivability of radios can be enhanced significantly by shielding afforded by a forest, whereas personnel survivability is enhanced more by shielding afforded by protective shelters.

Somewhat surprisingly, when the effects of dispersion of the tactical area were considered, we found that, as the unit area increased for all values of probability of survival greater than 50 percent, the

unit survivability for personnel decreased. However, as the unit area increased for all values of probability of survival less than 50 percent, the unit survivability for personnel increased.

Vulnerability data similar to those obtained are being generated for other tactical units in an assortment of deployment configurations. These data are in a form that can be readily added to a computer file and used with larger nuclear survivability assessment or wargaming programs. This process will facilitate the task of investigating the survivability of theater nuclear forces during conduct of tactical nuclear warfare studies.

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TITLE: Battalion Command and Control

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ABSTRACT

There is a lack of analytical information concerning the battalion echelon command and control process; no methodology is known to exist for the quantitative analysis of the process. To develop such a methodology, the battalion command and control process must first be analyzed and defined. This paper defines the current maneuver battalion command and control system and the information flow within this system. Several levels of hierarchical process flow of battalion operation are presented in flowchart format. These diagrams encompass the combat functions of the battalion together with the command and control functions. Prior to computer simulation, future efforts will be required to further detail the logic of battalion operations and to determine the parameters considered by the decision maker under combat conditions. Application of these results to combat simulations would allow quantitative analysis of the battalion command and control system(s) and permit trade-offs between competing materiel support systems.

I. INTRODUCTION

Published research concerning command and control systems either has been oriented toward organizations at division echelon and above or has tended to concentrate on individual hardware and software systems such as the Tactical Operations System. None is known to have analyzed the total system, including the individual decision maker, as it is employed in combat by a maneuver battalion. Principal analytical measures used previously were individual system performance and efficiency, (e.g., message failures, throughput, and targets acquired) rather than measurement of the change in combat effectiveness of the unit due to changes in the total command and control system. The result is a lack of systematic analysis of command and control below division. This has limited analyses of potential command and control system trade-offs and of trade-offs between dissimilar systems to qualitative methods. A simulation of maneuver battalion combat containing all organic systems would provide the Army analytical community with a tool to provide a better understanding of system trade-offs. Such a simulation could also provide, or verify, modifying factors for higher echelon combat simulations. This paper represents an initial step in the analysis of command and control in combat at echelons below division.

II. METHODOLOGY

Work on this project was initiated by a search of existing documentation on command and control processes. A literature search included the Army Study Documentation and Information Retrieval System and the

Defense Documentation Center computerized data bank. The literature reviewed is listed in the bibliography. In addition, contacts within the Army command and control research and development community were established to collect current documentation relative to command and control and to exchange views and concepts on command and control in a maneuver battalion. These contacts included representatives of:

(1) TRADOC Systems Analysis Activity

(2) Combined Arms Combat Development Activity

(3) Command and General Staff College

(4) Army Research Institute

(5) Combined Arms Tactical Training Simulator

(6) Armor School

(7) Infantry School

The basic decision-making process represented in FM 101-5 was charted and combined with the definition of Command and Control found in AR 310-25 to develop a system description for the battalion level; the system description developed is found in Section III. Subsequently, the logic of the command and control processes was synthesized and is described in Section V.

Concurrent with the development of system description and logic, research was conducted in the area of decision theory. In addition, the suitability of current models that might be adaptable to analyses of the Battalion Command and Control system was investigated. This research is summarized in Section IV and Appendix 1.

III. COMMAND AND CONTROL DEFINITION

A command and control system is defined as "the facilities, equipment, communications, procedures, and personnel essential to a commander for planning, directing, and controlling operations of assigned forces pursuant to the missions."* Analysis of this definition permits identification of its components: Elements, Decision-maker, Functions and Mission. The elements (facilities, equipment, personnel, communications, and procedures) are the tools required by the decision-maker (commander) to perform the management functions (planning, directing, and controlling) to accomplish his mission. This compartmental definition is illustrated in Figure 1. FM 71-2 states, "Command is a very personal thing,"** and, as a result, the individual Battalion Command and Control system will vary between commanders within the constraints of the formal definition.

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*AR 310-25, 1 June 1972, p 127
**FM 71-2, 23 July 1976 (draft), p 3-14, 3-15

ELEMENTS	DECISION MAKER	FUNCTIONS	MISSION
FACILITIES EQUIPMENT COMMUNICATIONS PROCEDURES PERSONNEL	COMMANDER	PLANNING DIRECTING CONTROLLING	OPERATIONS

Figure 1. Command and Control System Definition

The Battalion Command and Control system can be considered to encompass other types of systems for analytical purposes. Since there is a flow of information within the command and control system, without which the system presumably would not work, at least portions of the command and control system can be likened to an information processing system. The decision making system (commander) does not process information per se but rather uses information to formulate a plan which ultimately results in the issuance of orders--the directing function of command and control. The separation of command and staff actions into these two types of systems is shown in Figure 2. It should be noted that the use of the word "information" in this context is a broad one including both intelligence and combat information as well as directives from higher headquarters. A block diagram, Figure 2, shows the information flow within the command and control system. It is the total of all the pieces of information which enters into the general function which has been termed "Collection" in this figure. (This summation of information is a variable based on the quality of the information collection system). Through the use of the personnel and equipment within the allocated facilities this mass of information is analyzed, synthesized, and otherwise processed. The commander at each decision point may then call upon the system to provide pertinent data for each decision as it arises. Of course, he is not prevented from further diagnosing or amending the data he receives. Factors such as experience contribute to such amendments prior to the decision and subsequent issuance of the order or guidance.

Within the command and control system, information takes several forms. At its arrival into the system, it is a combination of raw data and synthesized data. This information is converted, where possible, to increase the proportion of synthesized data (data which have been analyzed and combined with previous data). The combination is stored for retrieval by the commander or his support staff. As new combat data are received, they are added to the existing store of knowlege, in a synthesized form if possible. As the commander retrieves information, he can, at his option, further change or add to it. Upon reaching his decision, the factors considered in that decision are melded into a new form as well as being retained in their previous form. The new form is his order.

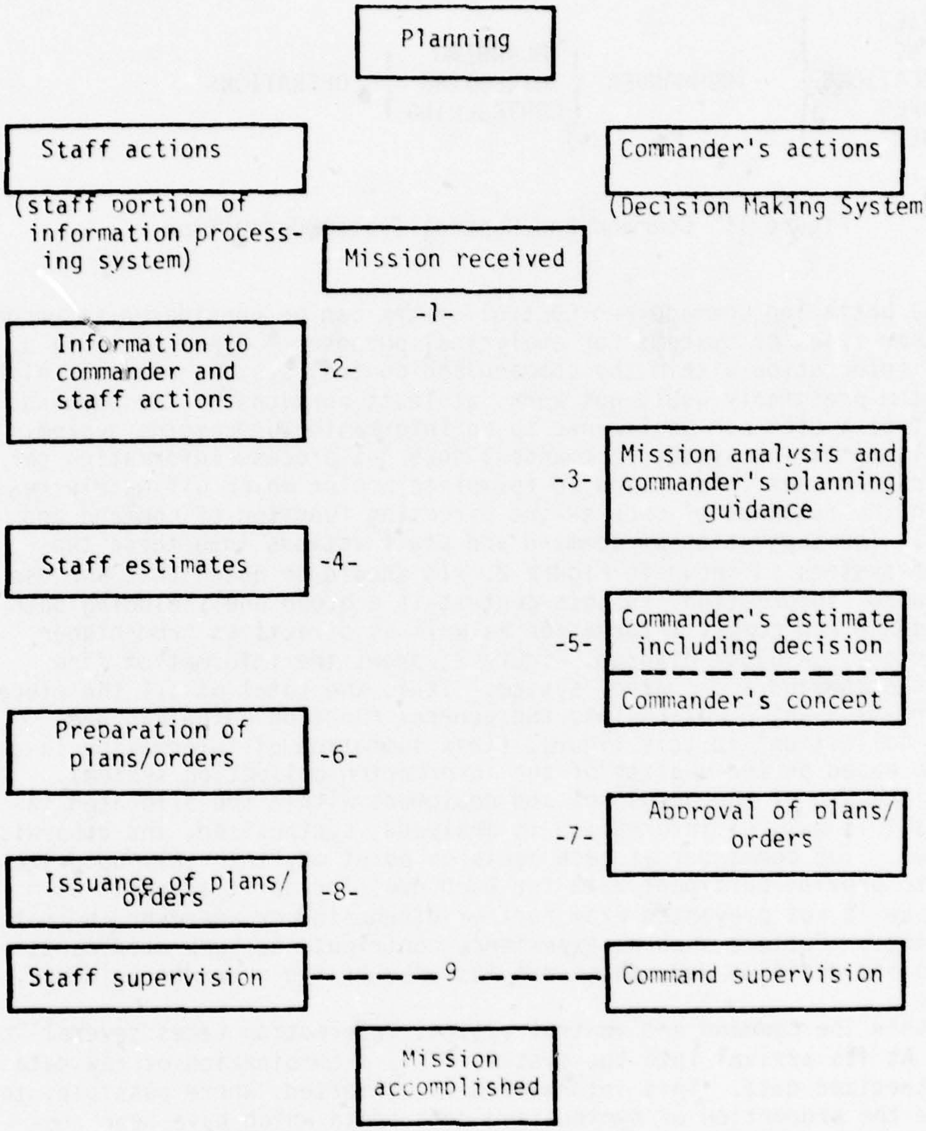

Figure 2 Command and Staff Actions[a]

[a]Field Manual 101-5, p 5-14

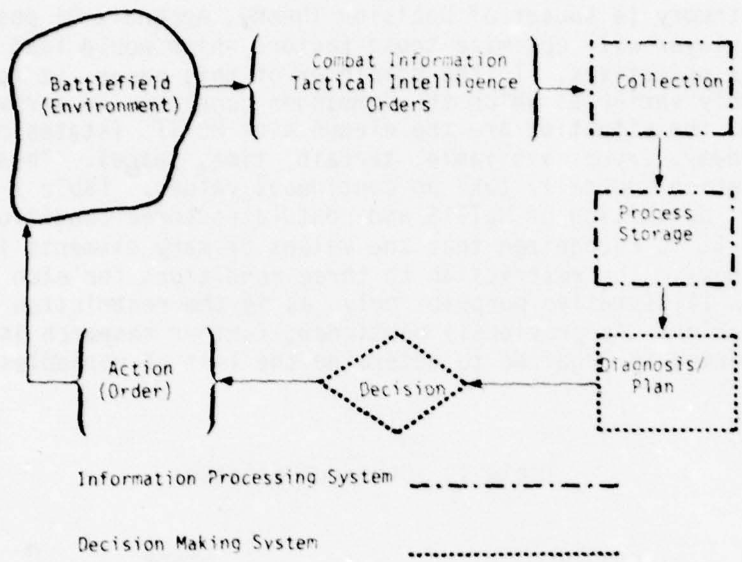

Figure 3. Command and Control Information Flow

The environment in Figure 3 is the commander's area of influence and interest. All forces must be accounted for in this environment: his superiors and subordinates, the enemy, those weapons and equipment outside his geographical area which can cause effects within it, and those forces which will shortly come within the geographical area. Thus, the commander's environment is the source of all information which he considers. In fact, one of the functions of combat information processing is that of filtering extraneous information from outside his environment.

IV. DECISION MAKING

The intent of the combat orders issued by the commander is to alter the environment in some manner, e.g., change unit locations, cause enemy attrition, reinforce units. This order, in combination with other orders, can be assessed as to its effect on the environment. This is the commander's basis for determining whether the mission has been accomplished. In modeling and studies this effect is the measure of combat/force effectiveness. In combat the effect on environment is assessed by the sensing system (e.g., radar, observations). Again, the broad definition of sensing is used, hence the inclusion of human eyes and other human senses with electronic systems. Each time information is sensed the process function operates to assimilate the new data into the existing "information file." This in turn may reset those parameters which the decision maker evaluates in arriving at his decision. This "information file" currently exists in the maps, journals, message logs, and memories of the commander and staff.

Game theory (a subset of Decision Theory, Appendix B) postulates that each player will optimize those factors which would lead to attainment of his objectives. In the remainder of this paper, it is assumed that the only variables which the commander considers in arriving at the estimate of the situation are the elements of METTTS (states of the mission, enemy, troops available, terrain, time, space). These are state variables which generally take on continuous values. Table 1 presents the current definition of METTTS and postulates three conditions for each variable. It is recognized that the values of many elements in real life are continuous. The restriction to three conditions for each state variable is for illustrative purposes only, as is the restriction to six state variables. As previously mentioned, further research in military decision making is required to determine the list of variables to be used.

Table 1. Command Decisions

	Variables	State
M	Mission	Attack, Defend, Retrograde
E	Enemy	Heavy, Moderate, Light
T	Troops Available	Heavy, Moderate, Light
T	Terrain/Weather	Good, Indifferent, Bad
T	Time Available	Long, Moderate, Short
S	Space (Maneuver)	Unrestricted, Moderately Restricted, Restricted

The assumptions suggested by METTTS are that the commander knows certain information and estimates certain information, thus leading to actual and inferred, or perceived, states respectively. One additional state can be considered: the desired state. The hypothesis is that if the commander's environment is the actual state, his imperfect knowledge degrades this actual state to the inferred state and his mission is planned to lead him toward the desired state. Any time the inferred and desired states are not the same he will make a combat decision to act. During combat the inferred and desired states will seldom, if ever, be congruent. This is due to the changing of the desired state by the commander's reassessment or by preparation for future courses of action. The inferred state can be altered also by new information concerning one or more of the decision variables. The commander will continue to make combat decisions based on the METTTS variables until the states are identical due to cessation of combat.

Consider the situation where the commander is given the mission to attack. The mission variable of METTTS is a "given" and hence actual and inferred states are equal, but could be changed at a later time by authority of higher echelon commanders or by the battalion commander's

analysis of potential future missions. An example of actual, inferred, and desired states is given in Table 2.

Table 2. Decision Variable Examples

Variable	Mission	Enemy	Troops	Terrain	Time	Space
Desired state	Attack	Light	Heavy	Good	Long	Unrestricted
Inferred state	Attack	Light	Moderate	Bad	Long	Unrestricted
Actual state	Attack	Moderate	Moderate	Bad	Moderate	Unrestricted

In this example it can be seen that the commander is facing a moderate strength enemy force with his own unit having moderate strength in bad terrain with a moderate amount of time available to react. Decisions he makes will be based upon the inferred state of the environment which is not identical to the actual state due to imprecise, imperfect, or otherwise faulty information. Therefore, given superior forces, the commander may first attempt to maneuver to better terrain. Had his information been perfect (or at least better), he would have known that the enemy force approximated his strength and was concentrated closer to his units, giving a shorter time element than he estimated. The outcome of the attempted maneuver toward better terrain is based on the reactions of this higher strength enemy force. The command and control system provided him with erroneous information, possibly causing him to execute the wrong decision which could result in a different outcome than desired.

If the command and control system which provided the information in Table 2 were different due to an alternative configuration, the two "competitive" systems could be compared based on resultant combat measures of effectiveness, i.e., the combat outcomes of two identical battalions each employing a different command and control system. The quality of the information provided to the decision maker directly affects the decision made, thus translating the performance measures (speed, efficiency, and quantity) into combat outcomes.

To further consider the previous example, the commander (in the inferred state) in general desires to maximize his forces and minimize the enemy forces in the best terrain with maximum time to react and space to maneuver. The situation may prevent such actions, however. An attack order contains many elements to be considered in the planning phase of the combat. It may not permit a change of terrain or an increase in forces. If it does not, the commander could decide to request such changes if they impact on the higher echelon's environment. If the request or orders prevent such actions, as is usually the case at battalion, the commander can never enter the desired state but will still try to optimize the remaining factors.

It should be remembered that the commander is not considering one sequence of decisions but rather a series of overlapping decisions and actions. The result of this sequence is the outcome of the combat. Such a multiplicity of information flow, both to and from the commander, is a cause of queuing in the various subsystems, e.g., fire requests to artillery and calls through the communications system in excess of its capacity.

V. COMMAND AND CONTROL LOGIC

From a logical viewpoint, the Battalion Command and Control system can be diagram as a process flowchart (Figure 4). From this overview, it is seen that the overall process begins with receipt of combat orders from higher echelons which are then processed through the planning phase resulting in directions to the battalion elements for their portions of combat. In the planning phase alternative plans are determined which must be evaluated with the resultant selection of the "best" plan for the battalion. Other plans considered may become contingencies to be acted upon when the original plan must be replaced or altered. Upon receipt of subsequent orders to "act" the EXECUTE process, which controls all processes needed during combat, is started. Movement is "begun" at any velocity: positive, negative, or zero (i.e., depending on METTTS factors, movement could be in any direction at any allowable velocity, including zero). During movement, personnel do not turn off their senses so that the human sensing process called OBSERVE is always providing additional information. Decisions are required to obtain electronic surveillance by the SENSOR process, to provide FIRE SUPPORT and DIRECT FIRE on targets, and to change plans. Upon completion of the optional processes the EXECUTE process is continued. Once the sequence of decisions and actions is completed, the EXECUTE process is repeated and each decision is reevaluated by the commander. It is assumed that within each process are communications effects, the expenditure/resupply process, and damage assessment. Of these only some communications delays will be illustrated in this paper.

Each of the processes and functions of the abbreviated logic diagram (Figure 4) has been flowcharted in greater detail. Future work must provide even greater detail if the command and control process is to be simulated by a computer model. The detailed logic is shown in Figures 5 through 10 with narrative explanations of each of these subroutines contained in the following paragraphs. It should be noted that certain details are not duplicated in each flowchart such as the decision to communicate and the question of ability to communicate. Alysis required for the decision is assumed to be included in the decision block. An additional levels of analysis should include these details.

a. PLAN

(1) The logic for the process called PLAN is contained in Figure 5. Should no orders be received, the logical system allows the battalion to assess the "prepare to" portion of the previously received order. If orders are received, the variables in the commander's "file of information" (consisting of journals, maps and staff memories) are adjusted to any new values contained in the orders. This file must contain

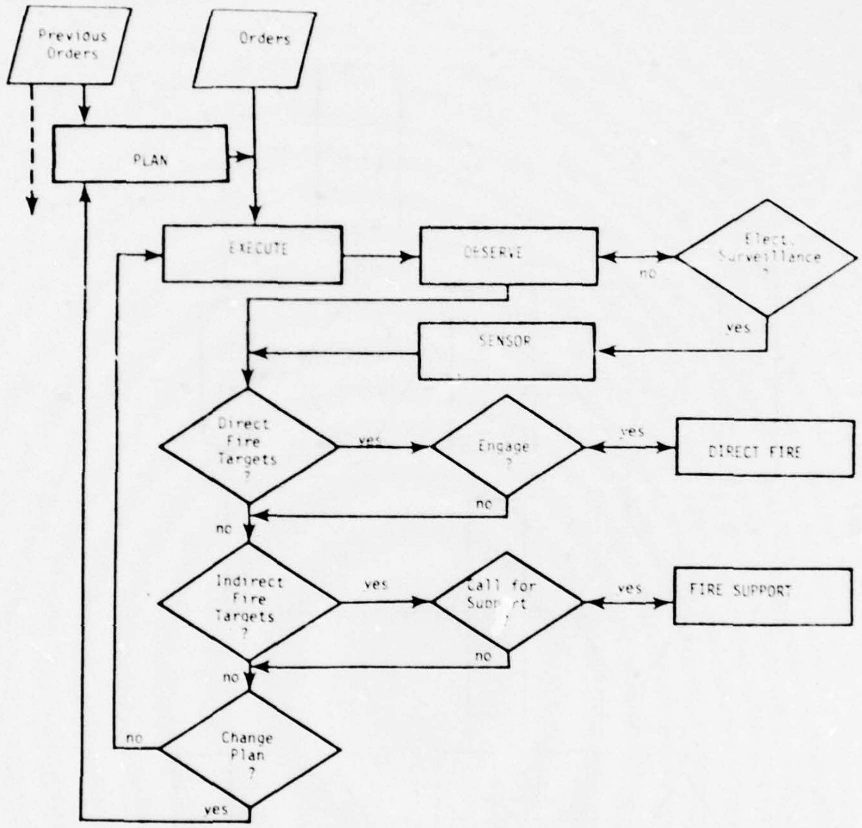

Figure 4 Process Flowchart, Top Level of Hierarchy

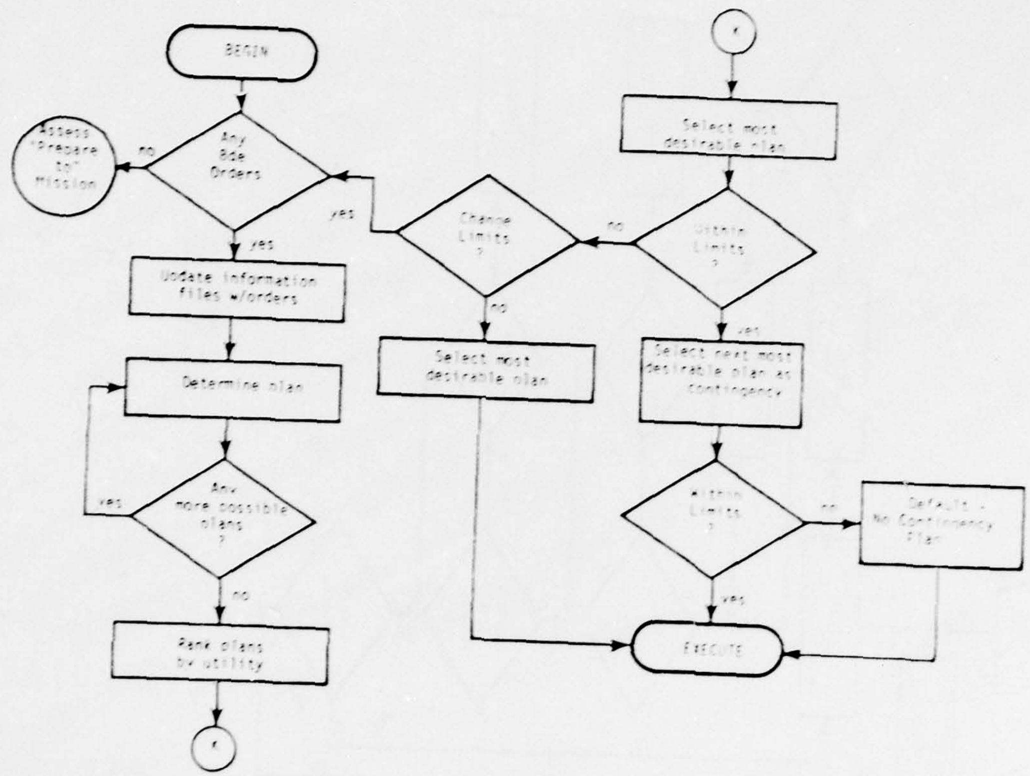

Figure 5 Flowchart of PLAN Process

all variables which the decision maker utilizes to select the proper alternative. Those variables which are determined by intelligence and other variables which are estimated are in the commander's information file as the inferred state. Any decisions made are based on the inferred values of variables while damage assessment and other combat results are based on the actual values of the variables.

(2) PLAN provides implementable orders to the elements of the Battalion. In it various alternative plans are developed, ordered by worth or utility (based on the commander's judgment) and checked against some minimal level of worth. Inputs to the evaluation process include all those variables he considers for other decisions. The plan with the highest worth becomes the plan to implement and the plan with second highest worth becomes a possible contingency plan. Should no plans be possible which are above the minimal worth, a default occurs. This could result in the implementation of one of the determined plans or of a hold order depending on the additional guidance given to the Battalion commander by Brigade.

b. EXECUTE

(1) EXECUTE is the central process for the combat portion of the logic system. EXECUTE (Figure 6) includes the movement of the units on the battlefield and decisions concerning the optional processes of combat. The speed of movement is adjusted to reflect those variables in the information file which impact on movement. Examples of these are mission, inferred enemy location and strength, friendly mode of transportation, and terrain characteristics. Both positive and negative movement (e.g., advance and retreat), as well as zero, movement is permitted. Command and control is exercised by a comparison of the locations of the subordinate units to the commander's control measures developed in the plan. Two bases for changing the plan are illustrated. The first is the change of information file values beyond some input values. The second is the loss of contact with a unit. Each of these events results in the commander doing additional planning via the PLAN process to develop a new plan based on the new data available.

(2) The non-movement portions of EXECUTE consist of several decisions on the part of the commander. If electronic surveillance is desired, the SENSOR process enters the logical system; if the use of such devices is not desired, the process continues with only OBSERVE (human) inputs available to update the information file. When enemy units which exceed a threshhold of target worth come within range of either indirect or direct fire systems, the FIRE SUPPORT or DIRECT FIRE processes are involved, depending on the information entering the commander's decisions. The communications effect on the process flow is a delay in transmission of information. Until updated information is transmitted and received, the combat proceeds without it.

c. OBSERVE. The flowchart for the OBSERVE process is shown in Figure 7. This process includes the human physical senses of sight, hearing, and smell used by personnel to provide information about the combat situation, specifically about the variables in the information file. Communication may not be permitted under certain conditions. If this is the case, only the local echelon information file is updated with

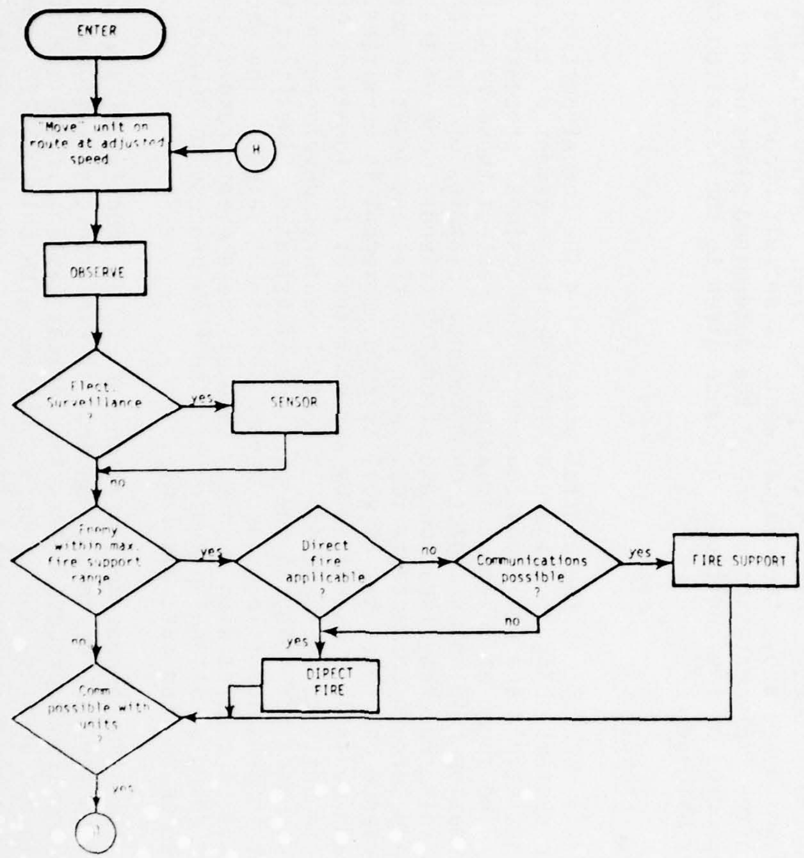

Figure 6 Part 1. Flowchart of EXECUTE Process

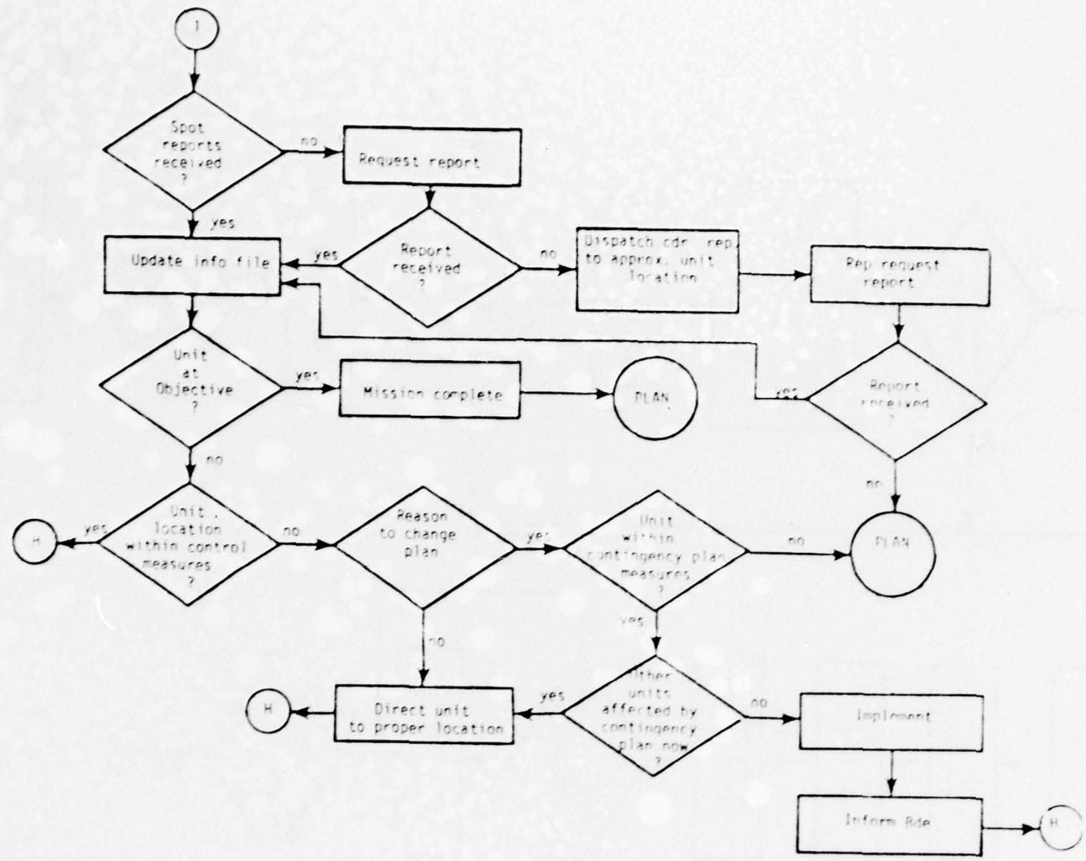

Figure 6 Part 2. Flowchart of EXECUTE Process

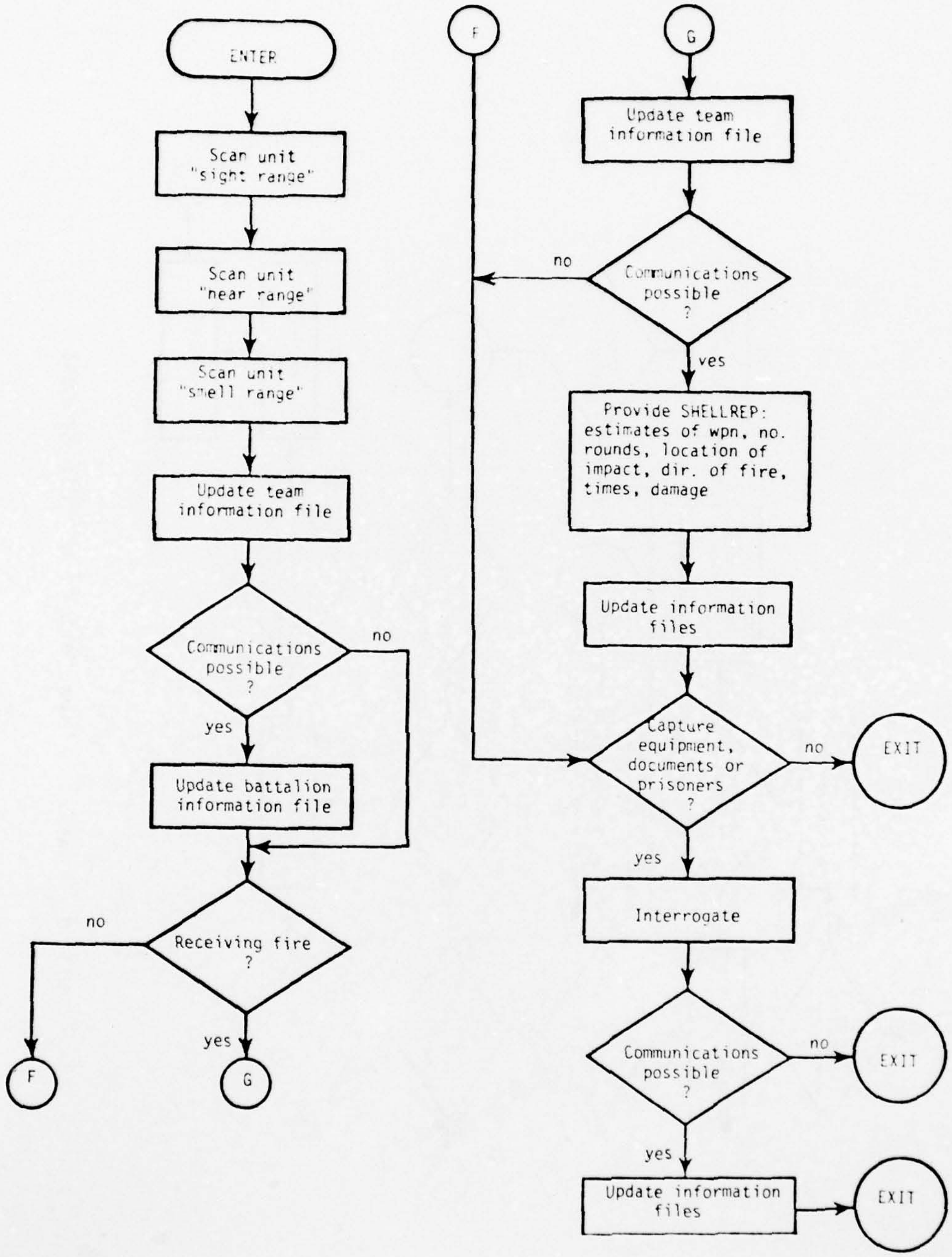

Figure 7 Flowchart of OBSERVOR Process

new information; the battalion commander and staff would continue operating on older information until the communications limitations are changed. Shell Reports should be provided when fire has been or is being received. Data gathered from captured equipment, interrogated prisoners and discovered documents are also included in this process.

d. SENSOR. When electronic surveillance is desired, the SENSOR process (Figure 8) is invoked. The first question asked is whether any devices for such surveillance are organically available to the battalion. If not, the battalion obviously cannot deploy them. Given that devices are available, the deployment of the teams with the devices is made. The "team" is also given the characteristics of the device, their mission and information from the file which might affect the mission. The device is then switched on; if not operational in some number of attempts as determined by the commander, the surveillance mission is abandoned. If the device is operational the mission is then carried out with obtained information communicated (if possible) to the appropriate information files for comparison with other data and file update. A return is made to the EXECUTE process if additional missions are not desired.

e. DIRECT FIRE. The DIRECT FIRE process is in Figure 9. This process is a straightforward sequence of computations to rank the targets by the threat, select the optimal target within range, select the ammunition for the effect desired, and fire all weapons within range of the target which are required to provide a desired probability of kill. Damage is assessed and ammunition used is calculated after each mission. Upon firing at the last target, the process is ended. Damage/logistics information is passed to the commander for file update if communication is possible. Not shown is a feadback to the OBSERVE, and possibly SENSOR, processes to provide the damage assessment.

f. FIRE SUPPORT. The FIRE SUPPORT process (Figure 10) is entered if desirable enemy targets are located by the elements of the battalion. First a check is made to determine if the targets are within designated restricted areas. If so, an attempt is made to coordinate with other units. Should no mission be possible after coordination, the process is ended. The next steps are the rank ordering of the targets as to the desirability to fire against each of them, and the selection of an indirect fire weapon system such as organic mortars, artillery, or close air support from Air Force, Navy, or Army air. Finally fire is requested from the desired system against the target(s). This portion of the activity must consider the response time of the weapon systems, the suitability of particular weapon systems against the type of target and the safety of friendly units. The unit responsible for mission implementation then checks the range, weapons appropriateness, and response time. If any of these are unacceptable, the request is denied. Approved targets are placed in a priority queue. After the mission, damage assessment is done and ammunition used is computed. Should all nonorganic indirect fire support systems respond with a denial of the fire request, the default weapon system organic to the battalion may be employed. A check, queue, fire and assess computation is then done in a manner similar to the nonorganic systems.

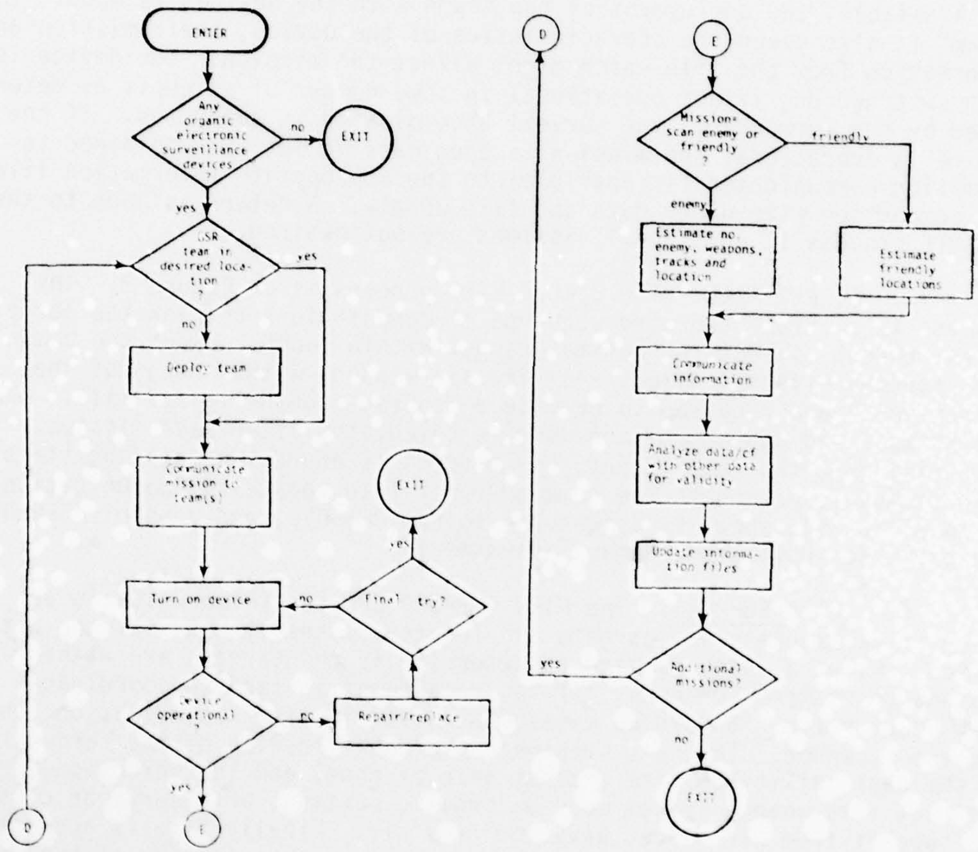

Figure 8 Flowchart of Process SENSOR

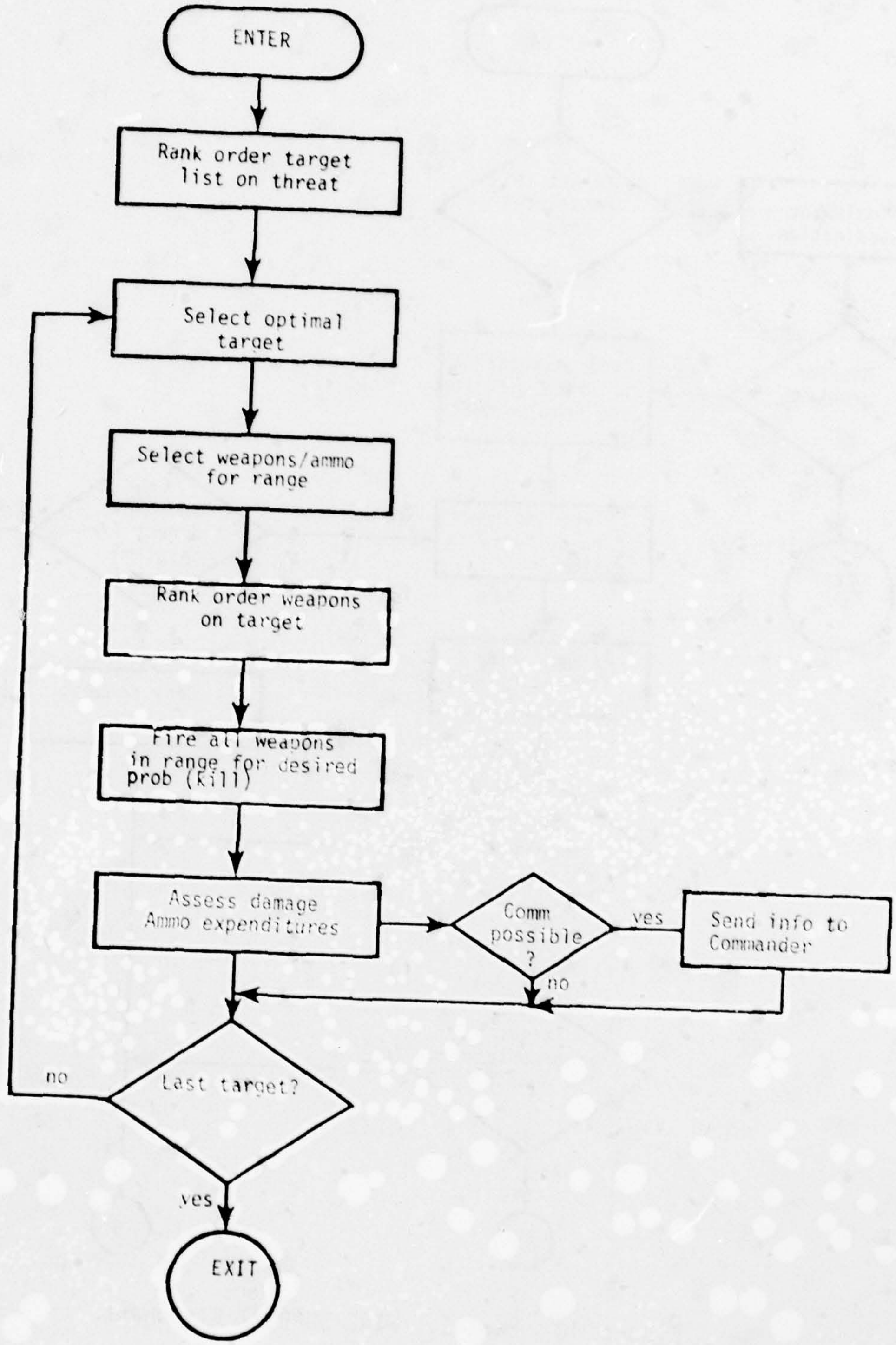

Figure 9 DIRECT FIRE Flowchart

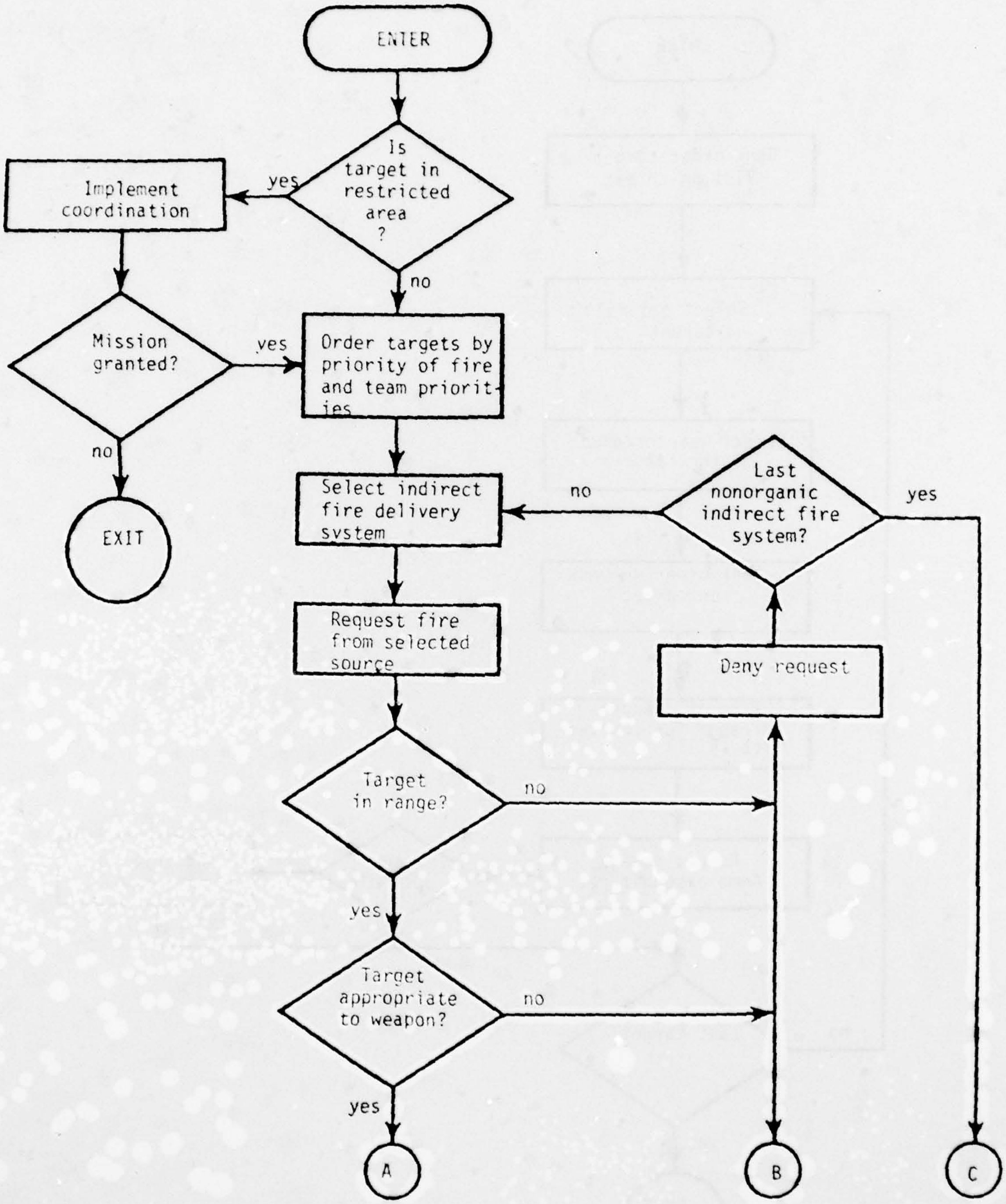

Figure 10 Part 1. FIRE SUPPORT Flowchart

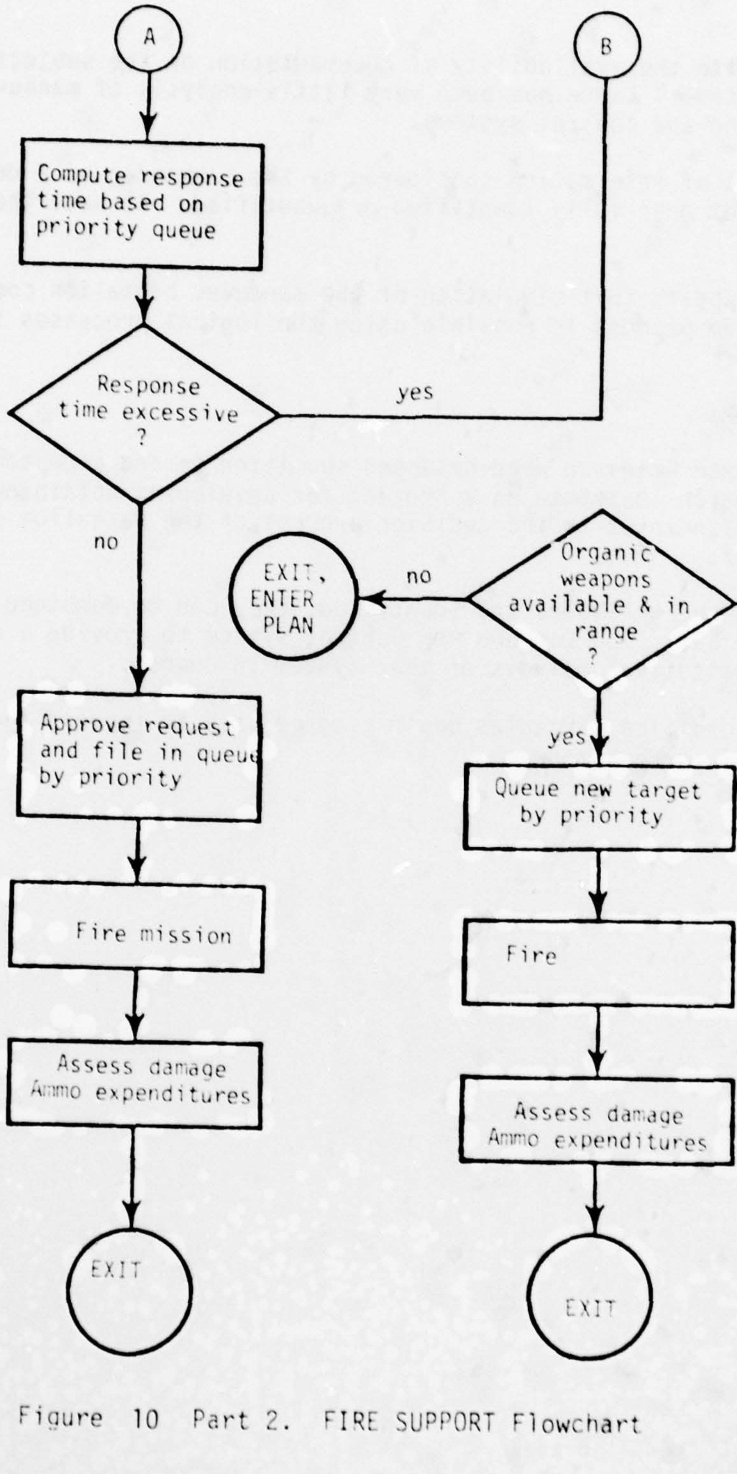

Figure 10 Part 2. FIRE SUPPORT Flowchart

VI. OBSERVATIONS AND FUTURE ACTIONS

OBSERVATIONS

o Despite the availability of documentation on the subject of "Command and Control," there has been very little analysis of maneuver battalion command and control systems.

o Types of information considered by the commander as a decision maker have not been fully identified or quantified. At best they are assumed.

o It appears that simulation of the maneuver battalion command and control system process is possible using the logical processes identified in this paper.

FUTURE ACTIONS

o A Human Research Need has been submitted to and accepted by the US Army Research Institute as a project for developing/obtaining the variables which input to the decision process of the battalion commander and his staff.

o Once the variables are identified, they can be combined with the logic of the Battalion Command and Control system to provide a methodology for quantitative analysis of that system in combat.

o The decision variables could also be used to improve the decision algorithms of current models.

APPENDIX 1 MODEL EVALUATION

Early war gaming models were totally manual affairs. These games permitted qualitative analyses of the command and control aspects of a staff. Table tops served as displays. Stochastic calculations were done based on the roll of dice. Manual war games were then very strong on logic but weak on computational aspects. With the advent of computers, the random throw of the die could be replaced by computer algorithms. For a time manual war games were interspersed with computer runs. Some of these games evolved to take the form of programs featuring humans at interactive consoles, usually in the role of decision makers.

In its presently evolved form, the computer war game is usually weak on command and control logic since humans have been taken out and replaced with table "lookups." Conversely, the games are strong on firepower calculations, calculations of logistics, and, in fact, all of those items which had once been table "lookups" in the early manual war games.

In order to reintroduce analysis of command and control into combat simulations, we must again include the staff. We could in effect go all the way back to the original manual war games. This is being done at the TRADOC Combined Arms Test Activity in the various command post exercises and workshops. However useful these manual exercises may be, an iterative analytical tool is to be desired as well. Such a computer alternative, or complement to these manual tests, would be a simulation of the combat process coupled with the command and control processes. An ideal model would be totally automated, include reasonable decision making algorithms, and include the components of a command and control system (detailed previously). This would then allow comparison of competitive systems in a combat environment.

A cursory review of existing computerized models was made to ascertain the ability of existing models to simulate command and control or the feasibility of their being modified to incorporate a command and control simulation. No attempt was made to totally analyze the applicability of each. A list of these models*, with evaluative comments, follows:

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*For a brief description of these (except DIVOPS and FOURCE) and other models, refer to Catalog of War Gaming and Military Simulation Models, Studies, Analysis and Gaming Agency, 6th Ed, June 1975, H. J. Walther, NTIS AD/A-012 803.

ADVICE II. A computerized division echelon manual game (i.e., containing an external decision-maker) of 1967 vintage which can demonstrate the efficiency of information flow but not its combat effectiveness. Run times are long: 2 days per 10 hour battle.

CAMMS. Brigade resolution computer assisted game designed as a training vehicle for an external decision maker (and staff).

CARMONETTE. Computerized two battalion model with tactics pre-determined by a "decision maker" and provided as part of the input data. Many conditions alter the implementation of the decisions. Designed and/or used to study tactics and mixes of forces.

COMMEL II.5. An improved versions of COMMEL (1963 vintage). Heavy on simulation of communications systems; coupled with a divisin combat simulator but with simplistic and deterministic decision logic based on abstract dimensionless intelligence levels and modified firepower ratios.

DIVSIFT. A COBOL simulation of the manual staff functions in a division TOC; does not consider combat decisions, cannot be used for automated command and control systems, and is not coupled with a combat simulator. Inoperable.

DIVOPS. Division level fully automated combat simulation with user provided decision rules based on perfect information. No command, control or communications.

DYNTACS. Battalion level combat simulator of extreme complexity. Decisions are as complex as CARMONETTE, but information flow is minimal although some tactical communications is inlcuded. Useful in comparisons of ground vehicular weapon systems.

FOURCE. TRASANA version of DIVOPS with extensive rebuild of subroutines to account for the command and control process; totally automated. Model development is currently in process with test and validation due for completion CY77. Documentation is incomplete, but the model is intended to include sufficient resolution to analyze the Division Command and Control system. Current test and production is in support of CACDA's CEATOS effort which analyzes Division Command and Control system alternatives.

TACOS II. Similar to COMMEL in many aspects but reportedly providing better decision algorithms based on each unit's estimate of the enemy situation drawn from a normal distribution whose mean is the true enemy location, size, speed, and direction with a variance related to the quality of the intelligence. Combat outcomes depend on the true enemy situation and the applied firepower. If it operates as claimed in the documentation, it would rank high among all models surveyed; TACOS II has not been located.

None of the above models appear to consider the pre-combat planning phase. They are concerned with the combat process and the possible modification of the plan. Except for FOURCE, none in their present state were judged suitable for evaluation of command and control systems; however, FOURCE is in the process of development. DYNTACS and CARMONETTE

may be suitable for modification to account for the Battalion Command and Control system.

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TITLE: How to Cook a Bayes

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ABSTRACT: Although there are four forms of Bayes theorem which intelligence analysts should know how to employ, normally usage has been restricted to the simplest form, that of two hypotheses and statistical independence. This probably is the case partly because most intelligence analysts lack familiarity with the various forms, and partly because the two hypotheses - statistical independence form is the easiest to employ. The objective of this student-oriented paper is to develop a cookbook approach that will enable a beginner to use the other, more general forms of the theorem, just as easily.

Bayesian Analysis
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HOW TO COOK A BAYES*

INTRODUCTION

Although there are four forms of Bayes theorem which intelligence analysts should know how to employ, normally usage has been restricted to the simplest form, that of two hypotheses and statistical independence. This probably is the case partly because most intelligence analysts lack familiarity with the various forms, and partly because the two hypotheses - statistical independence form is the easiest to employ. The objective of this student-oriented paper is to develop a cookbook approach that will enable a beginner to use the other, more general forms of the theorem, just as easily.

This objective seems of particular importance if analysts are to grapple with more complex problems. That is, while the two hypotheses - statistical independence form of Bayes may be highly appropriate for some I&W (Indications and Warning) problems, such as the one where hypothesis H_1 may be that "Country A will attack Country B within one week" and the second (and last) hypothesis H_2 is that "Country A will not attack country B within one week," many problems call for more hypotheses. One might, for example, be faced with a situation where hypothesis H_1 is that "there will be a large-scale attack by Country A on Country B within three months," H_2 is that "there will be small-scale provocations by Country A against Country B within three months," and H_3 is that "there are no planned hostilities by A against B for the next three months." Trying to force this situation into the two hypotheses mold of "attack" or "no attack" might be highly inappropriate. We will return to this example after we present the four forms of Bayes theorem and develop a tabular cookbook approach to calculating revised probabilities.

Let us turn then to a statement and discussion of Form I.

*or, "Recipes from the Bayes Cookbook"

FORM I: Any number of hypotheses; statistical dependence

This is the most general form because it places no limitations on the number of hypotheses and does not assume statistical independence.

The formula for this case is very long and complicated. It is given on this page and then explained. An example follows the explanation.

Formulas for FORM I

Let $H_1, H_2, \cdots, H_m$ be a set of "m" mutually exclusive and exhaustive hypotheses. Then, given a sequence of "n" events, $E_1, E_2, \cdots E_n$, the revised probability of a given hypothesis, H_i, is given by the formula

$$P(H_i | E_1 \cap E_2 \cap \cdots \cap E_n) = \frac{P(H_i) \cdot P(E_1 | H_i) \cdot P(E_2 | H_i \cap E_1) \cdots P(E_n | H_i \cap E_1 \cap E_2 \cap \cdots \cap E_{n-1})}{\sum_{j=1}^{m} P(H_j) \cdot P(E_1 | H_j) \cdot P(E_2 | H_j \cap E_1) \cdots P(E_n | H_j \cap E_1 \cap E_2 \cap \cdots \cap E_{n-1})}$$

We call this the first version of Form I.

A second way of writing this same formula, which is probably clearer to a student, is the following second version of Form I.

$$P(H_i | E_1 \cap E_2 \cap \cdots \cap E_n) = \frac{P(H_i) \cdot P(E_1 | H_i) \cdot P(E_2 | H_i \cap E_1) \cdots P(E_n | H_i \cap E_1 \cap E_2 \cap \cdots \cap E_{n-1})}{\begin{array}{l} P(H_1) \cdot P(E_1 | H_1) \cdot P(E_2 | H_1 \cap E_1) \cdots P(E_n | H_1 \cap E_1 \cap E_2 \cap \cdots \cap E_{n-1}) \\ +P(H_2) \cdot P(E_1 | H_2) \cdot P(E_2 | H_2 \cap E_1) \cdots P(E_n | H_2 \cap E_1 \cap E_2 \cap \cdots \cap E_{n-1}) \\ +P(H_3) \cdot P(E_1 | H_3) \cdot P(E_2 | H_3 \cap E_1) \cdots P(E_n | H_3 \cap E_1 \cap E_2 \cap \cdots \cap E_{n-1}) \\ \vdots \\ +P(H_m) \cdot P(E_1 | H_m) \cdot P(E_2 | H_m \cap E_1) \cdots P(E_n | H_m \cap E_1 \cap E_2 \cap \cdots \cap E_{n-1}) \end{array}}$$

There probably is still some question in the minds of many students about such things as the subscripts "i", "n", "m" and about the meaning of the "$\cdots$" that is used throughout the formula. These symbols will be explained now, but you may not understand the formula completely until you follow on through the first example.

The first question a student may have is what is meant by writing "$H_1, H_2, \cdots, H_m$" for the list of our hypotheses. All this is saying is that there are "m" hypotheses in number. If m=2, there are only two hypotheses, H_1 and H_2. If m=4, there are four hypotheses, H_1, H_2, H_3 and H_4. The use of the three dots "$\cdots$" in "$H_1, H_2, \cdots, H_m$" simply means that we have left out the listing of the hypotheses after the second one, i.e., H_2, because it is clear what we are doing, that is, it is clear that we are listing one hypothesis after another, until we get to the last one, which is H_m. It is traditional to list only the last item in a list after the three dots "$\cdots$". So we list H_m, the last hypothesis, after the three dots "$\cdots$".

In regard to the list of events, "$E_1, E_2, \cdots, E_n$" the same explanation holds. All we are saying is that there are "n" events in number. If n=1, there is only one event, E_1. If n=2, there are only two events, E_1 and E_2. If n=4, we have four events, E_1, E_2, E_3, and E_4. You might ask, why not let "m" be the last term in this list, i.e., why not talk about "$E_1, E_2, \cdots, E_m$". The reason is that this would imply that there necessarily were as many events as hypotheses, since we used "m" to list the number of hypotheses. So we pick another letter to stand for the last of our events. It may be, by chance, that in a practical example we have as many events as hypotheses, but by using this symbolism where we say there are "m" hypotheses and "n" events we allow for the number of hypotheses and events to be different.

Looking at the expression $P(H_i | E_1 \cap E_2 \cap \cdots \cap E_n)$ (located to the left of the "=" sign in the second version of Form I), the first thing to explain is the use of the term H_i. Why "i"? The term "H_i" stands for any of the hypotheses. That is, we are giving a general formula that will tell you how to calculate the revised probability for any one of the hypotheses. Thus "i" can be any

number between "1" and "m". That is, if $i=1$, we use the formula to calculate H_1. We do this by replacing all the "i"s everywhere they appear to the left and the right of the "=" sign by the number "1". So in the numerator of the second version of Form I, for example, we would replace the "i" in $P(H_i)\cdot P(E_1|H_i)\cdot P(E_2|H_i\cap E_1)\cdots P(E_n|H_i\cap E_1\cap E_2\cap\cdots\cap E_{n-1})$ by "1" and then have $P(H_1)\cdot P(E_1|H_1)\cdot P(E_2|H_1\cap E_1)\cdots P(E_n|H_1\cap E_1\cap E_2\cap\cdots\cap E_{n-1})$.

If $i=4$, to give another example, we would be calculating the revised probability of H_4, i.e., $P(H_4|E_1\cap E_2\cap\cdots\cap E_n)$ and the numerator would be $P(H_4)\cdot P(E_1|H_4)\cdot P(E_2|H_4\cap E_1)\cdots P(E_n|H_4\cap E_1\cap E_2\cap\cdots\cap E_{n-1})$. In the expression of the numerator: $P(H_i)\cdot P(E_1|H_i)\cdot P(E_2|H_i\cap E_1)\cdots P(E_n|H_i\cap E_1\cap E_2\cap\cdots\cap E_{n-1})$, the three dots "$\cdots$" are used again. The usage once more means that it is clear (hopefully) how we are listing terms so that after the first few we can put the dots "$\cdots$" down to indicate that we keep on listing terms until the last one is reached. The last term in the numerator is "$P(E_n|H_i\cap E_1\cap E_2\cap\cdots\cap E_{n-1})$". and it too uses the three dots to indicate that we keep listing "E"s until we get to the last term, E_{n-1}. To give an example, if $n=1$, the numerator, $P(H_i)\cdot P(E_1|H_i)\cdot P(E_2|H_i\cap E_1)\cdots P(E_n|H_i\cap E_1\cap E_2\cap\cdots\cap E_{n-1})$, becomes $P(H_i)\cdot P(E_1|H_i)$.

If $n=2$, the numerator becomes $P(H_i)\cdot P(E_1|H_i)\cdot P(E_2|H_i\cap E_1)$. All one is doing here is putting down "2" everywhere the "n" appears and listing terms until the last one.

If we have $n=4$, the numerator becomes

$$P(H_i)\cdot P(E_1|H_i)\cdot P(E_2|H_i\cap E_1)\cdot P(E_3|H_i\cap E_1\cap E_2)\cdot P(E_4|H_i\cap E_1\cap E_2\cap E_3)$$

Since in practice, of course, we have combinations of hypotheses and events, let us give a few examples of the two.

Suppose $m=3$, $n=4$. That is, we have three hypotheses, H_1, H_2, and H_3, and four events, E_1, E_2, E_3, and E_4. If we want to calculate the revised

probability of the second hypothesis, we would have to the left of the "=" sign $P(H_2|E_1 \cap E_2 \cap E_3 \cap E_4)$, while the numerator would become

$P(H_2)\cdot P(E_1|H_2)\cdot P(E_2|H_2 \cap E_1)\cdot P(E_3|H_2 \cap E_1 \cap E_2)\cdot P(E_4|H_2 \cap E_1 \cap E_2 \cap E_3)$.

Finally, look at the denominator in the second version of Form I. You can see that the denominator is pretty big, but each term should now be comprehensible and this is the version of Form I that you should use in working problems.

Indeed, the difference between the numerator and the denominator in the second verion of Form I is simply that the numerator is but one of the terms (i.e. rows) of the denominator. In the denominator we have a term (i.e. row) for every last one of the hypotheses.

The three dots "$\vdots$" between the term

$P(H_3)\cdot P(E_1|H_3)\cdot P(E_2|H_3 \cap E_1)\cdots P(E_n|H_3 \cap E_1 \cap E_2 \cap \cdots E_{n-1})$ and the term

$P(H_m)\cdot P(E_1|H_m)\cdot P(E_2|H_m \cap E_1)\cdots P(E_n|H_m \cap E_1 \cap E_2 \cap \cdots \cap E_{n-1})$ in the denominator indicate that we are to continue listing the terms (i.e. rows) until we reach the one for "H_m", the last of the hypotheses.

Let us turn to an example which will, hopefully, clear up any remaining confusion a student might have about using the formula.

Example:

Suppose that we have four hypotheses, call them H_1, H_2, H_3, and H_4 and that the hypotheses are mutually exclusive and exhaustive.

If we want to run a Bayesian analysis we start off by assessing subjective probabilities for the first three hypotheses, i.e., we assess $P(H_1)$, $P(H_2)$, $P(H_3)$. Then $P(H_4) = 1 - [P(H_1) + P(H_2) + P(H_3)]$ since the hypotheses are mutually exclusive and exhaustive.

For this example, suppose we decide that $P(H_1) = 0.4$, $P(H_2) = 0.2$, $P(H_3) = 0.1$. Then $P(H_4) = 1-0.7 = 0.3$.

Next, suppose we use these initial values in our Bayesian analysis and that we wish to revise the probabilities in light of an event E_1. According to the formula for the second version of Form I, the following is true:

$$P(H_1|E_1) = \frac{P(H_1)\cdot P(E_1|H_1)}{\begin{array}{l} P(H_1)\cdot P(E_1|H_1) \\ +P(H_2)\cdot P(E_1|H_2) \\ +P(H_3)\cdot P(E_1|H_3) \\ +P(H_4)\cdot P(E_1|H_4) \end{array}}$$

$$P(H_3|E_1) = \frac{P(H_3)\cdot P(E_1|H_3)}{\begin{array}{l} P(H_1)\cdot P(E_1|H_1) \\ +P(H_2)\cdot P(E_1|H_2) \\ +P(H_3)\cdot P(E_1|H_3) \\ +P(H_4)\cdot P(E_1|H_4) \end{array}}$$

$$P(H_2|E_1) = \frac{P(H_2)\cdot P(E_1|H_2)}{\begin{array}{l} P(H_1)\cdot P(E_1|H_1) \\ +P(H_2)\cdot P(E_1|H_2) \\ +P(H_3)\cdot P(E_1|H_3) \\ +P(H_4)\cdot P(E_1|H_4) \end{array}}$$

$$P(H_4|E_1) = \frac{P(H_4)\cdot P(E_1|H_4)}{\begin{array}{l} P(H_1)\cdot P(E_1|H_1) \\ P(H_2)\cdot P(E_1|H_2) \\ P(H_3)\cdot P(E_1|H_3) \\ P(H_4)\cdot P(E_1|H_4) \end{array}}$$

Strictly speaking, once we calculate $P(H_1|E_1)$, $P(H_2|E_1)$, and $P(H_3|E_1)$, we can calculate $P(H_4|E_1)$ by the formula $P(H_4|E_1) = 1-[P(H_1|E_1)+P(H_2|E_1)+P(H_3|E_1)]$. However, as a check on all our calculations, we really should go ahead and calculate $P(H_4|E_1)$ just to make sure the numbers come out right.

Note that in the formulas for $P(H_1|E_1)$, $P(H_2|E_1)$, $P(H_3|E_1)$, and $P(H_4|E_1)$, the denominators are all the same. Only the numerators vary.

In order to use the formulas, we now must assign these subjective conditional probabilities:

$P(E_1|H_1)$
$P(E_1|H_2)$
$P(E_1|H_3)$
$P(E_1|H_4)$.

In assigning these probabilities, we first ask ourself, "What is the probability event E_1 would occur if we assume (i.e. given) hypothesis H_1 to be true" in order to assess $P(E_1|H_1)$. Next we ask ourself, "What is the probability event E_1 would occur if we assume (i.e. given) hypothesis H_2 to be true" in order to assess $P(E_1|H_2)$. We continue to ask the same sort of questions, assuming H_3 to be true in order to assess $P(E_1|H_3)$ and then H_4 to be true in order to assess $P(E_1|H_4)$.

Suppose we decide on the following probabilities:

$P(E_1|H_1) = 0.8$

$P(E_1|H_2) = 0.9$

$P(E_1|H_3) = 0.4$

$P(E_1|H_4) = 0.5$

Note that there is no need for these probabilities to add up to "1".

Once these values are assigned we simply plug them into our equations. In this case, we get the following results:

$$P(H_1|E_1) = \frac{P(H_1)\cdot P(E_1|H_1)}{P(H_1)\cdot P(E_1|H_1) + P(H_2)\cdot P(E_1|H_2) + P(H_3)\cdot P(E_1|H_3) + P(H_4)\cdot P(E_1|H_4)}$$

$$= \frac{(0.4)(0.8)}{(0.4)(0.8) + (0.2)(0.9) + (0.1)(0.4) + (0.3)(0.5)}$$

$$= \frac{0.32}{\begin{array}{l}0.32\\+0.18\\+0.04\\+0.15\end{array}}$$

$$= \frac{0.32}{0.69}$$

$$= 0.464$$

Since, as noted before, the denominators are all the same for $P(H_1|E_1)$, $P(H_2|E_1)$, $P(H_3|E_1)$, $P(H_4|E_1)$, we can simply write:

$$P(H_2|E_1) = \frac{P(H_2)\cdot P(E_1|H_2)}{0.69}$$

$$= \frac{(0.2)(0.9)}{0.69} = \frac{0.18}{0.69} = 0.261$$

$$P(H_3|E_1) = \frac{P(H_3)\cdot P(E_1|H_3)}{0.69}$$

$$= \frac{(0.1)(0.4)}{0.69} = \frac{0.04}{0.69} = 0.058$$

$$P(H_4|E_1) = \frac{P(H_4)\cdot P(E_1|H_4)}{0.69} = \frac{(0.3)(0.5)}{0.69} = \frac{0.15}{0.69}$$

$$= 0.217$$

So the results are:

$$\begin{aligned} P(H_1|E_1) &= 0.464 \\ P(H_2|E_1) &= 0.261 \\ P(H_3|E_1) &= 0.058 \\ P(H_4|E_1) &= \underline{0.217} \\ & \quad 1.000 \end{aligned}$$

and the results, that is, the revised probabilities, add to "1" as they should.

Thus far, to tabulate our results, we have:

TABLE I

$P(H_1) = 0.4$	$P(E_1 \mid H_1) = 0.8$	$P(H_1 \mid E_1) = 0.464$
$P(H_2) = 0.2$	$P(E_1 \mid H_2) = 0.9$	$P(H_2 \mid E_1) = 0.261$
$P(H_3) = 0.1$	$P(E_1 \mid H_3) = 0.4$	$P(H_3 \mid E_1) = 0.058$
$P(H_4) = 0.3$	$P(E_1 \mid H_4) = 0.5$	$P(H_4 \mid E_1) = 0.217$

Suppose now that event E_2 occurs, and we wish to calculate our revised probabilities in light of the latest evidence.

According to the second version of Form I, we wish to calculate:

$$P(H_1 \mid E_1 \cap E_2) = \frac{P(H_1)\cdot P(E_1 \mid H_1)\cdot P(E_2 \mid H_1 \cap E_1)}{\begin{array}{l} P(H_1)\cdot P(E_1 \mid H_1)\cdot P(E_2 \mid H_1 \cap E_1) \\ +P(H_2)\cdot P(E_1 \mid H_2)\cdot P(E_2 \mid H_2 \cap E_1) \\ +P(H_3)\cdot P(E_1 \mid H_3)\cdot P(E_2 \mid H_3 \cap E_1) \\ +P(H_4)\cdot P(E_1 \mid H_4)\cdot P(E_2 \mid H_4 \cap E_1) \end{array}}$$

$$P(H_2 \mid E_1 \cap E_2) = \frac{P(H_2)\cdot P(E_1 \mid H_2)\cdot P(E_2 \mid H_2 \cap E_1)}{\begin{array}{l} P(H_1)\cdot P(E_1 \mid H_1)\cdot P(E_2 \mid H_1 \cap E_1) \\ +P(H_2)\cdot P(E_1 \mid H_2)\cdot P(E_2 \mid H_2 \cap E_1) \\ +P(H_3)\cdot P(E_1 \mid H_3)\cdot P(E_2 \mid H_3 \cap E_1) \\ +P(H_4)\cdot P(E_1 \mid H_4)\cdot P(E_2 \mid H_4 \cap E_1) \end{array}}$$

$$P(H_3 \mid E_1 \cap E_2) = \frac{P(H_3)\cdot P(E_1 \mid H_3)\cdot P(E_2 \mid H_3 \cap E_1)}{\begin{array}{l} P(H_1)\cdot P(E_1 \mid H_1)\cdot P(E_2 \mid H_1 \cap E_1) \\ P(H_2)\cdot P(E_1 \mid H_2)\cdot P(E_2 \mid H_2 \cap E_1) \\ P(H_3)\cdot P(E_1 \mid H_3)\cdot P(E_2 \mid H_3 \cap E_1) \\ P(H_4)\cdot P(E_1 \mid H_4)\cdot P(E_2 \mid H_4 \cap E_1) \end{array}}$$

$$P(H_4|E_1 \cap E_2) = \frac{P(H_4)\cdot P(E_1|H_4)\cdot P(E_2|H_4 \cap E_1)}{\begin{array}{l} P(H_1)\cdot P(E_1|H_1)\cdot P(E_2|H_1 \cap E_1) \\ P(H_2)\cdot P(E_1|H_2)\cdot P(E_2|H_2 \cap E_1) \\ P(H_3)\cdot P(E_1|H_3)\cdot P(E_2|H_3 \cap E_1) \\ P(H_4)\cdot P(E_1|H_4)\cdot P(E_2|H_4 \cap E_1) \end{array}}$$

As a check on our calculations, we should calculate $P(H_4|E_1 \cap E_2)$ as well as the other probabilities.

Note again, that all the denominators in the above equations are the same. Only the numerators differ.

In order to use the formulas, we now must assign these subjective conditional probabilities:

$P(E_2|H_1 \cap E_1)$

$P(E_2|H_2 \cap E_1)$

$P(E_2|H_3 \cap E_1)$

$P(E_2|H_4 \cap E_1)$.

In assigning these conditional probabilities, we first ask ourself, "What is the probability that event E_2 would occur if we assume (i.e. given) hypothesis H_1 to be true and if (i.e. given) event E_1 has occurred" in order to assess $P(E_2|H_1 \cap E_1)$. Next, we ask ourself, "What is the probability event E_2 would occur if we assume (i.e. given) hypothesis H_2 to be true and if (i.e. given) event E_1 has occurred" in order to assess $P(E_2|H_2 \cap E_1)$. We continue on to ask the same sorts of questions, first assuming H_3 to be true and E_1 to have occurred in order to assess $P(E_2|H_3 \cap E_1)$ and next assuming H_4 to be true and E_1 to have occurred in order to assess $P(E_2|H_4 \cap E_1)$.

Suppose we decide on these values:

$P(E_2 | H_1 \cap E_1) = 0.2$

$P(E_2 | H_2 \cap E_1) = 0.8$

$P(E_2 | H_3 \cap E_1) = 0.6$

$P(E_2 | H_4 \cap E_1) = 0.3$

Note again that there is no need for these values to add up to "1".

Once these values are assigned we simply plug them into our equations. To make our calculations easier to follow we will first repeat all the values we have assessed so far.

TABLE II

$P(H_1) = 0.4$	$P(E_1 \| H_1) = 0.8$	$P(E_2 \| H_1 \cap E_1) = 0.2$
$P(H_2) = 0.2$	$P(E_1 \| H_2) = 0.9$	$P(E_2 \| H_2 \cap E_1) = 0.8$
$P(H_3) = 0.1$	$P(E_1 \| H_3) = 0.4$	$P(E_2 \| H_3 \cap E_1) = 0.6$
$P(H_4) = 0.3$	$P(E_1 \| H_4) = 0.5$	$P(E_2 \| H_4 \cap E_1) = 0.3$

Taking these values then, we can plug the numbers into our formulas. We get:

$$P(H_1 | E_1 \cap E_2) = \frac{P(H_1)\cdot P(E_1|H_1)\cdot P(E_2|H_1 \cap E_1)}{\begin{array}{l} P(H_1)\cdot P(E_1|H_1)\cdot P(E_2|H_1 \cap E_1) \\ +P(H_2)\cdot P(E_1|H_2)\cdot P(E_2|H_2 \cap E_1) \\ +P(H_3)\cdot P(E_1|H_3)\cdot P(E_2|H_3 \cap E_1) \\ +P(H_4)\cdot P(E_1|H_4)\cdot P(E_2|H_4 \cap E_1) \end{array}}$$

$$= \frac{(0.4)(0.8)(0.2)}{\begin{array}{l} (0.4)(0.8)(0.2) \\ +(0.2)(0.9)(0.8) \\ +(0.1)(0.4)(0.6) \\ +(0.3)(0.5)(0.3) \end{array}}$$

$$= \frac{0.064}{0.064 + 0.144 + 0.024 + 0.045}$$

$$= \frac{0.064}{0.277} = 0.231$$

Noting again that since the denominators in the equations for calculating $P(H_1|E_1 \cap E_2)$, $P(H_2|E_1 \cap E_2)$, $P(H_3|E_1 \cap E_2)$, $P(H_4|E_1 \cap E_2)$ are the same, we can simply write:

$$P(H_2|E_1 \cap E_2) = \frac{P(H_2)\cdot P(E_1|H_2)\cdot P(E_2|H_2 \cap E_1)}{0.277}$$

$$= \frac{(0.2)(0.9)(0.8)}{0.277}$$

$$= \frac{0.144}{0.277}$$

$$= 0.520$$

$$P(H_3|E_1 \cap E_2) = \frac{P(H_3)\cdot P(E_1|H_3)\cdot P(E_2|H_3 \cap E_1)}{0.277}$$

$$= \frac{(0.1)(0.4)(0.6)}{0.277}$$

$$= \frac{0.024}{0.277}$$

$$= 0.087$$

$$P(H_4|E_1 \cap E_2) = \frac{P(H_4)\cdot P(E_1|H_4)\cdot P(E_2|H_4 \cap E_1)}{0.277}$$

$$= \frac{(0.3)(0.5)(0.3)}{0.277}$$

$$= \frac{0.045}{0.277}$$

$$= 0.162$$

So the results are:

$$P(H_1|E_1 \cap E_2) = 0.231$$

$$P(H_2|E_1 \cap E_2) = 0.520$$

$$P(H_3|E_1 \cap E_2) = 0.087$$

$$P(H_4|E_1 \cap E_2) = \underline{0.162}$$

$$1.000$$

and the results, that is, the revised probabilities, add to "1" as they should.

At this point the reader may have noted something very interesting. If you look back at Table II, then you can see that the denominators we just calculated were just what you would get if you multiplied the entries together for each row in Table II and added up the results. Look at the denominator in the equation for $P(H_1 | E_1 \cap E_2)$ following Table II. It matches the array we set out in Table II, i.e., each term (i.e. row) in the denominator matches a corresponding row in the table.

As for the numerators we just calculated, if we want the revised probability of any hypothesis, the numerator comes from multiplying the entries together for the row in Table II that corresponds to the hypothesis of interest. For example, we get the numerator for $P(H_3 | E_1 \cap E_2)$ from the third row entries of Table II - compare to the actual calculation we made for $P(H_3 | E_1 \cap E_2)$ to verify this.

All this is very important, because it means that we can simply plug our entries into a table and multiply out the rows and add up the results in order to calculate the denominators. Any numerator for a given hypothesis comes from the corresponding row for that hypothesis. Let us do this for the following example.

Suppose we keep on with the previous case, but now have four events. We have to assess the following entries for a table:

TABLE III

$P(H_j)$	$P(E_1\|H_j)$	$P(E_2\|H_j \cap E_1)$	$P(E_3\|H_j \cap E_1 \cap E_2)$	$P(E_4\|H_j \cap E_1 \cap E_2 \cap E_3)$	Row Product
$P(H_1)=0.4$	$P(E_1\|H_1)=0.8$	$P(E_2\|H_1 \cap E_1)=0.2$	$P(E_3\|H_1 \cap E_1 \cap E_2)=0.6$	$P(E_4\|H_1 \cap E_1 \cap E_2 \cap E_3)=0.3$	0.01152
$P(H_2)=0.2$	$P(E_1\|H_2)=0.9$	$P(E_2\|H_2 \cap E_1)=0.8$	$P(E_3\|H_2 \cap E_1 \cap E_2)=0.7$	$P(E_4\|H_2 \cap E_1 \cap E_2 \cap E_3)=0.6$	0.06048
$P(H_3)=0.1$	$P(E_1\|H_3)=0.4$	$P(E_2\|H_3 \cap E_1)=0.6$	$P(E_3\|H_3 \cap E_1 \cap E_2)=0.1$	$P(E_4\|H_3 \cap E_1 \cap E_2 \cap E_3)=0.2$	0.00048
$P(H_4)=0.3$	$P(E_1\|H_4)=0.5$	$P(E_2\|H_4 \cap E_1)=0.3$	$P(E_3\|H_4 \cap E_1 \cap E_2)=0.5$	$P(E_4\|H_4 \cap E_1 \cap E_2 \cap E_3)=0.9$	0.02025
					0.09273

To verify that this array really corresponds to the entries in the second version of Form I, look at that equation and substitute in $m=4$, $n=4$. It should be clear that each row of Table II corresponds to a row of the denominator of the second version. The row products in Table III correspond to the values of the terms (i.e. rows) in the denominator of the second version.

Using the table, since the sum of the row products is 0.09273, we have:

$$P(H_1|E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{0.01152}{0.09273} = 0.124$$

$$P(H_2|E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{0.06048}{0.09273} = 0.652$$

$$P(H_3|E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{0.00048}{0.09273} = 0.005$$

$$P(H_4|E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{0.02025}{0.09273} = \underline{0.218}$$

$$0.999$$

Our sum for the revised probabilities is essentially "1" as it should be.

This sort of tabular approach to Bayes greatly simplifies calculations. One simply lists as many rows as there are hypotheses. The first column will be the initial, or prior probabilities assessed for the probabilities of

the hypotheses. The following columns correspond to the entries assessed as the events occur. If n=4, as in the above example, one simply fills in the appropriate entries - what is appropriate is quickly clear from an examination of the denominator of the second version of Form I.

This tabular approach allows us to easily carry out a sensitivity analysis of our figures. It is easy to keep straight what we are doing.

FORM II - Any number of hypotheses; statistical independence

This form differs from Form I in that we assume statistical independence. Again, the formula is somewhat long and complicated, but as will be seen in a moment it is very similar to Form I and permits the same sort of tabular solution

Formulas for FORM II.

Let $H_1, H_2, \cdots, H_m$ be a set of "m" mutually exclusive and exhaustive hypotheses. Then, given a sequence of "n" independent events $E_1, E_2, \cdots, E_n$ the revised probability of a given hypotheses, H_i, is given by the formula:

$$P(H_i | E_1 \cap E_2 \cap \cdots \cap E_n) = \frac{P(H_i) \cdot P(E_1 | H_i) \cdot P(E_2 | H_i) \cdots P(E_n | H_i)}{\sum_{j=1}^{m} P(H_j) \cdot P(E_1 | H_j) \cdot P(E_2 | H_j) \cdots P(E_n | H_j)}$$

Again, a second way of writing this same formula, which is not only probably clearer to the student but also lends itself to a tabular arrangement can be given. We will call it the second version of FORM II. It is as follows:

$$P(H_i | E_1 \cap E_2 \cap \cdots \cap E_n) = \frac{P(H_i) \cdot P(E_1 | H_i) \cdot P(E_2 | H_i) \cdots P(E_n | H_i)}{\begin{aligned} &P(H_1) \cdot P(E_1 | H_1) \cdot P(E_2 | H_1) \cdots P(E_n | H_1) \\ &+P(H_2) \cdot P(E_1 | H_2) \cdot P(E_2 | H_2) \cdots P(E_n | H_2) \\ &+P(H_3) \cdot P(E_1 | H_3) \cdot P(E_2 | H_3) \cdots P(E_n | H_3) \\ &\quad \vdots \\ &+P(H_m) \cdot P(E_1 | H_m) \cdot P(E_2 | H_m) \cdots P(E_n | H_m) \end{aligned}}$$

The major difference between this second version of Form II and the second version of Form I, from the user's point of view, comes in the way one phrases the conditional probabilities to be assessed.

For example, instead of assessing (i.e. assigning a subjective probability to) $P(E_2 | H_1 \cap E_1)$ as we had to do in the second version of Form I, we now assess $P(E_2 | H_1)$ and ask ourself, "What is the probability that event E_2 would occur if we assume (i.e. given) hypotheses H_1 to be true." No attention is paid to the occurrence of E_1. Also, it clearly makes no difference, as to the probability assessed, in what order we consider the events.

The same sort of tabular approach used before can be used here to solve for the revised probabilities. Suppose we have five hypotheses and four events. Suppose we have assigned these subjective probabilities:

TABLE IV

$P(H_j)$	$P(E_1 \| H_j)$	$P(E_2 \| H_j)$	$P(E_3 \| H_j)$	$P(E_4 \| H_j)$	Row Product
$P(H_1)=0.10$	$P(E_1 \| H_1)=0.5$	$P(E_2 \| H_1)=0.9$	$P(E_3 \| H_1)=0.1$	$P(E_4 \| H_1)=0.7$	0.00315
$P(H_2)=0.05$	$P(E_1 \| H_2)=0.3$	$P(E_2 \| H_2)=0.8$	$P(E_3 \| H_2)=0.2$	$P(E_4 \| H_2)=0.8$	0.00192
$P(H_3)=0.15$	$P(E_1 \| H_3)=0.4$	$P(E_2 \| H_3)=0.4$	$P(E_3 \| H_3)=0.3$	$P(E_4 \| H_3)=0.1$	0.00072
$P(H_4)=0.30$	$P(E_1 \| H_4)=0.2$	$P(E_2 \| H_4)=0.1$	$P(E_3 \| H_4)=0.2$	$P(E_4 \| H_4)=0.2$	0.00024
$P(H_5)=0.40$	$P(E_1 \| H_5)=0.9$	$P(E_2 \| H_5)=0.2$	$P(E_3 \| H_5)=0.2$	$P(E_4 \| H_5)=0.9$	0.01296
					0.01899

So the denominator, in any of the equations to solve for the revised probabilities, is 0.01899.

To solve for $P(H_1 | E_1 \cap E_2 \cap E_3 \cap E_4)$ we have

$$P(H_1 | E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{0.00315}{0.01899} = 0.166$$

$$P(H_2 | E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{0.00192}{0.01899} = 0.101$$

$$P(H_3 | E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{0.00072}{0.01899} = 0.038$$

$$P(H_4 | E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{0.00024}{0.01899} = 0.013$$

$$P(H_5 | E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{0.01296}{0.01899} = \underline{0.682}$$

$$1.000$$

And again, the revised probabilities add to "1" as they should.

After a while, in using the tabular arrangement, one might simplify it and write it this way, for example:

TABLE V

j	$P(H_j)$	$P(E_1 \| H_j)$	$P(E_2 \| H_j)$	$P(E_3 \| H_j)$	$P(E_4 \| H_j)$	$P(E_5 \| H_j)$	Row Product
1	0.1	0.7	0.6	0.1	0.6	0.1	0.000252
2	0.7	0.5	0.2	0.2	0.7	0.4	0.003920
3	0.1	0.9	0.7	0.7	0.8	0.9	0.031752
4	0.05	0.1	0.4	0.9	0.1	0.7	0.000126
5	0.05	0.3	0.3	0.8	0.2	0.6	0.000432
							0.036482

That is, once you get used to using the tabular arrangement, you can just say to yourself what entry you are putting into a given cell, e.g., $P(E_4 | H_3) = 0.8$, but you just put down the 0.8.

<u>FORM III: 2 hypotheses; statistical dependence</u>

In contrast with the previous cases, the formulas here are quite easy. Because the tabular arrangement here is so easy, we prefer <u>not</u> to present

the usual "odds - likelihood" form of Bayes Theorem. Also, the "odds - likelihood" form does not lend itself quite as easily to sensitivity analysis, in our opinion.

Formulas for FORM III.

Let H_1 and H_2 be two mutually exclusive and exhaustive hypotheses. Then, given a sequence of "n" events, $E_1, E_2, \cdots E_n$, the revised probability of a given hypothesis, H_i, is given by the formula:

$$P(H_i \mid E_1 \cap E_2 \cap \cdots \cap E_n) = \frac{P(H_i)\cdot P(E_1 \mid H_i)\cdot P(E_2 \mid H_i \cap E_1)\cdots P(E_n \mid H_i \cap E_1 \cap E_2 \cap \cdots \cap E_{n-1})}{\begin{gathered} P(H_1)\cdot P(E_1 \mid H_1)\cdot P(E_2 \mid H_1 \cap E_1)\cdots P(E_n \mid H_1 \cap E_1 \cap E_2 \cap \cdots \cap E_{n-1}) \\ P(H_2)\cdot P(E_1 \mid H_2)\cdot P(E_2 \mid H_2 \cap E_1)\cdots P(E_n \mid H_2 \cap E_1 \cap E_2 \cap \cdots \cap E_{n-1}) \end{gathered}}$$

In this formula, "i" can only take on the values "1" and "2" since there are only two hypotheses, H_1 and H_2.

Note that once we assess $P(H_1)$, it follows that $P(H_2) = 1 - P(H_1)$ since the hypotheses are mutually exclusive and exhaustive.

To give an example that uses the usual table format, suppose we have n=5, and that we make the assessments shown here:

Table VI

$P(H_j)$	$P(E_1 \mid H_j)$	$P(E_2 \mid H_j \cap E_1)$	$P(E_3 \mid H_j \cap E_1 \cap E_2)$	$P(E_4 \mid H_j \cap E_1 \cap E_2 \cap E_3)$	$P(E_5 \mid H_j \cap E_1 \cap E_2 \cap E_3 \cap E_4)$	Row Product
$P(H_1)=0.6$	$P(E_1 \mid H_1)=0.3$	$P(E_2 \mid H_1 \cap E_1)=0.8$	$P(E_3 \mid H_1 \cap E_1 \cap E_2)=0.8$	$P(E_4 \mid H_1 \cap E_1 \cap E_2 \cap E_3)=0.7$	$P(E_5 \mid H_1 \cap E_1 \cap E_2 \cap E_3 \cap E_4)=0.4$	0.032256
$P(H_2)=0.4$	$P(E_1 \mid H_2)=0.7$	$P(E_2 \mid H_2 \cap E_1)=0.3$	$P(E_3 \mid H_2 \cap E_1 \cap E_2)=0.4$	$P(E_4 \mid H_2 \cap E_1 \cap E_2 \cap E_3)=0.8$	$P(E_5 \mid H_2 \cap E_1 \cap E_2 \cap E_3 \cap E_4)=0.8$	0.021504
						0.053760

So our denominator is 0.053760. Then

$$P(H_1 \mid E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5) = \frac{0.032256}{0.053760} = 0.6$$

$$P(H_2 \mid E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5) = \frac{0.021504}{0.053760} = \underline{0.4}$$

$$1.0$$

Once more the revised probabilities add to "1" as they should.

This formulation can be put in a tabular form that is almost as fast as the "odds - likelihood" form, and is better suited for a sensitivity analysis.

For example, taking the data in Table VI above, let us compute the revised probabilities as each piece of data comes along. We will use this format:

j	$P(H_j)$	$P(E_1 \mid H_j)$	Row Product	Revised Probabilities for H_j
1	0.6	0.3	0.18	$P(H_1 \mid E_1) = \frac{0.18}{0.46} = 0.39$
2	0.4	0.7	0.28	$P(H_2 \mid E_1) = \frac{0.28}{0.46} = 0.61$
			0.46	1.00

For the next piece of data we have:

j	$P(H_j)$	$P(E_1 \mid H_j)$	$P(E_2 \mid H_j \cap E_1)$	Row Product
1	0.6	0.3	0.8	0.144
2	0.4	0.7	0.3	0.084
				0.228

$$P(H_1 \mid E_1 \cap E_2) = \frac{0.144}{0.228} = 0.63$$

$$P(H_2 \mid E_1 \cap E_2) = \frac{0.084}{0.228} = \underline{0.37}$$

$$1.00$$

For the next piece of data we have:

j	$P(H_j)$	$P(E_1 \mid H_j)$	$P(E_2 \mid H_j \cap E_1)$	$P(E_3 \mid H_j \cap E_1 \cap E_2)$	Row Product
1	0.6	0.3	0.8	0.8	0.1152
2	0.4	0.7	0.3	0.4	0.0336
					0.1488

Note that we can take our latest entry in row 1, namely "0.8," and multiply it times the row product for the previous calculation, namely "0.144," to get our latest row product of "0.1152." This sort of process greatly speeds up the use of the technique. For this latest event E_3 we calculate:

$$P(H_1 | E_1 \cap E_2 \cap E_3) = \frac{0.1152}{0.1488} = 0.77$$

$$P(H_2 | E_1 \cap E_2 \cap E_3) = \frac{0.0336}{0.1488} = \underline{0.23}$$

$$1.00$$

Finally, for the latest event E_4 we have, using the shortcut suggested in the last paragraph:

j	Previous Product	$P(E_4 \vert H_j \cap E_1 \cap E_2 \cap E_3)$	Row Product
1	0.1152	0.7	0.08064
2	0.0336	0.8	0.02688
			0.10752

So $P(H_1 | E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{0.08064}{0.10758} = 0.75$

$$P(H_2 | E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{0.02688}{0.10752} = \underline{0.25}$$

$$1.00$$

After one becomes used to the process one could build an even shorter arrangement. For example, to do the calculation for E_5, we could have this format:

j	Previous Product	$P(E_5 \vert H_j \cap E_1 \cap E_2 \cap E_3 \cap E_4)$	Row Product	Revised Probabilities for H_j
1	0.08064	0.4	0.032256	$\frac{0.032256}{0.053696} = 0.60$
2	0.02688	0.8	0.021140	$\frac{0.021440}{0.053696} = 0.40$
			0.053696	1.00

)RM IV: 2 hypotheses; statistical independence

The simplest form of all, and the most used, this has a formulation very similar to the previous one.

Let H_1 and H_2 be two mutually exclusive and exhaustive hypotheses. Then, given a sequence of "n" independent events, E_1, E_2, $\cdots E_n$, the revised probability of a given hypothesis, H_i, is given by the formula:

$$P(H_i|E_1 \cap E_2 \cap \cdots \cap E_n) = \frac{P(H_i)\cdot P(E_1|H_i)\cdot P(E_2|H_i)\cdots P(E_n|H_i)}{P(H_1)\cdot P(E_1|H_1)\cdot P(E_2|H_1)\cdots P(E_n|H_1) + P(H_2)\cdot P(E_1|H_2)\cdot P(E_2|H_2)\cdots P(E_n|H_2)}$$

Again, "i" can only take on the values "1" and "2" since there are only two hypotheses, H_1 and H_2.

Note again that once we assess $P(H_1)$, it follows that $P(H_2) = 1-P(H_1)$.

A tabular arrangement similar to the previous one makes calculations go very rapidly.

We can start off this way:

j	$P(H_j)$	$P(E_1\|H_j)$	Row Product	Revised Probabilities for H_j
1	0.65	0.3	0.195	$\frac{0.195}{0.440} = 0.44$
2	0.35	0.7	0.245	$\frac{0.245}{0.440} = 0.56$
			0.440	1.00

For the next event, E_2, we would have:

j	Previous Product	$P(E_2\|H_j)$	Row Product	Revised Probabilities for H_j
1	0.195	0.8	0.1560	$\frac{0.1560}{0.2295} = 0.68$
2	0.245	0.3	0.0735	$\frac{0.0735}{0.2295} = 0.32$
			0.2295	1.00

For the next event E_3, we would have

j	Previous Product	$P(E_3 \mid H_j)$	Row Product	Revised Probabilities for H_j
1	0.1560	0.8	0.1248	$\frac{0.1248}{0.1542} = 0.81$
2	0.735	0.4	0.0294	$\frac{0.0294}{0.1542} = 0.19$
			0.1542	1.00

EXAMPLE

Now that we have presented the four forms of Bayes theorem and the tabular cookbook approach for quickly calculating the revised probabilities, let us return to the three hypotheses example mentioned in the introduction.

Let H_1 = There will be a large-scale attack by Country A on Country B within three months,

H_2 = There will be small-scale provocations by Country A against Country B within three months.

H_3 = There are no planned hostilities by A against B for the next three months.

Suppose that we have assigned the initial probabilities of $P(H_1)=0.2$, $P(H_2)=0.6$. Then $P(H_3)=0.2$.

Next, suppose that E_1 is that A has secretly ordered a reserve army division to be mobilized as rapidly as possible. Suppose further that A has a total force of four active and eight reserve divisions and B a force of six active and eight reserve divisions. Using our tabular arrangement, we might make the following assessments and calculations.

j	$P(H_j)$	$P(E_1 \mid H_j)$	Row Product	Revised Probabilities for H_j
1	0.2	0.7	0.14	$\frac{0.14}{0.66} = 0.21$
2	0.6	0.7	0.42	$\frac{0.42}{0.66} = 0.64$
3	0.2	0.5	0.10	$\frac{0.10}{0.66} = 0.15$
			0.66	1.00

Next suppose that E_2 is that Country A's army chief of staff has visited Country C, which is an ally of A's, and also has a border with Country B. Again, using our tabular arrangement, we might have the

following assessments and calculations, where we assume statistical dependence.

j	Previous Product	$P(E_2 \mid H_j \cap E_1)$	Row Product	Revised Probabilities for H_j
1	0.14	0.9	0.126	$\frac{0.126}{0.356} = 0.35$
2	0.42	0.5	0.210	$\frac{0.210}{0.356} = 0.59$
3	0.10	0.2	0.020	$\frac{0.020}{0.356} = 0.06$
			0.356	1.00

Next, suppose that E_3 is that Country A has announced that it will be holding its annual army maneuvers on time in the second of the three months covered in the hypotheses. The maneuvers this year, however, will be near the natural invasion route into Country B. Maneuvers have not been held there for the last 10 years, largely because when they were held there ten years ago, the result was a major protest by Country B and a general souring of relations that has continued to the present time.

Again, using our tabular arrangement, we might have the following assessments and calculations:

j	Previous Product	$P(E_3 \mid H_j \cap E_1 \cap E_2)$	Row Product	Revised Probabilities for H_j
1	0.126	0.9	0.113	$\frac{0.113}{0.291} = 0.39$
2	0.210	0.8	0.168	$\frac{0.168}{0.291} = 0.58$
3	0.020	0.5	0.010	$\frac{0.010}{0.291} = 0.03$
			0.291	1.00

At this point, after three items of evidence, our estimate of the probability of a large scale attack has almost doubled, going from 0.2

to 0.39. The probability of small scale provocations is almost the same, going from 0.6 to 0.58. The probability of no hostilities has declined from 0.2 to 0.03. So some form of conflict is extremely likely.

We could go on with this, but the point really was to show how easy it is to use the tabular arrangement in recording subjective probability assessments and in calculating revised probabilities of hypotheses.

TITLE: A SOFTWARE COST ESTIMATING TECHNIQUE FOR COMMUNICATIONS SYSTEMS

AUTHOR: STAN DUNN

ABSTRACT: A software cost estimating technique is presented for large-scale communications systems. A modified Delphi approach in conjunction with a Software Cost Estimating model is used to arrive at a principal cost estimate. Then, due to the nebulous nature of software development in complex systems, a risk analysis model is used to characterize the high-risk elements associated with a generalized communications system. The results of the risk analysis are then applied to the principal cost estimate in order to arrive at a cost estimate. After developing a technique which provides a cost estimate tailored to a specific project, but not tailored to any particular developer, a second technique is presented which derives a customized cost estimate. This is accomplished through the use of regression analysis based on previously estimated software development costs and schedules. In addition, the risk analysis model is re-run with the high-risk elements adjusted to reflect the capabilities of a specific developer. The results of the regression technique and the adjusted risk analysis techniques are then combined to establish a cost estimate tailored to both the project and the developer of the software.

A SOFTWARE COST ESTIMATING TECHNIQUE
FOR COMMUNICATIONS SYSTEMS*

STAN DUNN

U. S. ARMY COMMUNICATIONS COMMAND

There is no universal method for estimating the costs of software. In actuality, attempts to make these estimates have been far from adequate. It is not unusual to find projects in both government and business costing 10 times the original estimate and two to three times is common!

There are several factors which complicate estimating the cost of software development:

1) Software development is largely an iterative process in a complex system caused by engineering changes, technology advances, etc., which historically impose changes in software. Also, work completed on a process step may have to be repeated because of later developments in the process. For example, the failure of a program to pass an integration test will require repeating some part of the program design, code, and test step; or the systems analysis and design step; or both. This recycling within the programing process is widely recognized as a major factor in both the magnitude of total costs and the uncertainty of predicting programing costs.

2) Secondly, software development, as a field, is very new and has been an unwilling stepchild of computer hardware technical advances. Thus, just when software analysts and programers begin to feel comfortable with a technical level of computer hardware, a new level comes along and their methods become obsolete. Software technology is also growing both from a managerial and a technical standpoint. For example, at the same time the analyst or programer is learning to deal with the latest in data base management or file structuring, he is hit with a new technique for managing software development (e.g., structured programing).

3) A third significant factor complicating software cost estimating is the elusiveness of productivity itself. "Programing productivity has been shown to vary by as much as a factor of 26 between individuals" 1/

4) The fourth factor affecting software cost estimating is equally as significant as the first three. However, it differs in one important aspect. Whereas the first three are probably unavoidable in a rapidly growing field, the fourth, the lack of software engineering data base, does not necessarily

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*This paper was based upon an actual system. Although specific numbers were changed to prevent disclosure of the system, ratios between the numbers were retained, thus retaining the relationships between cost estimates and actual costs.

1/Boehm, Barry W., Datamation, May 1973: "Software and Its Impact; A Quantitative Assessment."

have to be. Certainly, enough systems, large and small, have been developed in many disciplines to have acquired a rather extensive set of cost estimating relationships and standards which could be used in estimating future software costs. However, recording of this information has been sparce, and as a result, a good engineering data base still does not exist.

In view of the above complications, a cost estimate for software development generally requires some combination of techniques including the following:

1) Specific Analogy. Instances in which costs of software components can be related to software development cost of similar components. Because of the lack of a software engineering data base, the use of this technique may be significantly limited.

2) Unit Price. Instances in which the number of units and their per unit costs can be determined (e.g., lines of code and cost per line of code).

3) Percent of Other Item. This technique may be used when the cost of some portion of the software development or maintenance can be estimated as a predetermined percentage of the cost of another item (e.g., the cost of computer program design, code, or test might be some relatively predictable percentage of total software development cost).

4) Parametric Equations. Instances in which the costs of portions of the software development can be estimated from an equation based upon various characteristics or requirements.

METHODOLOGY

Based upon the above estimating techniques, a methodology was established for estimating software costs. The estimate for the software is predicated upon there being basically nine phases of software development: Systems Requirements Analysis; Systems Design; Program Design; Coding; Program Test; System Test; User Training; System Implementation; and Systems Maintenance. Those costs incurred during the first eight phases are considered software development costs for purposes of this analysis. Systems maintenance, on the other hand, is considered sufficiently different to be considered separately. The following discussion is limited to software development.

The primary step in estimating the cost of software development was to review the literature for methodology, available software engineering data bases, cost estimating relationships and standards. As mentioned previously, there is a lack of appropriate software engineering data bases available. Accordingly, valid cost estimating relationships and standards, at lower levels of detail, are scarce. However, at a more general level, the literature is somewhat more generous, particularly with regard to the relationship between the various major software development functions. Partial results of this research are presented in Table 1.

SOFTWARE COST FACTORS
COST FACTORS/EQUATION

SOURCE	ACTIVITIES/% OF TOTAL PROJECT COSTS
1. E. A. Nelson, "Management Handbook for the Estimation of Computer Programmer Costs", 20 Mar 67 (based on 169 completed computer programing projects).	Analysis & Design, 34.5% Program Coding & Checking, 18.0% Checkout & Test, 47.5%
2. F. P. Brooks, Jr., "Why is the Software Late?," Datamation, Aug 71 (former Project Manager for IBM 360 development has applied these percentages effectively).	Planning, 33-1/3% Coding, 16-2/3% Component Test & Early System Test, 25.0% System Test all Components in Hand, 25.0%
3. R. A. Findley, "Computer Software Development Costs, Predictable or Not?" (Study Report Per 74-1)(Higher Order Languages).	Analysis & Design, 40.0% Coding, 15.0% Testing, 45.0%
4. R. W. Wolverton, IEEE Transactions on Computers, VOL C-23, No. 6, Jun 74, "The Cost of Developing Large-Scale Software."	
a. Case History A.	Software Requirements Analysis & Preliminary Design, 29.8% Detailed Design, 16.4% Code and Debug, 20.0% Validation Test & Operational Demonstration, 13.5% Development Test, 20.3%
b. Case History B.	Analysis & Design, 42.0% Code & Debug, 33.0% QA Test & System Tests, 25.0%
c. "Rule of Thumb" discovered by several researchers.	Analysis & Design, 40.0% Coding & Debugging, 20.0% Checkout & Test, 40.0%

d.

SYSTEM	ANALYSIS & DESIGN	CODING & AUDITING	CHECKOUT & TEST
SAGE	39.0%	14.0%	47.0%
NTDS	30.0%	20.0%	50.0%
GEMINI	36.0%	17.0%	47.0%
SATURN V	32.0%	24.0%	44.0%
OS/360	33.0%	17.0%	50.0%
TRW SURVEY	46.0%	20.0%	34.0%

TABLE 1

Modified Delphi Technique. The Delphi technique was used because, although the problem did not lend itself to precise analytical techniques, the subjective judgments of a group of experienced personnel could be obtained for evaluation.

1) A group of experienced senior software managers was assembled and given a presentation and general specifications for a large automated telecommunications system (hereafter referred to as System X), including information on difficulty and complexity.

2) The general approach used to obtain contributions from the group on each item was:

a) The monitor made a presentation describing the problem and what was desired as a contribution.

b) The group was encouraged to ask questions and discuss the problem until each had a good understanding of the problem and the required contribution.

c) Each member then prepared an independent contribution and identified the principal factors considered and their impact on the problem.

d) The monitor analyzed the contributions from the group and prepared a candidate description of the data.

e) The monitor then made point checks where possible, and further reviewed the candidate data.

3) The problems presented to the group for consideration are identified below:

a) Allocate the software effort to eight basic functional tasks.

b) Discuss and identify each party's problem in consumating all tasks.

c) Defend/support decisions with verbal arguments.

The information provided through the above process was analyzed and further evaluated to provide the basis for the distribution of effort. After the data were organized, several spot checks were made to validate data points.

Based on the results of the above process, the objective of estimating software development costs was pursued in two phases. The first phase consisted of running a Software Cost Estimate (SCE) model to develop an independent parametric estimate; and the second phase consisted of applying risk analysis via the use of a Risk Analysis model to the results of the first phase. The SCE model accepts as input several factors developed in the modified delphi technique or obtained from technical documents and reports. From this input, the model derives the expended funds, cost to complete, and total cost at completion for the software program. Major inputs to the model are:

1) Percentage of software module development allocated to the various software development activities.

2) Complexity element. The primary indicator of complexity for System X was determined to be the size of the program module (number of instructions at completion). Thus, it was necessary to identify the principal software modules for a specified level of detail and estimate the number of instructions required to perform each task of the specified software modules

3) Percentage Completion.

The major output of the SCE model is an estimate of software development (including documentation) costs. However, due to the nebulous nature of software development in complex systems, it became apparent that a risk analysis model would be needed to enhance the credibility of the cost estimate. This model was used to characterize the high-risk elements associated with System X. The model basically estimated a mean risk factor, its standard deviation, and the range of risk factors considering the principal high-risk elements of System X software.

The approach taken for the risk analysis was to identify the principal risk elements and to estimate for each the associated distribution of the factor increase in final cost. The assembly of these high-risk elements and their associated increase cost factor distribution is called the raw risk factor matrix (see Tables 2 and 3 for subsystems A and B respectively).

The 13 risk areas (program growth, core growth, etc.) were selected as those which could conceivably result in cost growth in the software development. The identified high-risk factor elements are characteristic of software programs in general. The raw risk factor matrix was developed considering other software programs with special attention to System X. The 20 factors across each row reflect an assessment based on experience with similar systems as to the magnitude and likelihood of this cost growth. For example, for Subsystem A, Table 2 indicates that in seven out of 20 occurrences, (35% of the time), "program growth" could be expected to increase the cost of software development by at least 40 percent. In two out of 20 times (10%), it would increase it by 30 percent, etc. Multiplying the risk factors for any given column will give the final risk factor for the described conditions.

An analysis of the Subsystem A software program for the specified risk factors shows the expected average increase in cost for a program of the complexity of System X is 241 percent of the principal cost. An analysis of the subsystem B software program for the specified risk factors shows the expected average increase in cost for a program of the complexity of System X is 199 percent of the principal cost.

Once the basic cost estimate was developed for System X subsystems, and the risk factors derived, the following equation was used to develop a final cost estimate.

Principal Cost x Risk Factor = Final Cost

SUBSYSTEM A RAW RISK FACTORS

COST GROWTH FACTORS

RISK AREA	PROBABILITY OF OCCURRENCE (%) 5	10	15	20	25	30	35	40	45	50
01 PROGRAM GROWTH	1.40	1.40	1.40	1.40	1.40	1.40	1.40	1.30	1.30	1.20
02 CORE GROWTH	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.03
03 OUT OF CORE GROWTH	1.50	1.30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
04 COMPUTE BOUND-SOFT/HARD BOUND	1.20	1.15	1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00
05 REAL TIME EXECUTIVE	1.08	1.08	1.08	1.08	1.08	1.08	1.08	1.08	1.08	1.08
06 SOFTWARE CONTROL/DYNAMIC RECONFIGURATION	1.50	1.35	1.20	1.10	1.10	1.10	1.05	1.05	1.03	1.02
07 TIMING ANALYSIS-SOFT/HARD PROBLEM	1.10	1.10	1.05	1.05	1.00	1.00	1.00	1.00	1.00	1.00
08 HARDWARE/SOFTWARE INTERFACE RESOLUTION	1.30	1.30	1.30	1.30	1.30	1.30	1.20	1.20	1.20	1.09
09 SYSTEM INTEGRATION	1.30	1.20	1.15	1.15	1.13	1.12	1.11	1.10	1.09	1.08
10 PERIPHERAL HANDLING	1.20	1.17	1.15	1.10	1.07	1.06	1.06	1.06	1.06	1.06
11 MAN/MACHINE INTERFACE	1.06	1.06	1.06	1.05	1.05	1.05	1.04	1.03	1.03	1.02
12 OPERATIONAL REAL TIME DIAGNOSTICS	1.20	1.16	1.15	1.14	1.13	1.12	1.11	1.10	1.09	1.08
13 CPU/CPU CONTROL	1.09	1.08	1.07	1.06	1.06	1.06	1.06	1.05	1.04	1.04

RISK AREA	55	60	65	70	75	80	85	90	95	100
01 PROGRAM GROWTH	1.20	1.20	1.20	1.20	1.20	1.10	1.10	1.10	1.10	1.10
02 CORE GROWTH	1.03	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
03 OUT OF CORE GROWTH	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
04 COMPUTE BOUND-SOFT/HARD BOUND	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
05 REAL TIME EXECUTIVE	1.08	1.08	1.07	1.07	1.06	1.05	1.04	1.03	1.01	1.00
06 SOFTWARE CONTROL/DYNAMIC RECONFIGURATION	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
07 TIMING ANALYSIS-SOFT/HARD PROBLEM	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
08 HARDWARE/SOFTWARE INTERFACE RESOLUTION	1.09	1.09	1.09	1.05	1.05	1.05	1.05	1.05	1.05	1.00
09 SYSTEM INTEGRATION	1.07	1.07	1.07	1.07	1.06	1.06	1.06	1.04	1.02	1.00
10 PERIPHERAL HANDLING	1.05	1.05	1.05	1.04	1.04	1.04	1.04	1.03	1.02	1.00
11 MAN/MACHINE INTERFACE	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12 OPERATIONAL REAL TIME DIAGNOSTICS	1.07	1.06	1.05	1.05	1.04	1.02	1.00	1.00	1.00	1.00
13 CPU/CPU CONTROL	1.03	1.02	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 2

SUBSYSTEM B RAW RISK FACTORS

COST GROWTH FACTORS

RISK AREA	5	10	15	20	25	30	35	40	45	50
01 PROGRAM GROWTH	1.30	1.30	1.30	1.30	1.30	1.30	1.30	1.25	1.25	1.15
02 CORE GROWTH	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.03
03 OUT OF CORE GROWTH	1.20	1.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
04 COMPUTE BOUND-SOFT/HARD BOUND	1.10	1.05	1.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00
05 REAL TIME EXECUTIVE	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.05	1.05
06 SOFTWARE CONTROL/DYNAMIC RECONFIGURATION	1.50	1.35	1.20	1.10	1.10	1.10	1.05	1.05	1.03	1.02
07 TIMING ANALYSIS-SOFT/HARD PROBLEM	1.10	1.10	1.05	1.05	1.00	1.00	1.00	1.00	1.00	1.00
08 HARDWARE/SOFTWARE INTERFACE RESOLUTION	1.30	1.30	1.30	1.30	1.30	1.30	1.20	1.20	1.20	1.09
09 SYSTEM INTEGRATION	1.20	1.15	1.10	1.10	1.09	1.09	1.08	1.07	1.06	1.06
10 PERIPHERAL HANDLING	1.30	1.20	1.15	1.10	1.07	1.06	1.06	1.06	1.06	1.06
11 MAN/MACHINE INTERFACE	1.06	1.05	1.05	1.05	1.04	1.04	1.03	1.02	1.02	1.01
12 OPERATIONAL REAL TIME DIAGNOSTICS	1.10	1.08	1.07	1.06	1.06	1.05	1.05	1.05	1.04	1.04
13 CPU/CPU CONTROL	1.09	1.08	1.07	1.06	1.06	1.06	1.06	1.05	1.04	1.04

RISK AREA	55	60	65	70	75	80	85	90	95	100
01 PROGRAM GROWTH	1.10	1.10	1.10	1.05	1.05	1.05	1.05	1.03	1.02	1.00
02 CORE GROWTH	1.03	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
03 OUT OF CORE GROWTH	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
04 COMPUTE BOUND-SOFT/HARD BOUND	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
05 REAL TIME EXECUTIVE	1.05	1.05	1.05	1.05	1.04	1.04	1.04	1.03	1.02	1.00
06 SOFTWARE CONTROL/DYNAMIC CONFIGURATION	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
07 TIMING ANALYSIS-SOFT/HARD PROBLEM	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
08 HARDWARE/SOFTWARE INTERFACE RESOLUTION	1.09	1.09	1.09	1.05	1.05	1.05	1.05	1.05	1.05	1.00
09 SYSTEM INTEGRATION	1.05	1.05	1.05	1.05	1.05	1.05	1.04	1.04	1.03	1.00
10 PERIPHERAL HANDLING	1.05	1.05	1.04	1.04	1.04	1.03	1.03	1.02	1.00	1.00
11 MAN/MACHINE INTERFACE	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
12 OPERATIONAL REAL TIME DIAGNOSTICS	1.03	1.02	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00
13 CPU/CPU CONTROL	1.03	1.02	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00

PROBABILITY OF OCCURRENCE (%)

TABLE 3

REGRESSION ANALYSIS

This approach consists of the development of a trend line on the basis of previously estimated contract prices and projection of this trend to future costs. The following data were obtained from Cost Performance Reports prepared by the System X contractor.

ESTIMATED SOFTWARE COSTS ($ MILLION)

AS OF DATE	SUBSYSTEM A	SUBSYSTEM B	TOTAL
31 May 74	1.025	.893	1.918
31 Dec 74	1.150	1.073	2.224
31 Jul 75	1.363	1.124	2.487
31 Dec 75	1.378	1.447	2.825
31 Dec 76	2.730	3.926	6.656
31 Dec 76	2.689	4.742	7.432
31 Jan 77	2.690	4.742	7.432

As of 31 May 1974, the contractor's estimated software development cost was 1.918 million. These estimates have progressively increased over the last 2 1/2 years to the most recent estimate of 7.432 million.

The assumption behind the trend line approach is that a discernible trend of escalating costs has already been established, which is an historical composite of estimating error, contract redefinition, mismanagement and rework. It is, in effect, a summarization of the technical and human elements at work on this project, both anticipated (the initial 1.918 million), and unanticipated (the additional 5.514 million), to date. It further assumes that the same elements will continue to impact on this project until its completion. Thus, if the assumption holds, the extension of the trend line using regression analysis techniques should provide a reasonable estimate of the final cost.

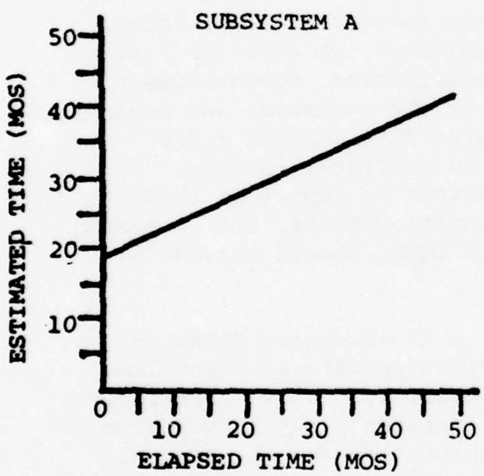
The method selected for fitting the curves is the Stepwise Multiple Linear Regression (least squares). The correlation coefficients for Subsystem A and Subsystem B were .92 and .93, respectively. These correlation coefficients are significant at the 95 percent confidence level indicating that a probable cause and effect relationship exists between the time of the estimate and estimated costs to complete the software. Analysis showed a significant linear regression was present and that the slope is significantly different from zero.

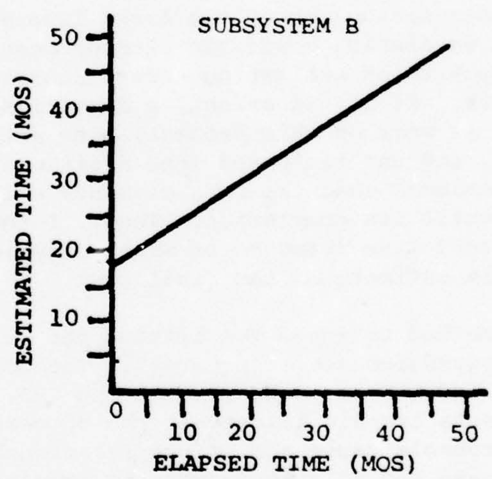
Based on the above projections of software development costs for current elapsed time estimates of 28.1 months (Subsystem A) and 31.9 months (Subsystem B) are $3.536 million and $7.267 million respectively for a total of $10.803 million.

It should be noted, at this point, that the regression equations in the above analysis were plotted only up to the scheduled completion date. No adjustment was made for progressive schedule slippage. In order to correct for this, a trend line similar to the above was developed on the basis of previously estimated contract completion time for software development. The following data represents estimates of elapsed time to completion at various points in time, up to the current time. The same regression method and significant levels used for costs are applicable to this analysis of scheduled slippage.

ESTIMATE NUMBER	ESTIMATED MONTHS TO COMPLETE SUBSYSTEM A	SUBSYSTEM B
1	20.0	20.0
2	21.1	21.1
3	23.06	23.06
4	28.06	31.94

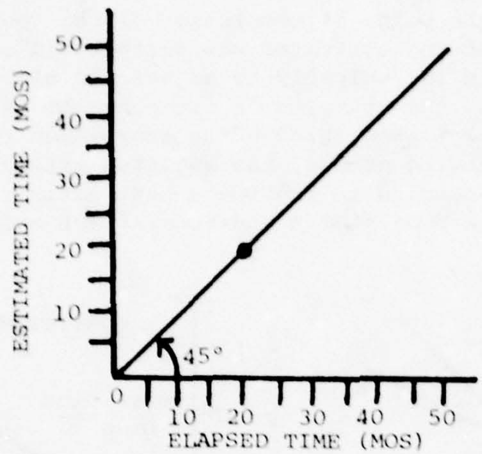
As was the case for projecting escalating costs, it is necessary to assume that a discernible trend of progressive schedule slippage has already been established. The regression lines for Subsystem A and Subsystem B are shown below·



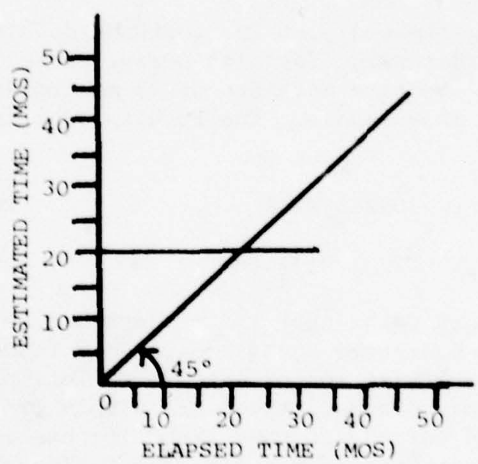


The above regression lines reflect the relationship between the passage of time and progressive schedule slippage in software development for System X. The positive slope of the lines indicate that slippage has occurred and is expected to continue occurring. The objective of this analysis, however, is to actually predict the number of elapsed months which will be required to complete the two subsystems based on the established trend. This prediction required the development of another curve, which when coupled with the above regression curves, provides this prediction.

This additional curve which will be called the "perfect vision" curve is shown below:

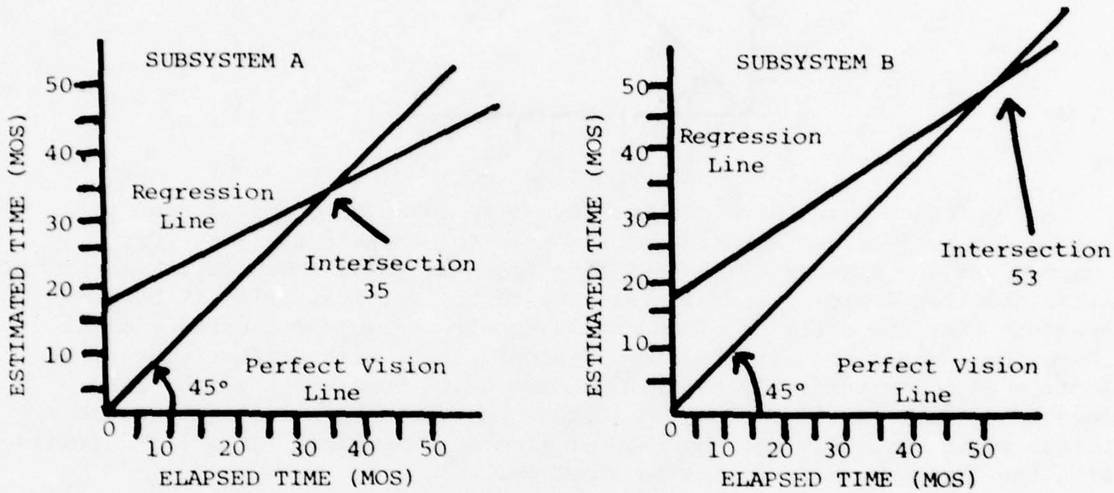


Any point on this curve reflects equality between estimated time to complete a project and actual time required to complete the job. For example, if one follows a vertical line from the point shown to the horizontal axis, and then follows a horizontal line to the vertical axis, it becomes apparent that the estimate of 20 months equals the actual elapsed time of 20 months. Thus, the term "perfect vision" line. With perfect vision, an individual would initially estimate 20 months to complete the job. Any further estimate would also be 20 months. A regression line through these points would obviously be a horizontal line at 20 months. This line, along with the "perfect vision" line is shown below:



As shown above, at the point of intersection (20 months), the job is complete as was estimated. Now assume that the estimator does not have perfect vision, but for various reasons, under-estimates the time required to complete the job and that the developer of these curves is aware of this. The developer may know this either by historical observation of the

estimator, or as in the case of System X, by actually observing several progressively increasing estimates. In either case, the developer can use this knowledge to predict the point of completion of the job just as he could use the knowledge that the estimator has perfect vision. The method for utilizing this knowledge is basically to adjust the slope of the horizontal line running from the estimator's estimates to the "perfect vision" line. This is accomplished through the regression technique for this analysis. In the following graphs, the adjusted estimator lines and "perfect vision" lines are coupled to provide a best estimate of the actual elapsed time to complete the Subsystem A and Subsystem B software.



In order to develop an estimated cost for software development, it is now necessary to apply the regression formulas derived from the previous cost regression analysis to the time estimate of 35 months (Subsystem A) and 53 months (Subsystem B) shown above. The results of this are shown below:

SUBSYSTEM A	SUBSYSTEM B	TOTAL
4.22 Million	12.01 Million	16.23 Million

It should be noted at this point that the projected time estimates of 35 months and 53 months for Subsystem A and Subsystem B respectively are not intended to be used as a projection of actual completion times for System X software development. Rather, these are simply projective of the time which would be required per the current trend for use in the cost regression. Obvious by its absence is the management alternative of providing an additional resource at some point to reduce the elapsed time. However, this is seen as a separate problem which might either increase or decrease the above cost estimate. It is not considered worthwhile to project this potentiality at this point as it would require second guessing management reaction to events which have not occurred.

The software experts who conducted the previously described Modified Delphi Technique, were asked to assess risk factors for the current System X contract specifically with regard to software from the Modified Delphi Technique, their two independent assessments resulted in the following estimates:

	SUBSYSTEM A	SUBSYSTEM B	TOTAL
Expert #1	\$4.924 M	\$10.443 M	\$15.367 M
Expert #2	\$5.936	\$12.266	\$18.201

The validity of each expert's opinion was judged to be equal, therefore, a simple average of the two is considered an adequate composite. This results in an estimated total cost of \$16.784 M. This estimate is surprisingly close to the \$16.233 M derived via the regression technique. In either case, the estimated total cost is over eight times the original contractor estimate of \$1.918 M!

RECOMMENDATIONS

The lessons learned during this study suggest a set of goals which should be extremely valuable to any government agency requiring software development. In fact, the degree of success achieved should be directly proportional to the nearness with which reality compares to the set of goals. These goals are presented as a list of recommendations below.

1) Emphasize Front-End Planning, Analysis and Design. Much of the cost growth in software development is a natural outgrowth of ill-defined systems and resulting attempts tc plan, design, and develop systems simultaneously. Plar on 35-45 percent of the software development effort occurring before a single line of code is written!

2) Minimize Modifications. The number of modifications required will probably be reduced significantly by adhering to the first recommendation. Tight controls should be exercised on remaining modifications.

3) Hold Software Developers To Their Estimates. By adhering to the first two recommendations, it should be possible to present the software developer with a complete set of plans and specifications and impose severe penalties for significant deviations from their initial cost estimates.

4) Develop an In-House Software Cost Estimating Capability. Do not entrust this important function to software developers whose judgment may be tainted by the fact that they are competing for the award of a contract.

5) Develop Cost Estimating Relationships (CER's) and Standards. Communicate experiences in software development so that CER's and standards may be developed for use in future efforts.

SUMMARY

It is imperative that the validity of software cost estimates be improved. The experiences and resultant recommendations generated by this report will facilitate the accomplishment of this improvement.

TITLE: "RISK ANALYSIS AND COMMUNICATIONS COSTING: A PRACTICAL APPLICATION"

AUTHOR: Mr. John Bezner

ABSTRACT: This paper outlines the risk analysis techniques used to aid in developing a cost estimate for a large scale communications system. Included is a discussion of the methods used to quantify the uncertainty associated with both schedule and technical aspects of the project. The methods used include Advanced Solvnet, Delphi, a specially tailored monte carlo simulation and subjective judgment. Special emphasis is placed on discussion of data collection, analysis and interpretation of results of the schedule risk analysis. The risk quantifications are related to cost factors to provide a measure of risk in dollars. The results of the dollar quantification are then in a form useful for sensitivity analysis and determination of a management reserve. Finally, comments concerning the apparent worth of the information obtained are provided.

"RISK ANALYSIS AND COMMUNICATIONS COSTING: A PRACTICAL APPLICATION"

Mr. John S. Bezner

U.S. Army Communications Command

1.0 PREFACE. The development of a detailed cost estimate for a large scale communications system is an exacting task. The number and variety of component members of such a system provide more than ample opportunity for errors of duplication or omission to occur. In addition, the data sources, inflation factors, cost estimating relationships, learning curve slopes, cost factors and expert judgment all contribute to the complexity of a final cost estimate. The accuracy of these individual inputs provide the boundaries of accuracy for the total. Surrounding all of these factors is something called risk, the quantifiable uncertainty associated with the future of the project. The purpose of this paper is to demonstrate the importance of a risk analysis in the development of a total cost estimate. The basis for this demonstration is an actual application of the techniques described.

1.1 INTRODUCTION. During the spring of 1977, a team of analysts developed an Independent Parametric Cost Estimate (IPCE) for a Large Scale Communications System*. In order to develop the cost estimate, basic assumptions concerning a program schedule and system configuration were required. These assumptions made it possible to organize and structure the cost analysis procedure. However, a serious cost estimate that does not include realistic research into the effect of these basic assumptions is considered to lack an important ingredient of credibility. Therefore, a risk analysis which addresses those questions of concern to the IPCE Study Team was conducted. The objective of this risk analysis was to support the costing effort as opposed to supplanting the normal close project scrutiny expected of a formal Decision Risk Analysis.

1.2 METHODOLOGY. This analysis is divided into two phases; quantification of uncertainty and application of this quantification to the costing effort. Within the first phase, there exists a further subdivision, schedule and technical risk are each given separate attention. Though techniques vary and the level of accuracy achievable is not constant, the central theme is maintained throughout the analysis. That theme is to arrive at some measurable determination of risk which will permit intelligent discussion of the impact on the cost of the project. For the purposes of this paper, one part of the risk quantification (schedule) will be addressed in detail, the other part (technical) will be covered only to the extent required to explain phase II (application to costing).

*The name of the system is withheld and the study results have been generalized in order to present the techniques without divulging privileged information.

a. Schedule Risk Analysis. The risk analysis procedure employed to examine the project schedule consists of seven parts. These parts, along with the organizational elements that contributed to the completion of each part, are summarized in Table 1. Discussion of each of these parts is provided in more detail in the paragraphs following Table 1.

SCHEDULE RISK ANALYSIS STRUCTURE	
TASK	ORGANIZATIONAL INPUT
Subtask Identification	Independent Communications Group Project Office IPCE Study Team
Schedule Model Development	Independent Communications Group Project Office IPCE Study Team
Schedule Data	Independent Communications Group Project Office IPCE Study Team
Probability Data	Independent Communications Group Project Office
Analysis Proper	IPCE Study Team
Sensitivity Analysis	Independent Communications Group Project Office IPCE Study Team
Report	IPCE Study Team

TABLE 1

(1) Subtask identification consists of determining the set of discrete activities which are required for completion of the project. This set of activities was chosen subject to constraints which require analytical manageability and a reasonably accurate representation of project reality.

(2) Schedule model development consists of identifying the proper interrelationship of the subtasks in terms of constraints and milestones. In the course of developing network models of the project, six basic alternatives were identified. (For an example, see Figure 1.) These alternatives include variations of dependency between two major subsystems (A & B), ideas offered by both data sources (the Independent Communications Group and the Project Office), and varied methods of handling administrative matters (contract award, ASARC/DSARC decision processes, etc.).

ALTERNATIVES #1 AND #2

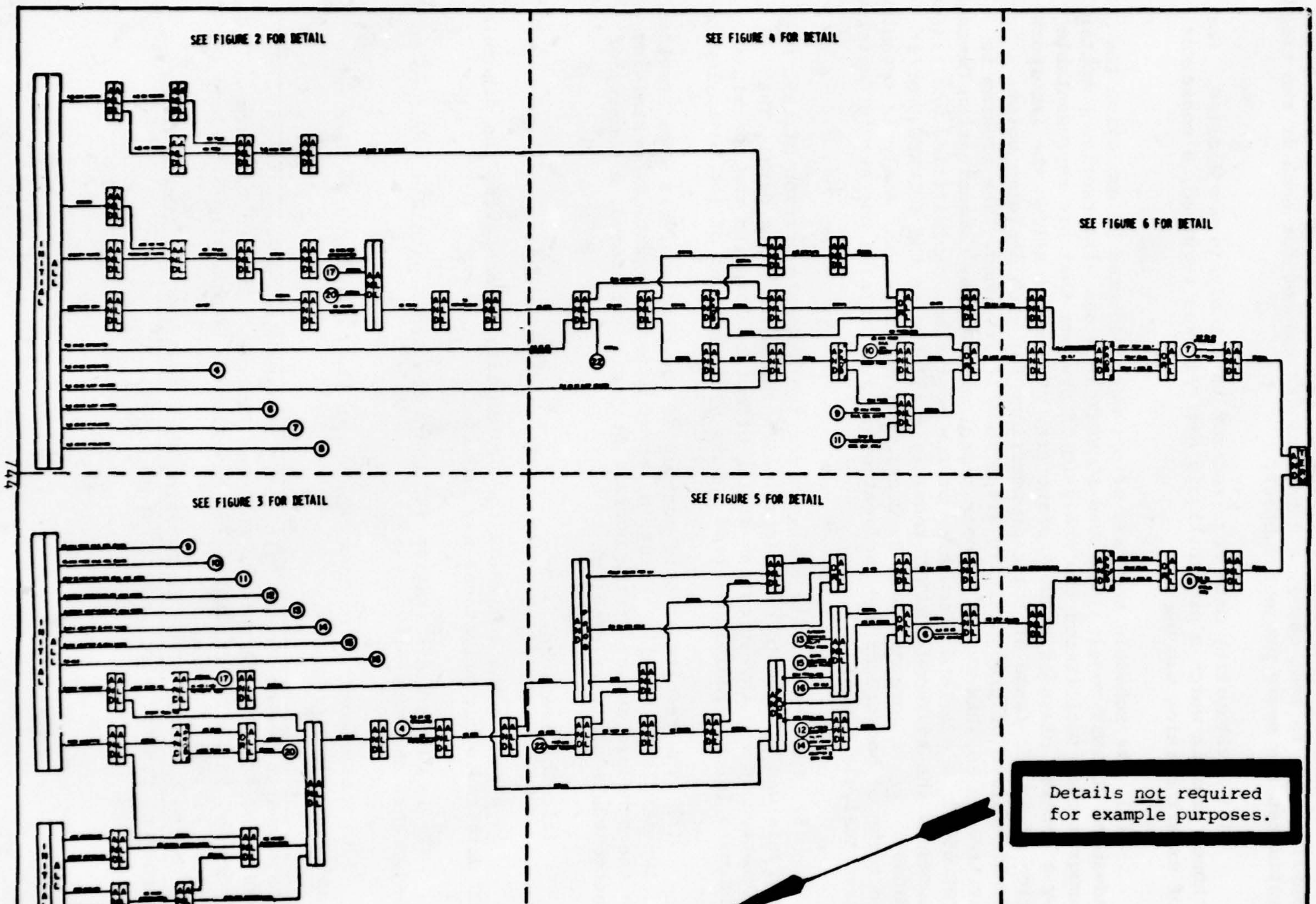


FIGURE 1 (Figures 2-6 not included)

(3) Schedule data was collected by employing a modification of the Delphi technique. 1/ Representatives from both of the data source groups participated in supplying the required schedule information. In addition, data available in the form of official project schedules were used when applicable. In most cases, a consensus was reached for each of the time estimates for every project subtask.

(4) Probability data required for this analysis was limited. For those cases in which a probabilistic estimate was required, a consensus of expert opinion was used.

(5) The schedule analysis effort was conducted by employing the Advanced SOLVNET Model. Advanced SOLVNET is a user-interactive computer program that facilitates the analysis of systems that are representable by a general class of network structure. The model allows the incorporation of a wide range of logic propositions for each decision point. Consideration of probabilistic situations is included. The program is written in FORTRAN IV and is operational on the Army Communication Command's CDC 6500. A complete discussion of the operation, capabilities and limitations of the Advanced SOLVNET Model is available in the SOLVNET User's Manual. 2/ A more complete discussion of Schedule Risk Analysis techniques in terms of background and application is available in the Army's Decision Risk Analysis Manual. 3/

(6) The sensitivity analysis for the schedule portion of this risk analysis was limited to one variation on the basic input data. That variation is the investigation of the effect of lowering the optimistic estimate for all individual time inputs by 10 percent of the basic input.

(7) The results of this portion of the risk analysis were provided in two levels of detail: First, a set of major observations pertaining to the schedule aspect of the overall project; and second, a summary of the schedule analysis results.

(a) Observations.

1 Based on the analysis, the probability of achieving the scheduled IOC date and other milestones was provided.

2 In all but Alternative #3, the subsystem A path was found to be longer than the subsystem B path.

3 The support activities related to the parent program were not constraining the program.

1/"Systematic Use of Expert Opinions," Olaf Helmer, November 1967.
2/Advanced SOLVNET, "A Network Analyzer Program," Stephen R. Pearcy, Report No. PAPAS-14, January 1973, Revision 2, February 1975. Plans Office, Picatinny Arsenal.
3/A Course of Instruction in Decision Risk Analysis, US Army Logistics Management Center, Fort Lee, Virginia, 1974.

4 Based on the analysis, the project schedule was considered optimistic.

5 In addition, a number of detailed observations concerning various stages of the project were provided.

(b) Schedule Summary. The results of the Schedule Risk Analysis were provided in graph, table and computer printout form. Massive amounts of detail are generated in such an analysis, the critical requirement is to effectively summarize the data. Table 2 is an example of summarized data.

<table>
<tr><th colspan="8">INDEPENDENT COMMUNICATIONS GROUP</th></tr>
<tr><th colspan="3">ALTERNATIVE #1
BASIC DATA</th><th colspan="5">SUMMARY DATA SHEET
SCHEDULE RISK ANALYSIS</th></tr>
<tr><th colspan="3">MILESTONE</th><th>85th PERCENTILE</th><th>AVERAGE</th><th>EARLIEST</th><th>SCHEDULED DATE</th><th>MOST LIKELY</th></tr>
<tr><td rowspan="4">SUBSYSTEM</td><td rowspan="2">A</td><td>DSARC</td><td rowspan="5">SCHEDULE DATE (MONTH & YEAR) FOR WHICH THERE EXISTS AN 85% PROBABILITY OF MEETING OR BEATING</td><td rowspan="5">MEAN COMPLETION DATE</td><td rowspan="5">MOST OPTIMISTIC COMPLETION DATE</td><td rowspan="5">TARGET MONTH & YEAR</td><td rowspan="5">MODAL COMPLETION DATE</td></tr>
<tr><td>FIELD PRODUCT</td></tr>
<tr><td rowspan="2">B</td><td>DSARC</td></tr>
<tr><td>FIELD PRODUCT</td></tr>
<tr><td colspan="3">PROJECT COMPLETION</td></tr>
</table>

TABLE 2

b. Technical Risk Analysis. The risk procedure used to examine technical risk consists of two parts; software and hardware.

(1) Software Risk Analysis Results. The nature of software development is one which inherently involves high risk. Even the simplest of programs can be plagued by false starts, program growth, system integration, timing analysis problems, etc. In fact, the complexity of analyzing software development is sufficient to have generated a special paper devoted entirely to this subject. 4/ For this reason, details of the software risk analysis are not provided here. However, it is pertinent to mention that

4/"A Software Cost Estimating Technique for Communications Systems," Stan Dunn, May 1977.

the resultant product is a set of risk factors to apply against basic cost estimates. These risk factors quantify the uncertainty associated with the software development problems mentioned above.

(2) Hardware Risk Analysis Results.

(a) Risks in the hardware segment not related to software problems or schedule problems were considered small. However, to insure no items of potential risk were overlooked in the hardware segment of the project, an analysis system based on expert opinion was developed. This system included a review of project status and a delphi-like assessment by an independent set of experts. Among the areas of hardware risk investigated using this technique are:

Engineering,
Percent Completion,
Modifications,
Space, Weight and Power,
Hardware/Software Integration,
Learning Experience,
Power Processor, and
Mix of Space Division Multiplexers and Time Division Multiplexers.

(b) From this examination, hardware risk areas identified include the need for expanded capacity, apparent excessive modifications, and uncertainty associated with the learning curve slope.

1.3 RESULTS, WHAT NEXT? At this point, a set of risk results ranging from numerical probabilities to qualitative remarks was available for schedule and technical factors. The next key task was the conversion of risk data into an assessment of its impact upon cost.

1.4 APPLICATION TO COSTING. The results of the risk analysis were used in the IPCE in five major ways.

*As a factor to apply to basic cost estimates.

*As a quantification of costs expected to occur outside the then current scope of the project.

*As a method of determining valid ranges for the sensitivity analysis.

*As a method of determining an adequate management reserve for the project.

*As a means of identifying and highlighting management issues.

Amplifying discussions and examples for each of the five uses are provided in the following paragraphs.

a. Risk Factor. In many instances, it was possible to identify a factor to apply to cost estimates developed for "all's well" situations. This process is especially applicable to labor intensive activities involved in the development of highly complex outputs.

EXAMPLE: Software Development

Total Cost = Basic Cost Estimate x Risk Factor

NOTE: For this application, the risk factor was developed by taking the mean value from a distribution developed through Monte Carlo simulation (2,000 iterations with 13 variables).

b. Outside Scope. This category consists of costs for items of equipment or work efforts not fully recognized at the time of the cost exercise.

EXAMPLE: In one instance, the investigation revealed that, though not planned for, an additional component would be required for effective operation of the system. This component involved expanding capacity of existing equipment and the entire cost of the new element was added to the total system cost.

c. Sensitivity Ranges. Typically, the ranges employed in a sensitivity analysis degrade to an arbitrary ± X%. In most cases, this results from the absence of any better way of assigning end points. Contrast this to the process used for this study in which the risk analysis results were used to examine each critical variable to determine the most reasonable range.

EXAMPLE: Consider the schedule question. What variation from the planned schedule is meaningful and how is this time variation converted to cost?

After careful evaluation of the six schedule alternatives, Alternatives #1 and #5 were selected as the most realistic and appropriate for this analysis. Alternative #1 represents the Independent Communications Group's more optimistic input (still pessimistic compared to the Project Office's input). Alternative #5 was selected as the most acceptable input from the Project Office. These two inputs were selected based on their merits (e.g., project knowledge, bias, effort expended, understanding of the SOLVNET technique, etc.).

Then, for purposes of establishing a range of values for schedule slippage, the quickest date experienced for Alternative #5 was selected as the earliest possible completion date, and the latest date experienced for Alternative #1 was selected as the latest possible completion date (see Figure 2).

Now, given the maximum reasonable end points of schedule range, these end points were compared to the expected completion dates within the R&D Phase and the Investment Phase of the project.

Finally, a cost factor per month for schedule slippage was determined by cost phase and applied against the number of months of expected variance from the official schedule.

d. Management Reserve. After a set of sensitivity ranges have been developed to demonstrate the potential effect of the unknown, a critical question must be asked. How much of what can go wrong will go wrong? More directly, what portion of possible cost increase should be set aside as management reserve to avoid critical funding problems? The results of a risk analysis go a long way towards helping to answer this question. Quantification of uncertainty makes it possible for decisionmakers to supplement subjective judgment in a positive manner.

EXAMPLE: Consider again Figure 2. For determination of management reserve for schedule, the arithmetic average of the modes of Alternative #1 and #5 was recommended. That is, the number of months difference in the averaged modes of #1 and #5 and the scheduled IOC was multiplied by the appropriate cost per month factor to arrive at a management reserve for schedule.

e. Management Issues. In many cases, it was possible due to the risk examination process to point out to management various potential problems that given immediate attention, would obviate the need for drastic corrective procedures.

EXAMPLE: In one instance, the need for advance planning for billeting requirements to support testing was identified. This identification will make it possible to avoid a potentially very expensive solution.

1.5 SUMMARY AND CONCLUSION.

a. The results of the Risk Analysis were used to provide several important ingredients to the completed Independent Parametric Cost Estimate. Among these ingredients are basic cost factors, sensitivity ranges and point estimates for management reserve.

VALUES FOR SCHEDULE SENSITIVITY RANGE AND FOR MANAGEMENT RESERVE

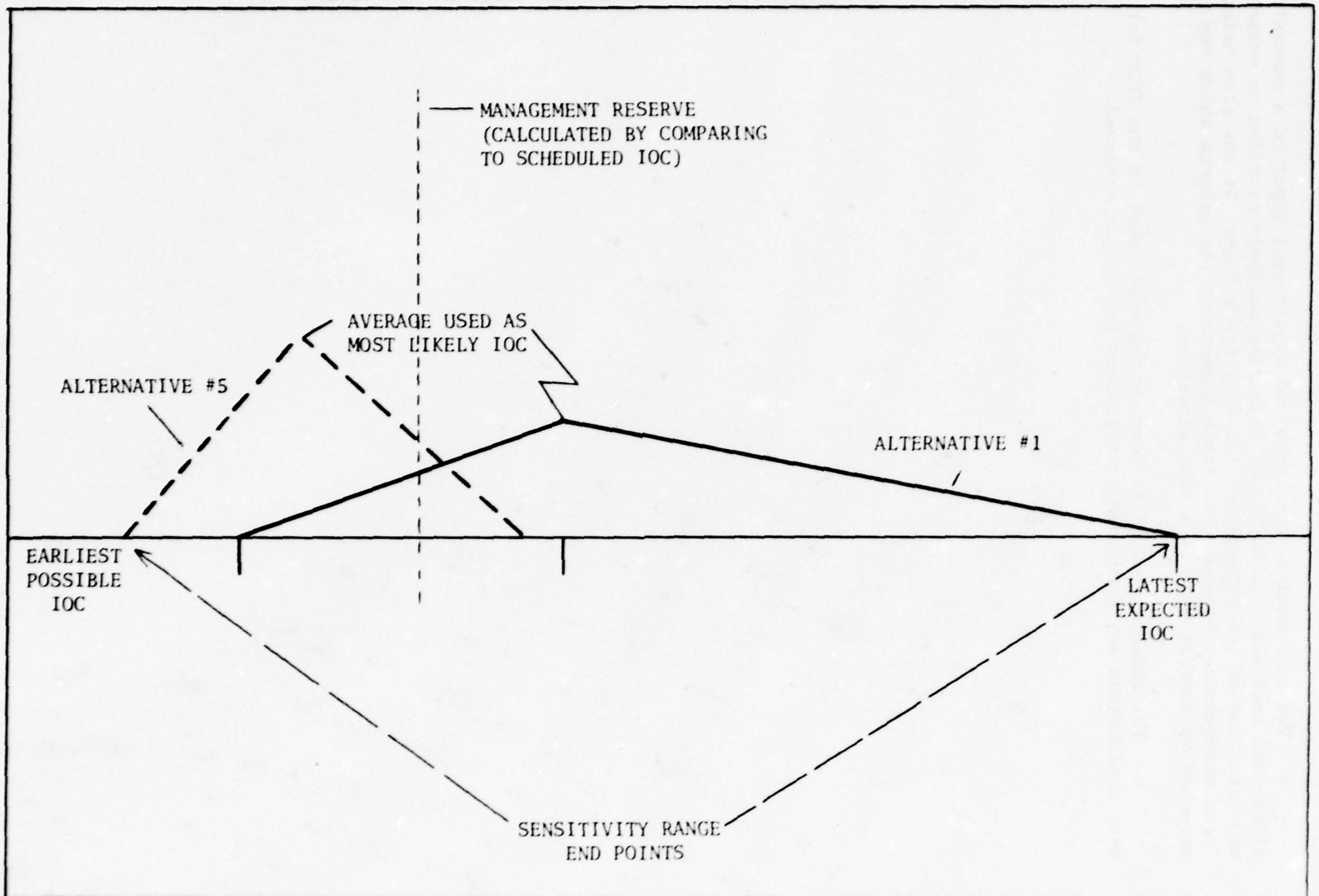


FIGURE 2

b. The risk analysis has provided significant input to a communications cost analysis. In addition, this risk analysis provided an enhanced understanding of the project for the IPCE Study Team. It has also helped focus management attention upon those aspects of the program which can be expected to benefit most from that attention.

c. The effort required to develop the risk input to the IPCE for this application was more than justified by the output achieved.

TITLE: A Decision making game to establish the value of various types of Battlefield data.

Author: D W Daniel Defence Operational Analysis Establishment UK

ABSTRACT: The purpose of developing and investing in those systems that acquire and process battlefield data is to improve the quality and timeliness of decisions made by the battle-field commanders. 'Data' in this context describe symbols, words or numbers which only become 'information' if they influence the course of action taken by a decision maker. It follows that not all 'Data' are necessarily 'information'. The problem facing any analysis of future Command and Control systems, or those parts of them concerned with collecting,transmitting and processing data, is to find a way of assessing their value in terms of how they improve battle outcome. What will be described in the paper is the development and application of a highly controlled, yet very simple, Operational Research gaming technique. This technique has been successfully applied to discovering how Military Officers are influenced in making deployment decisions by various types and level of data.

A DECISION MAKING GAME TO ESTABLISH THE VALUE OF VARIOUS TYPES OF BATTLE FIELD DATA

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INTRODUCTION

1. Ackoff and Emery in their book (1) 'On Purposeful Systems' define information as: a communication that produces a change in any of the receiver's probabilities of choice, informs him and hence transmits information. If the word 'data' is used to describe the contents - symbols, words or numbers - of a communication then it follows that not all 'data' are necessarily 'information' for not every part of every communication influences a receiver's perception of the choice of option open to him.

2. In Defence there is considerable momentum behind the development of so-called 'information' systems (systems that acquire, transmit and process battlefield data). Their purpose is to improve the quality and timeliness of decisions made by battlefield commanders. But do they?

3. It seems fair comment at the moment to say that Operational Analysts can do little about assessing the value of investment in these systems if by value one means how they improve the outcome of battle. This is for lack of an understanding of what 'data' are 'information'. Because data become information only in the mind and perception of the recipient - a person - such an understanding can only be obtained from a basis of experiments conducted on people. In Operational Research terminology these experiments are called games. Although games have been played in military contexts for years, few of them have been structured in a way that enables quantitative analysis of decision making to be made.

4. One game designed to do this is described in this paper. The ideas behind it rely heavily on those of K C Bowen who in a monograph, now in press (2), reviews the use of games and presents two classific-ations, one for their purpose, the other for their structure. The purpose of the game described in this paper is research. The object is to establish a technique to investigate the effect of data on the quality of Command decision making and hence on the outcome of battle. To obtain quantitative results the structure of the game is highly controlled, it is also simple. In the game one person represents a BLUE (friendly) Commander, plays against a pre-determined RED threat (the enemy) and makes one decision, at this point the game is terminated and analysed. In Bowen's classification (2) this is described as a 'one person non-modifiable game'.

5. No doubt it will be thought that such a simple game is unrealistic. It is hoped that this paper will dispel any such ideas. In fact there are other examples of games of similar structure quoted in (2) all of which have been played successfully, though in different contexts.

And with a little ingenuity the player may be unaware of the constraints placed upon him. The reason for choosing such a simple approach is that, having reviewed the possible spectrum of games from purely automaton or deterministic ones (playing with decision rules in models) to those involving both RED and BLUE players (traditional war games), it appears to be the one that can produce quantitative results in practical time-scales. Purely automaton games in which various decision rules are examined, have been used in deciding what decisions have the greatest potential payoff and are, therefore, worth analysing. But automaton games, by their nature, do not reveal anything about the behaviour of people nor what use they make of data. Two sided war gaming has been rejected because of the effort needed to replicate sufficient games for statistical validity. Indeed the thought of any form of inter-active gaming has been put to one side until some basic understanding can be gained of how to play and control simple games.

6. The rest of this paper will describe one particular 'one person non-modifiable' game, how it is played and what sort of results can be obtained from it. Some results are presented from the first series of games played. For these games players were drawn from readily available personnel. They did not have direct experience of the Command role they were asked to play. No conclusions drawn from their results, therefore, can be regarded as applying to the real world until they are validated. For the moment they demonstrate a technique, stimulate discussion and suggest hypotheses that might be the subject of future games.

DESCRIPTION OF THE GAME

7. During the game a player sits at a table in a room which has suitable maps pinned to the wall. He is told that he is to play the role of a land force Commander and to decide when and where to deploy a reserve formation in such a way as to halt the enemy advance, or delay it for as long as possible, and to inflict the maximum casualties, within nominated boundaries. He is given data on the progress of the battle in the form of map overlays and written briefs. The written briefs simulate what, in reality, would be mostly verbal Intelligence or Operations reports. They are written so that the experimenter knows that each player receives exactly the same data. It is felt that verbal briefs, though more realistic, are difficult to control and more liable to transmit nuances.

8. Battlefield data are heterogenous and for an initial study it was decided to group all types into one of four sets:

a. Prior Intelligence - data on RED strengths, locations, activities and intentions collected before battle commences.

b. Far RED Intelligence - data on the strength, locations, activities, intentions and groupings of RED units out of contact with BLUE forces.

c. Near RED Intelligence - as in b. above but for RED units in contact.

d. BLUE Intelligence - Data concerning friendly forces.

In this initial study two levels of each of the data sets were investigated, they are described in this paper as 'High' and 'Low', and they correspond to a military appreciation of the maximum and minimum that could be realistically played. As a rough guide 'High' corresponds to 80% of all the possible data and 'Low' corresponds to 20%.

9. At an early stage of game design it was decided that some random inaccuracies or errors should be introduced into the data, and players warned of this, to reduce the artificiality that would result if players believed that every item of data was accurate. In the event, it was found that the inevitable errors that cropped up during the initial data preparation were sufficiently numerous, and so realistic, that no deliberate errors were introduced. Proof reading of data briefs and maps overlays was, therefore, confined to correcting only obvious map and message errors. However, whether or not inaccuracies are introduced into any game, it is suggested as a precaution that players are always told not to regard every piece of data as necessarily correct.

Data Presentation

10. A preliminary brief covering BLUE forces, battle plans and deployments, plus whatever level of prior intelligence on RED forces are being played, is given to players on the day before playing the game. On the day of the game all data are supplied as a series of written briefs followed by an updated map overlay. Overlays are provided at fixed intervals of battle and between overlays players receive 9 data sheets. These 9 sheets are always presented in order:

a. Sheets 1, 4 and 7 - G Ops report (BLUE data).

b. Sheets 2, 5 and 8 - G Int report (near RED data).

c. Sheets 3, 6 and 9 - G Int report (far RED data).

Although each data sheet is given a separate date/time group they are bunched into three sets of three and not spread evenly in time so that, at any moment in game time, the player has a complete picture of what is going on. They are presented separately so that different combinations of data level can be played by simply exchanging data sheets. Map overlays, for similar reasons, are constructed as a base displaying low data with supplementary flips which convert low data to high data.

11. Combination of Data Sets Examined. With four data sets at two levels there are 16 possible combinations of data that could be played. In order to keep the initial study to reasonable proportions it was decided to study only nine of these. All combinations that have three data sets at a low level were excluded on the grounds that the experimenters had a preconception that combinations with more data than this are needed for good quality decisions to be made. However the combinations with four low data levels was included as a control. In addition the combinations in which BLUE data are at a low level and Near RED are

at a high level were discarded because it was felt that players would find this combination unrealistic. The data combinations used in the initial study are shown in Table I.

TABLE I

DATA COMBINATIONS FOR PART I OF THE STUDY

Number	Data Set			
	Prior	Far RED	Near RED	BLUE
1	High (H)	H	H	H
2	Low (L)	H	H	H
3	H	L	H	H
4	H	H	L	H
5	L	L	H	H
6	L	H	L	H
7	H	L	L	H
8	H	H	L	L
9	L	L	L	L

Experimental Design

12. In any experiment of this sort the analyst must be able to identify the effect on the results that the natural variations between players will cause as well as the primary subject under investigation which, in this case, is the level and type of data. To do this, either many games with each data combination are played, and the results averaged over the players, or each player plays more than one game. In the latter case to avoid unwanted interactions between the data presented to a player in one game and data used in the next, different scenarios must be constructed, with players playing one game in each scenario. For the initial study it was not practical to play many games so the games had to be arranged in such a way as to be amenable to analysis. Many formal statistical designs exist and after much discussion it was decided to use a Lattice Square design. This design requires only six players and three scenarios for some analysis to be carried out on the nine data combinations. It also has the benefit that it can be repeated with six more players at a later date to give more detailed results if required. An understanding of the design is not necessary in order to follow the arguments in this paper, however for those interested details can be found in (3) and (4). Table II shows which players played which data combination in which scenario. It is recognised that at this stage it is not possible to check the validity of assumptions on normality, additivity of errors etc. But the choice of this design does not preclude the use of non-parametric tests which do not rely on these assumptions and a number of such tests have been used.

TABLE II

PLAYER - SCENARIO - DATA COMBINATIONS

Player	Scenario		
	1	2	3
A	3	7	1
B	2	9	5
C	6	4	8
D	1	9	6
E	4	3	5
F	2	8	7

NOTE: The data combinations played by player B are repeated by other players in the same scenarios. This is an accident of the design and is of no special significance.

Scenario Preparation

13. In the three scenarios a different RED threat was posed against an unchanging initial BLUE deployment. A Model of Combat was used to simulate the progress of battle up to a point at which RED broke through the forward BLUE forces with the reserves uncommitted. This provided a basis for constructing the briefs and map overlays for the games as well as a means for analysing, in a consistent quantitative way after the games had been played, the deployment decisions made by the players.

Planning Aids

14. Each player is required to deploy the reserves from an initial concentration area in such a way as to delay the enemy and cause maximum casualties within fixed boundaries; in doing this he is not permitted to change the concepts of operations or plans he has "inherited". Two methods of planning the deployment of the reserves from the concentration area are available to players:

a. Contingency Plans. The initial Planning Instructions supplied to the player before the game give details of a number of realistic contingency plans and specifies the deployment time for each.

b. New Plans. In order to give the player flexibility to deploy reserves into areas not included in the contingency plan, he is given a simple planning aid. This is a measuring rule giving movement times with distance, on the same scale as the map, for all or part of the reserve, plus a statement of the amount

of time that must be added for reconnaissance, digging in etc. By pinning this on the map during the game the player can rapidly calculate how long it will take to deploy to any location.

End of the Game

15. The game ends when BLUE reserves are inevitably committed to fighting RED in a position selected by a player, or RED breaks through the forward BLUE defences. Games continue until this point for, in some situations, players' deployment plans could be altered as a result of fresh information that becomes available between the reserve being ordered to move and its arrival at the selected point. If conditions would realistically allow a change in plan to be put into effect, then this is allowed. In some cases it is quite difficult to decide when to terminate or, alternatively, to continue a game in which a player has made an early decision, without giving him information or making him suspicious of his decision. The reason for wanting to terminate a game as soon as possible is to minimise any feed-back to the player until he has played all three games. Although only one decision is being analysed players tend to make the decision in stages by committing reserves piecemeal.

PLAYER REACTIONS

Subjective

16. At a discussion following the game series all players said that they enjoyed the games and thought that they were worthwhile. They thought that the scenarios were realistic and that the problem was not trivial. Every effort was made to inform the players that some theories on the use of data were under examination and they were not being tested as individuals. However players still felt that they were under pressure to do well and were observed to be particularly tense in their first game. Every game was monitored by an observer from the experimental team and, surprisingly, there appeared to be no obvious "rule-playing" or gamesmanship.

Quantitative

17. To help gauge the reaction of players to various data levels, to assess possible learning effects and check on consistency of players, they were all timed whilst reading the data. It was emphasised to players that although they were being timed it was not a race and they were to take their own time. A considerable quantity of data on the time taken by players to play games and read the data briefs was collected and analysed. In summary these data show a high degree of consistency in the approach to the game by the individual players. There is also strong evidence to suggest that any learning effect is absorbed during the early part of a player's first game. This occurs well before he has to take a decision and is based on the observation that players took a longer time to play the earlier stages of their first game compared to the series as a whole. There is also some evidence, reinforced by subsequent experiments, that players who make the best decision take longer than average to play the games.

RESULTS

Introduction

18. There are a number of ways of analysing the results of the quality of decisions taken by the players. The one presented in this paper is derived from simulating the players' decisions in the Model of Combat which is used to generate the scenarios. During this analysis it became apparent that a player could make good judgments about the direction of the threat but poor judgments on timings, so that although he knew where to send his reserves he did not know when to send them. This can be deduced because players were asked to give warning orders to their reserves as the game progressed, as a good Commander would do in reality. It is, therefore, possible to analyse intended deployment as well as actual deployment.

Presentation of Results

19. All of the results from the initial experiment are shown in figure 1. Two measure of performances are used:

a. the additional number of RED units destroyed;

b. the additional delay caused;

compared to what would have happened if the reserve had not been deployed. These are expressed on a percentage of the maximum possible scores. They are the usual measures of battle employed with the model. The reason for using two measures is that they are not always consistent though they appear to be so in these results. The figure includes results for a test player and an additional point for each scenario. This additional point is an arbitrary basis for each scenario obtained when the reserves are deployed evenly across the front. The reason for including this is explained in paragraph 21. It can be seen from this figure that some results stand out as being worse than the others. The points 3T, 2D and 2B are the cases where players received all low levels of data, these results clearly show that data does have some effect on the player's ability to deploy his reserve. The result of Player D in Scenario 1 (point 1D) seems anomalous. In the first scenario with all high levels of data Player D deployed too late for the reserves to play any useful part in the battle. In the second, with all low levels of data he again deployed late, in the third his deployment was better, and achieved a good result. The poor result in scenario 2 might be expected with the all low data level, but the reason for the first result is unknown. It is impossible to say from the games played whether the player is a late deployer, or whether he was affected by nerves or just took longer to learn his role in the game. It is most likely to be the first as he performed well in assessing the threat (see paragraph 18) and seemed no more on edge than the other players.

Analysis of Results

20. The analysis of the results that follow is almost all based on the measure of additional RED units destroyed, similar tests carried out on

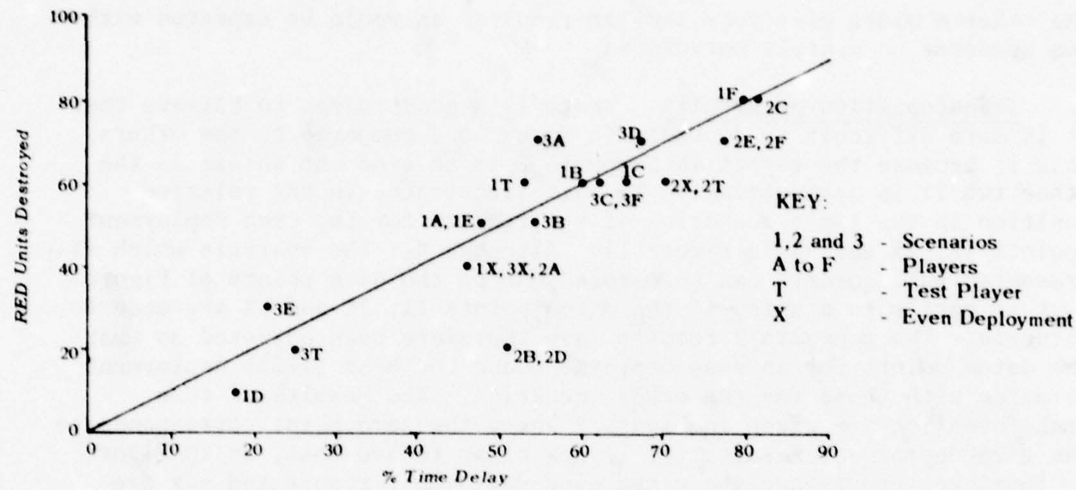

FIGURE 1: QUALITY OF DECISION - MODEL RESULTS (INCLUDING TEST PLAYER)

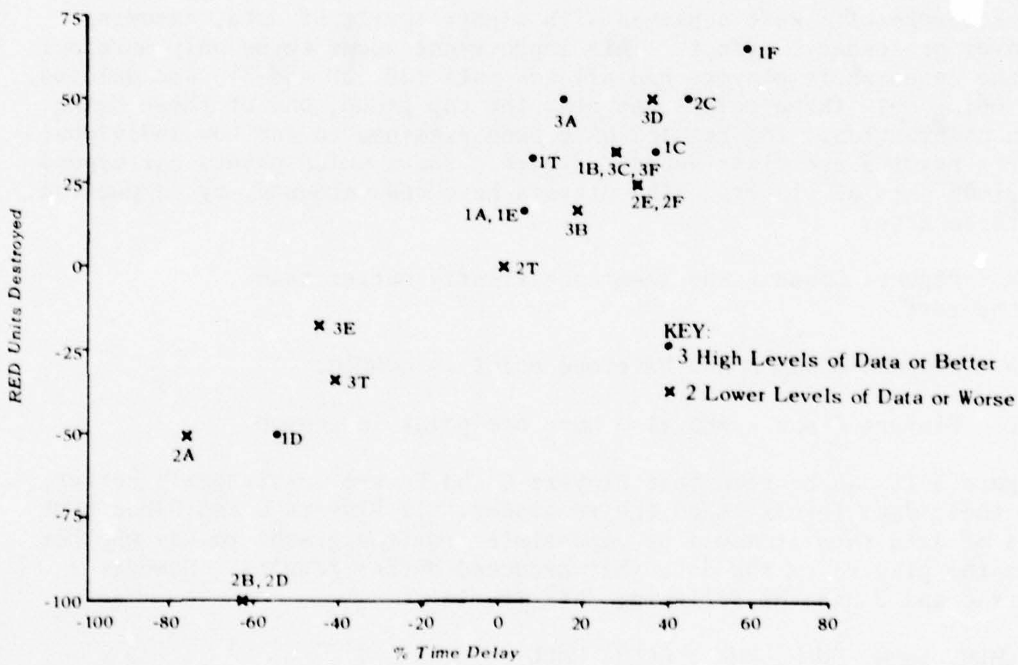

FIGURE 2: QUALITY OF DECISION - RESULTS ADJUSTED FOR SCENARIO 2 (INCLUDING TEST PLAYER)

the other measure give very similar results, as would be expected with two measures so closely correlated.

21. Transformation of Results. There is a good reason to believe that it is more difficult to do badly in scenario 2 compared to the others. This is because the threat in Scenario 2 is an even one whilst in the other two it is asymmetrical. This is illustrated in the relative position in the three scenarios of the results for the even deployment (points 1X, 2X and 3X in Figure 1). Although all the analysis which is presented subsequently can be carried out on the data points of Figure 1 it is easier to display if the datum points 1X, 2X and 3X are made to coincide. The scenario 2 results have therefore been adjusted so that the datum points for an even deployment and the best (100%) deployment coincide with those for the other scenarios. The results of this transformation are given in Figure 2 where the zero point corresponds to the even deployment result. It is now clear to see that, as in Figure 1, fourteen results are above the even deployment result and six are below.

22. Some Observations on the Results. The points have been marked to show those which were achieved with a fairly high level of data (3 or 4 high levels) and those with low levels. Apart from 1D it can be seen that better results were achieved with higher levels of data, assuming no player or scenario effect. This improvement seems to be only marginal when the cases where players had all low data (2B, 2D and 3T) are deleted, there being only three points not near the top group, one of these being a high combination. The results have been examined to see how individual player's results are distributed. Figure 3 shows which points correspond to various sets of players. The players have been grouped, by inspection, into three sets:

a. Players C and F who seem consistently better than the rest.

b. Players B and D who have one point in common.

c. Players A and E who also have one point in common.

In Figure 3 it can be seen that Players C and F were consistently better, given their data levels, than the remainder. If Players C and F had high levels of data then it would be impossible from the graphs to say whether it was the players or the data thatmproduced better results. However Players C and F had the following data levels:

LHLH, LHHH, HHLH, HHLL, HLLH, HHLL

that is four with two of the data sets at a low level and two with only one data set at a low level. This indicates that Players C and F did well despite the levels of data supplied and that, given no scenario effect, it seems that the better the data the better their decisions. As for the remainder, they did no better, on average, than they would have done had they followed the simple rule of deploying evenly across the front. Figure 4 shows which results correspond to each scenario. From

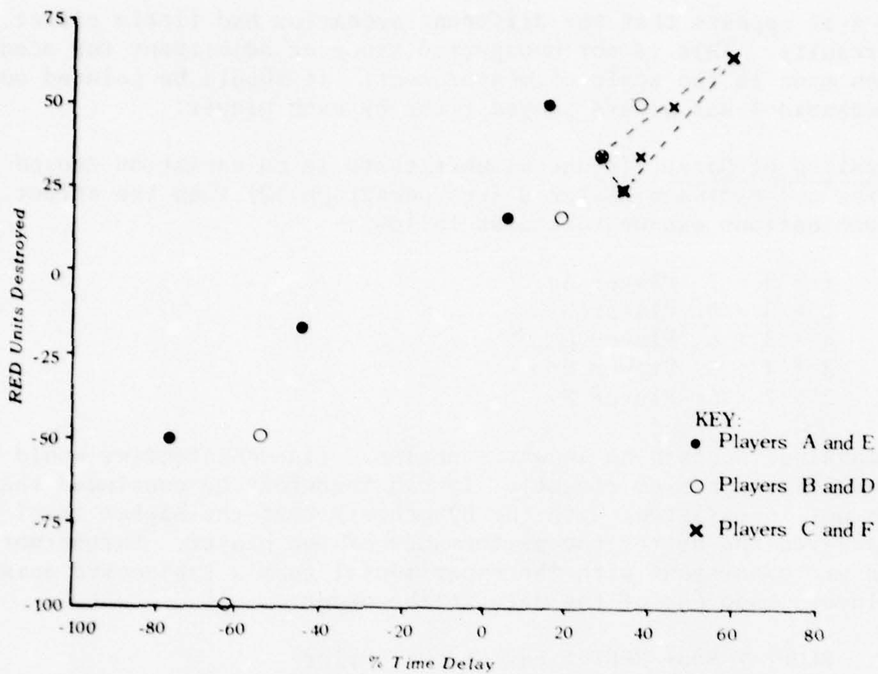

FIGURE 3: QUALITY OF DECISION - RESULTS GROUPED BY PLAYERS (EXCLUDING TEST PLAYER)

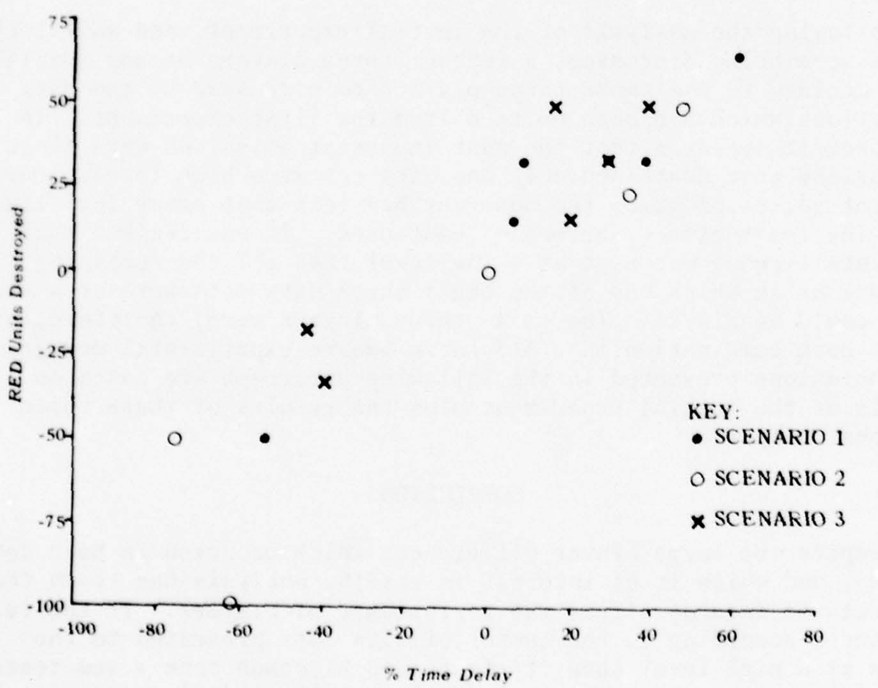

FIGURE 4: QUALITY OF DECISION - RESULTS GROUPED BY SCENARIO (INCLUDING TEST PLAYER)

Figure 4 it appears that the different scenarios had little effect on these results. This is not unexpected since an adjustment for scenarios has been made in the scale of measurement. It should be pointed out that Scenario 1 was always played first by each player.

23. Ranking of Data. If one assumes there is no variation caused by scenarios and excludes Player D (see paragraph 19) then the effect of data combinations can be ranked as follows:

1 > 3 > 7 Player A
2 > 5 > 9 Player B
4 > 8 > 6 Player C
3 > 4 > 5 Player E
2 > 7 > 8 Player F

These rankings contain no inconsistencies. (Inconsistencies would be expected if data has no effect). It can therefore be concluded that the data is not inconsistent with the hypothesis that the higher level of data received the better the performance of the player. Furthermore the results are consistent with the experimental team's subjective opinion that players made use of the data in the order:

BLUE > Near RED > Far RED > Prior

Further results

24. Following the analysis of the initial experiment, and whilst the results were being discussed, a further three players became available. It was decided to use these three players to play some of the data combinations which had been omitted from the first experiment. In retrospect it appeared that the most important omissions were those data combinations that contained only one data set at a high level. During the first series of games the observer had felt that prior intelligence was having least effect, or being least used. It was decided that if prior intelligence was kept at a low level then all the remaining combinations in which one of the other three data sets were at a high level, could be played. The next three players were, therefore, used to play each combination in a 3X3 Latin Square experimental design. The conclusions presented in the following paragraph are based on analysis of the initial experiment plus the results of these three additional players.

CONCLUSIONS

25. Despite the large player differences which occurred in both sets of games, and which is of interest in itself, analysis has shown that the levels of data do affect the performance of players. If the results are grouped according to the number of data sets presented to the players at a high level then 't' tests and Wilcoxon rank - sum tests (the latter being non-parametric) show significant differences (at the 97.5% level). Significant differences occur between games played with all data sets at a low level and those with one high and three low sets. Significant differences also occur between the 'one high three low'

games and the 'three high one low' games. The average result for the 'two high' games lies between the 'one high' and 'three high' cases but the differences between them are not significant. From this it can be firmly postulated that players, irrespective of the quality of their own overall achievement, perform significantly better the higher the level of data supplied.

26. In addition the results have been analysed to try to detect any significant differences between data types. The tests show that raising one data type (other than prior data which was not tested) from the all low case always leads to a significant (95% level) improvement in the results but that no one data set gives rise to a more significant increase than any other. It is clear from this that one type of data is being substituted for another and that in the particular command decision examined there is no one set of data which is of overriding importance. Because of the small sample it is impossible to analyse whether the value of one type of data is affected by the presence or absence of another set. Many players believe that combination effects such as these are important and have proposed various hypotheses One such hypothesis is that high levels of far RED data are only of value when combined with high levels of prior intelligence. This particular hypothesis can be shown to be false from the conclusion reached at the beginning of this paragraph. Other hypotheses will go untested until more data is available.

CAVEATS

27. As the games are intended to evaluate only certain types of data other factors that would normally influence the decision-maker need to be stated and clearly identified as not being part of the game. While this is apparently quite straight forward from the analytical point of view, it must be appreciated that command entails the complex balancing of numerous factors and when restricted to considering only three or four of them, the player acting as commander may produce a different answer to that which he would have produced as a real commander.

28. The games are designed so that the commander can make the critical decisions in the absence of the normal headquarter staff. This situation is unrealistic as the player/commander is devoid of normal specialist advisers and he has no filter between him and a certain amount of unprocessed data. This may possibly lead a player to perform the task of intelligence staff officer, perhaps at the expense of his ability to make command decisions.

FOOTNOTE

29. The author and his study team are convinced that this type of game is sufficiently realistic to provide a framework for the study of battle-field decision making. It is simple, quick and cheap to develop and run. It provides the opportunity for establishing a foundation of knowledge of human decision taking, based on quantitative, statistically valid, results. But even so the amount of painstaking analysis that needs to be done on

resolving fundamental questions is daunting. One way in which it may be possible to extract more information from each game is to ask the player to say what decision he would have taken at various decision points in the game even though these decisions, unless they coincide with the pre-determined scenario, are not implemented. In other words to get the player to act as an adviser rather than decision taker. The device or trick of making a player believe he is advising another player, remote from him,was first proposed in a Sussex University paper (5) as a way of introducing players to pre-determined situations and to keep a player interested in playing a game in which his decisions are not implemented. If it can be shown that in these circumstances the advice a player gives coincides with the decision he would have taken then more than one decision point can be investigated in each game. This possibility is the subject of an investigation by the study team at the moment.

Any views expressed are those of the author and do not necessarily represent those of the Ministry of Defence.

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THE SELECTION AND ANALYSIS

OF

TRAINING MEASURES OF EFFECTIVENESS

by

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ABSTRACT

In January 1976, CDEC was directed to expand its testing program to include the evaluation of training programs, devices and techniques. Consequently, we now incorporate training objectives in our Combat Developments Experiments and have begun to design discrete training tests. Since there have been few prior attempts to collect quantative training data, the selection of training MOEs can be a difficult task. In addition, qualitative training information can provide valuable assistance in the evaluation of training programs, and should, therefore, also be collected and reported as part of the field test. This paper discusses how training has been incorporated into CDEC's mission; gives examples of training EEAs and MOEs which have been used at CDEC; and concludes with a presentation of training results from some recent CDEC experiments.

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I. INTRODUCTION

In January 1976, TRADOC tasked CDEC to expand its testing program to include the collection of data which can be used to support the development and evaluation of training programs, devices and techniques. Specifically, CDEC training support now involves: The conduct of discrete training tests; the inclusion of training add-ons to Combat Development Tests (for example, the collection of ARTEP information); the review of selected prior experimentsfor the purpose of abstracting training information.

- Conduct Discrete Training Tests
- Include Training Objectives in Combat Development Tests

CDEC Training Support

In order to comply with our new task, CDEC training activities have focused on four major areas. They are: Attendance at various coordination meetings; compilation of training data from prior CDEC tests; incorporation of training objectives in combat development experiments; analysis of training data from selected experiments. Each of these areas will now be addressed in detail.

- Coordination with other TRADOC Agencies
- Compilation and Dissemination of Training Information from Prior CDEC Tests
- Collection of Training Data
- Analysis/Evaluation of Training Data

CDEC Training Activities

II. COORDINATION ACTIVITIES

Initial CDEC Coordination Activities were for the purpose of determining how we could support the training community. As a result of our early activity, we developed lines of communication, established points of contact and identified potential users of our training data. Following this initial period, we then undertook to brief interested TRADOC agencies on

our testing facility and to explain which training areas could best be supported by CDEC (e.g., data from instrumented engagement simulation exercises). Our recent activity has involved participation in meetings designed to coordinate the efforts of various TRADOC agencies involved in the development, collection and use of training information. This latest activity should greatly assist the training community by minimizing the duplication of effort which can result when programs are developed in isolation.

DATE	AGENCY	PURPOSE
JAN 76	HQ TRADOC	Determine CDEC Involvement in Training
JAN 76	TCATA	Observe Reorganization
FEB 76	USAIS	Engagement Simulation Conference (MILES)
MAR 76	HQ TRADOC	TEA Guidance
MAR 76	HQ TRADOC	WSTEA Conference
MAY 76	CACDA	WSTEA Conference
OCT 76	USAARMS	Determine Major TD Projects
MAY 77	TCATA	Review Operations
MAY 77	Ft Sill	TRADOC/FORSCOM Training Conference
JUN 77	HQ TRADOC	Meet Action Officers - Discuss Training Activities
MAY 77	USAADS	Training Briefing/Action Officer Coordination
JUN 77	USAFAS	Training Briefing/Action Officer Coordination
JUN 77	USAARMS	Training Briefing/Action Officer Coordination
JUL 77	HQ TRADOC	Training Briefing/Action Officer Coordination
JUL 77	TSM-TES/ARI	ARTEP Working Group
JUL 77	Nellis AFB	TRADOC/FORSCOM Training Conference
JUL 77	TSM-TES/ARI	TDWG for ATES-FYTP
SEP 77	ARI	IPR on Evaluator's Guidebook for Conducting ARTEPs
SEP 77	HQ TRADOC	ARTEP Coordination Meeting
SEP 77	CAC	NTC Development Conference

Prior CDEC Training Coordination Activities

III. DATA COMPILATION ACTIVITIES.

In light of our new assignment, prior CDEC tests were reviewed in order to determine if additional information, specifically addressing training, could be abstracted. We identified five areas for further review and currently plan to publish the following Training Reports this year:

- TOW Training Report (Published March 1977)
- Tank Training Report (Draft Completed)
- Detection/Acquisition/Ranging Training Report (Draft Completed)
- Aviation Training Report (Research in Progress)
- DRAGON Training Report

Although each report is organized in a slightly different manner, all will contain both quantitative and qualitative training or training related data. Each training report is intended to provide a concise review of training information collected in prior CDEC tests. For example, the TOW Training Report which was completed in March, contains two sections: The first describes the TOW training program conducted for the Tactical Effectiveness of Mine Fields in the Anti-Armor Weapons System (TEMAWS) Experiment; the second section lists TOW training information collected from four previous CDEC experiments. Some examples are listed below.

- TOW gunner performance is degraded when a blast simulator is used (TEMAWS TOW Training Program)
- More difficult for ATM gunners to track target during live fire than during training (TETAM)
- Expended ATM guidance wire on battlefield will be hazardous (TETAM)
- Need portable TOW battery charging system (ATMT)
- Aim point for target whose aspect is continually changing not specified in training programs (ATMT)
- Ranging more difficult than tracking (TAHOE)
- Difficulty transferring from visual to optical acquisition (TAHOE)
- Two weeks required to transfer skills of qualified TOW gunner to erectable systems (ITV)
- Tracking proficiency was increased by use of inexpensive training aid (Emerson)(ITV)

IV. COLLECTION OF TRAINING DATA

The third, and most important area of CDEC activity, is the collection of training data from our field experiments. This has involved supplementing previously scheduled Combat Development tests with training add-on requirements and the development and execution of discrete training tests. Both quantitative and qualitative training information can be collected from these tests. However, since there have been few prior attempts to collect quantitative training data, the selection of training MOEs has been a difficult task.

A. Training Add-Ons

Initially, CDC Pam 71-1, "Force Developments, the Measurement of Effectiveness", was used as a reference and relatively simple training add-on objectives were specified. Examples of some early training EEAs and MOEs are listed below:

TOW Against Helicopter Operational Experiment (TAHOE)

EEA 1 • How is TOW gunner performance affected by learning throughout conduct of the test?

- MOE 1 • Average gunner tracking error at end of record trails-
  Average gunner tracking error at beginning of record trials.

- MOE 2 • Distribution of gunner tracking error as a function of number of trials per player or group.

EEA 2 o How does fatigue affect TOW gunner performance?

- MOE 3 • Average gunner tracking error during 1st trials of the day-
  Average gunner tracking error during last 2 trials of the day.

- MOE 4 • Distribution of gunner tracking error as a function of number of trials per player or group per day.

Evaluation of the Frontal Parapet Foxhole, Part VII (PARFOX VII)

EEA 1 • How is player survivability affected by training over the duration of the trials?

MOE 1 • Multiple evaluation (e.g., time until casualty, range until casualty, etc.).

EEA 2 • Are the attitudes of player personnel towards the effectiveness of the parapet foxhole changed as they become better trained and more experienced?

MOE 2 • The player average ranking of the three foxholes (1 = most effective; 3 = least effective) at the beginning of record trials and at the conclusion of record trials.

EEA 3 • Does player performance/attitude (enthusiasm for training) improve or remain at a high state with the use of IDFS (real time casualty assessment)?

MOE 3 • The number of players who subjectively think training with the IDFS is better than previous training they have experienced, at the start of, the middle of, and the end of record trials.

As we gained experience and had an opportunity to interface with more of the training community, our collection of training data was better able to reflect specific user needs. For example, the following HIMAG IIA and MAPPRO III training add-ons were developed in conjunction with the proponent.

High Mobility/Agility, Phase IIA (HIMAG IIA)

EEA 1 • What amount of retraining is required to requalify TOW gunners to their former level?

MOE 1 • Training time (days) to requalify TOW gunners (equal or higher level).

EEA 2 • Can center of mass (COM) training improve the gunner's ability to determine the target's true center of mass?

MOE 2 • Average aim error from true center of mass (COM) before COM training (azimuth and elevation error).
minus
Average aim error from true center of mass (COM) after COM training (azimuth and elevation error).

EEA 3 • How effective is M60A1 training?

MOE 3 • Average error in lead angle before lead training.
minus
Average error in lead angle after lead training.

Test of New and Improved Maps and Map Products, Phase III (MAPPRO III)

EEA 1 • How effective is the training program?

MOE 1 • TEC test scores after training-
TEC test scores before training.

In the Tank and Mechanized Infantry (TAMI) test, formerly scheduled for fall 77, a slightly different approach was taken (please see Appendix B for a description of this test). Since many of the TAMI missions came from ARTEP 71-2, our TAMI training objectives were designed to: Investigate the adequacy of selected ARTEP standards; identify required subtasks; and provide guidance for future ARTEP modification. Data by task and by area had been scheduled for collection.

TAMI Training Data by Task

- Are procedures satisfactory or unsatisfactory?
- Was mission accomplished?

- If procedures were satisfactory and mission failed, what caused failure?
- If procedures were unsatisfactory and mission was accomplished, to what is success attributed?
- What are required subtasks?
- What are bottlenecks?
- What aspects of task need additional/less emphasis in training programs?

TAMI Training Data by Area

Planning

- Did the team commander develop a sound tactical plan based on the information available?
- Were the essential elements of this plan communicated to platoon leaders, the FO and separate section leaders?

Movement Techniques

- How far ahead of the main attack force was the point element traveling?
- What percentage of the attack force (combat vehicles) were visible prior to contact being initiated at the final objective? (Observations taken from several enemy vantage points).

Intelligence

- Did the team commander determine the location of the main defensive force (accurately enough to employ effective supporting fires) prior to his own attacking elements becoming decisively engaged?

Execution

- Did the team effectively suppress the main defensive force during the final attack (direct or indirect fires)?

- Did the team commander adjust his maneuver elements subsequent to contact to maximize his combat power (maneuver to one flank or the other, concentrate his force against a weak point in the defense)?

- What percentage of the major enemy weapons were destroyed by indirect fire, tanks, TOW's, other infantry weapons?

We worked closely with TSM-TES, ARI, the Armor School and those involved in the development of National Training Centers in order to develop TAMI training objectives. Additional training information was to have been derived from the objective data (kill/casualty ratios, player location as a function of time, player posture as a function of time, etc.), that would have been collected during the conduct of a trial. The final disposition of TAMI will be decided at the November TSARC. Currently, it has been postponed indefinitely.

B. DISCRETE TRAINING TESTS

CDEC has conducted one stand-alone or discrete training test, Helicopter Operations in a Night Environment Against a Simulated Threat (HONEST I), and plans to field DRAGON Training Effectiveness Analysis (TEA) in 78-79, and MECH ARTEP in 4th Q 78. In HONEST I, a training program for pilots using night tactical operations in terrain level flying, was developed.

- Developed training program for pilots using night tactical operations in terrain level flying (with and without night vision goggles)
- Identified short-coming of an AN/PVS-5 goggles

RESULTS OF HONEST I

The DRAGON training effectiveness analysis is a three phased experiment designed to: Compare alternate training programs; to assist in the evaluation of proposed gunner selection criteria; to evaluate selected training scoring methods; to compare the effect of the addition of one training round; to compare refresher training programs; and to identify gunner retention curves. An Outline Test Plan for this test has been developed in conjunction with the Infantry School and submitted to HQ, TRADOC.

- Compare training programs
- Assist in the validation of proposed gunner selection criteria
- Evaluate selected training scoring methods
- Compare effect of addition of one training missile
- Compare refresher training programs
- Identify retention curve

OBJECTIVES OF DRAGON TEA TEST

USAIS is also the proponent for the Mechanized ARTEP test. The purpose of MECH ARTEP is to generate data from which to assess the ability of umpires to evaluate a mechanized infantry platoon's execution of selected ARTEP missions. The results of this experiment may be used to revise or modify the evaluator training program and evaluation techniques. The specific objectives of the test are listed below.

- Determine how well evaluators judge unit's performance
- Identify areas of selected ARTEP missions that are most difficult to evaluate
- Recommend changes to evaluator training programs
- Provide insights into techniques of evaluation
- Provide insight into validity of selected aspects of Mechanized Infantry ARTEPS

OBJECTIVES OF MECH ARTEP TEST

V. ANALYSIS AND EVALUATION OF TRAINING DATA

The analysis and evaluation of training data has been initiated at CDEC during the last year and a half; quantitative and qualitative training information can be found in both the respective Final Reports and in the Final Report Supplements titled "Military Observations". The following four tests have been chosen to demonstrate the type of training information which has been obtained from a field test environment. For your convenience, I've also included a brief description of the experiment. For a more detailed explanation of the test and its results, please refer to the experiments' final report which can be obtained through DDC.

A. CDEC Experiment FC 033, Evaluation of the Frontal Parapet Foxhole, Part 7 (PARFOX VII), Dec 76.

1. Purpose. PARFOX VII was an infantry force-on-force scientific field experiment conducted to collect data in a realistic combat environment for the purpose of making a comparative evaluation of the frontal parapet foxhole, the split parapet foxhole, and the standard foxhole. The results of this experiment have been used by the USAIS and TRADOC to determine doctrine for the construction and employment of infantry fighting positions.

2. Concept. The experiment consisted of a series of trials in which an infantry platoon consisting of three squads conducted coordinated attacks against a prepared squad defensive position. Laser-equipped weapons, hand grenades and indirect fire simulations, and continuous communication between each soldier and a computer enabled CDEC to create a realistic battlefield environment in which casualties were assessed as they occurred. Suppressive cues were also fed to the players as the tactical situation dictated.

3. Design. Each trial generated two-sided, real-time, target acquisition and engagement sequences. These resulted in real-time casualty assessment and attrition in a free play environment. Trials examined only that portion of the battle which took place within 200-250 meters of the FEBA. Three defending and three attacking (threat) player sets conducted approximately 150 trials over a three month period with each player set participating almost daily in one-third of those trials. Both day and night trials were conducted on three different terrain sites.

4. Player Proficiency. Every effort was made to provide tactical realism for the player personnel provided by a TO&E infantry battalion from the 7th Infantry Division. This battalion had been trained in the conduct of day and night attack and defense and was determined to be proficient to level 2 in these areas as defined in ARTEP 7-15. The performance of each player was closely observed during the conduct of all trials in order to satisfy the purpose of the experiment. Players were critiqued at the conclusion of each trial on tactical deficiencies noted during the trial. It is doubtful that the tactical proficiency

of any combat unit has ever before been so closely scrutinized over such an extended period of time. The platoons and squads providing player personnel for this experiment were probably the best trained infantry units of their size in the US Army when the experiment was concluded.

5. Observations Concerning Training.

a. The probability of the attacking platoon leader becoming a casualty during the last 200 meters of an attack is sufficiently high to justify a special training effort to insure that his immediate successor is well trained in the platoon leader's duties and responsibilities.

b. During the conduct of the attack, the radio-telephone operator (RTO) was much more of an asset to the attacking platoon leader when he was well trained in indirect fire procedures.

c. Player movement was the single most important factor in revealing the location of defensive positions to attacking platoon players.

d. The hand grenade became an effective offensive weapon only after attacking platoon players understood its employment and gained proficiency in its use.

e. Defensive players who failed to select and engage priority targets greatly reduced the overall effectiveness of the defense.

B. CDEC Experiment FC 026, Tactical Effectiveness of Minefields in the Antiarmor Weapons System (TEMAWS), Apr 77.

1. Purpose. TEMAWS was an armor-infantry force-on-force field experiment conducted to collect data in a realistic combat enviornment to evaluate the effectiveness of scatterable antitank minefields. The results of this experiment will be used by the USAIS and CACDA as a basis for operational and production recommendations for scatterable Minefields in the Antiarmor Weapons System (TEMAWS).

2. Concept. The experiment consisted of a series of trials portraying a quick attack by a simulated Soviet reinforced tank company against antitank elements of a mechanized infantry/tank team defending in the main battle area. Laser-equipped direct fire weapons, simulated scatterable mines, indirect fire simulations, and continuous communication between each weapon system and a computer enabled CDEC to create a realistic battlefield environment in which casualties were assessed as they occurred. Side tests were conducted to evaluate detection of mines at night, to compare the effectiveness of night vision devices to assist detection, to observe the effects of smoke on detection, and to measure the effect of tank speed on mine detection and avoidance.

3. Design. Each trial generated two-sided, real-time, target acquisition and engagement sequences. Trials examined only that portion of the battle which took place within a 3000-meter wide by 1500-meter deep portion of the battlefield. Six different player sets conducted 143 record trials from August to December 1976. Only daylight record trials were conducted. Three different terrain sites were used.

4. Player Proficiency. Player personnel were provided by the 2d Battalion 63d Armor (1st Infantry Division), 6th Battalion 32d Armor (4th Mechanized Division), 2d Battalion 32d Infantry (7th Infantry Division) and other selected FORSCOM units. The armor units were trained at Fort Hunter Liggett (FHL) to employ specific threat tactics and to operate buttoned-up. The APC mounted TOW elements were given weapon qualification training by CDEC. The DRAGON gunners were given weapon qualification training by the USAIS at Fort Benning. Human factors data were gathered by monitoring the performance of the player force during all trials, and from player debriefings and tactical critiques at the conclusion of each trial. By the end of the experiment, the threat armor players were undoubtedly the best trained armor units in the US Army at executing buttoned-up operations. Similarly, the TOW and DRAGON gunners probably had more tactical training than most other US Army gunners.

5. Observations Concerning Training.

a. Tank crews required 3-5 weeks of training and practice to become proficient in cross country driving and tactical operations while buttoned-up.

b. Present Tank Commander (TC) battlesight gunnery techniques are too slow when M60A1 drivers and gunners acquire targets while buttoned-up.

c. The tank driver used a clock reference system to speed handoff of targets to the gunner and tank commander.

d. The plow and line charge Countermeasure (CM) devices can successfully breach a scatterable minefield only if the defense direct fire antiarmor weapons are suppressed, and if the hasty breaching procedures are carefully rehearsed and vigorously executed.

e. Significant command and control problems that resulted from buttoned-up operations and need corrective training emphasis were:

(1) Reduced visibility and depth perception through vision blocks altered perspective of the terrain, and made command and control generally more difficult. The company commander could seldom see more than one platoon at a time.

(2) Limited vision made it difficult to maintain unit formations.

(3) Total dependence on radio further complicated command and control because buttoned-up operations precluded hand and arm signals and limited other visual control techniques.

(4) Volume of fire was reduced because target detection was very difficult. Tank crews were reluctant to stop momentarily and search for targets while in the minefield.

f. In buttoned-up operations, except for the very high density minefields, terrain appeared to be more significant than minefield density in determing threat tank speed.

g. The M60A1 driver is the only crew member who can effectively detect and avoid mines when the tank is buttoned-up.

h. Tactical employment considerations should be incorporated into TOW and DRAGON gunner training courses.

i. Meteorological conditions (haze, glare, dust, shadow) should be considered in selecting antiarmor missile firing positions.

j. Defensive positions were most often revealed by firing and movement signatures.

k. Antiarmor weapons crews must be trained to select and prioritize targets within their assigned sector.

C. <u>CDEC Experiment FC 045, High Mobility/Agility - Phase IIA (HIMAG IIA), Jun 77</u>.

1. <u>Purpose</u>. HIMAG IIA was a scientific field experiment conducted to collect TOW and M60A1 gunner tracking performance data and M60A1 gunner aim error data against a highly mobile/agile target vehicle. The results of this experiment will be used by the USAARENBD in the design of a HIMAG test chassis which will be used to determine desirable characteristics for future armored fighting vehicles.

2. <u>Concept</u>. The experiment consisted of a series of trials in which static weapons systems tracked and simulated engagement of a target vehicle executing specified serpentine maneuvers. The experiment was designed to investigate the effects on gunner tracking performance of target vehicle speed, changes in lateral acceleration, range, angular offset to the maneuver path, and type of tracking system (i.e., TOW, M60A1 tracking Center of Mass (COM), and M60A1 tracking with lead applied).

3. <u>Design</u>. Each trial generated continuous tracking data and periodic engagement data. The wheeled target vehicle maneuvered in controlled serpentine wave patterns on a paved airstrip and developed

lateral accelerations to .7g, at speeds up to 58 mph. The tracking weapons systems (TOW's and M60A1's) were rotated between various ranges and offsets throughout the experiment. Each TOW and M60A1 was placed on a prepared platform with a 5-degree left or right cant which required gunners to simultaneously manipulate traverse and elevation controls while tracking. Gunner performance was recorded by motion picture film, video tape, and/or analog strip charts.

4. Player Proficiency. Players/gunners were provided by Armor Company C, Experimentation Support Command, CDEC. Most of the players had participated in previous field experimentation and were very familiar with their respective weapons systems. Every gunner attended over 50 hours of pre-experimentation training which consisted of refresher training on basic gunner skills as well as special training on experiment-peculiar aimpoints and lead angles. During the conduct of trials, each player's performance was closely observed and a debriefing was conducted prior to player rotation to insure pertinent gunner comments were recorded.

5. Observations Concerning Training.

a. The average HIMAG IIA gunner's ability to perceive correct target COM showed no significant improvement despite a major training effort devoted solely to that end.

b. Many M60A1 gunners experienced difficulty in simultaneously applying azimuth and elevation corrections while tracking a target executing serpentine maneuvers at speeds of from 15 to 55 miles per hour.

c. Video Through-the-Sight (TTS) Closed Circuit Television (CCTV) recording systems provided an excellent means of training and evaluating M60A1 gunners.

d. The CDEC-developed system for generating M60A1 tracking error data is an efficient means of recording and evaluating tank gunner tracking performance.

e. The task load for an M60A1 gunner who must apply lead to hit a moving target was so excessive in HIMAG IIA trial conditions that a target hit was highly improbable.

D. CDEC Experiment FC 016, Test of New and Improved Maps and Map Products - Phase III (MAPPRO III), Jul 77.

1. Purpose. The purpose of Phase III of the Test of New and Improved Maps and Map Products (MAPPRO III) was to evaluate the relative utility of four map products when used by armor personnel for identification of specified Military Geographic Information (MGI), for day and night cross-country navigation, and for remote target location.

2. Concept. MAPPRO III was a three part experiment consisting of classroom practical exercises (Part 1), cross-country navigation and target location during daylight (Part 2), and night (Part 3). The concept for Phase III emphasized the functional military uses of a map in conjunction with normal supplemental aids.

3. Design.

a. Part 1 was designed to evaluate the readability of Military Geographical Information (MGI) features as depicted on each map product. Using the map products, players were required to describe and answer questions concerning preselected MGI.

b. Part 2 was designed to evaluate the relative utility of four different map products on route planning, navigational accuracy, and target location during daylight. The navigational accuracy of players was evaluated primarily on the basis of player self-location when navigating between prescribed checkpoints. Measurements were obtained by comparing the player's estimate of his location to actual location in the vicinity of checkpoints. Measurements of target location accuracy were obtained by comparing the player's estimated location of the target with the target's true location.

c. Part 3 was designed to evaluate the relative utility of the four different map products on navigational accuracy along specified routes and target location during darkness. Design of Part 3 was essentially the same as that of Part 2.

4. Player Proficiency. The 36 players participating in Part 1 were junior officers (2LTs) of various branches, and mid-grade NCOs with infantry and reconnaissance MOS's. Twelve armor officers and 13 armor NCOs were selected from the original 36 players to participate in Parts 2 and 3. Prior to Part 1, all players participated in a classroom training program which emphasized familiarization with the various map products and to review basic map reading skills. The goal was to bring all players to reasonably equal levels of familiarity, confidence and ability with each of the map products.

5. Observations Concerning Training.

a. Although there was a high positive correlation between the written pre and post training tests and a positive correlation between day and night navigation errors, there was no correlation between the performance of an individual on written tests and that individual's performance in field navigation.

b. Training should emphasize the use of all map reading techniques and navigational aids.

c. Odometer readings must be supplemented for accurate cross-country navigation.

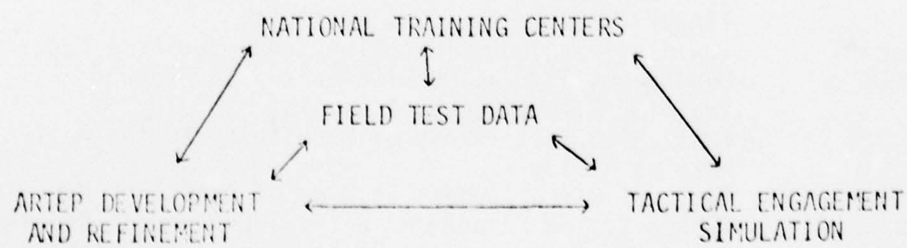
d. Different map reading training programs are probably required to bring junior grade officers and NCO's to the same level of overall expertise.

e. Training should emphasize the use of relief.

f. Periodic refresher training in target location technique is needed.

VI. SUMMARY

TRADOC is currently involved in numerous programs designed to identify and eventually reduce the loss in unit proficiency due to inadequate or inappropriate training - the "Training Gap" (some examples: WSTEAs; CTEAs; ARTEPs; engagement simulation; the National Training Centers and the Army Training System Study). Most or all of these studies will require both qualitative and quantitative training data to support them. Since the collection of quantitative training data, in such a broad sense, is a new requirement, Training Measure of Effectiveness must often be developed as the need arises. Unlike the combat development area, there are few "proven" training MOEs. Consequently, there is a need for those in the training community to coordinate their efforts and document their results. This documentation is needed in order to establish a list of "reliable" training MOEs that can be employed and refined by other users. Since new training systems, techniques and equipment will be developed, flexibility in the choice of a training MOE must still be maintained. In addition, the type of unit (combat versus combat support), size of unit (squad versus battalion), and type of training (individual versus collective) will often require different MOEs. These problems further increase the need for the training community to share information on the development of Training MOEs. An example of one such coordinated effort is depicted on the following diagram:



In order to develop training MOEs for use in two-sided, instrumented engagement simulation experiments, CDEC has been working with representatives from DCST (National Training Center), CATRADA (National Training Center), TSM-TES (Tactical Engagement Simulation and ARTEP Development), the Service Schools (ARTEP Development) and ARI (ARTEP Development). In return, CDEC has provided each of these agencies with training and/or instrumentation information from prior tests and plans to coordinate the collection of future training data with them as well. This sharing of information has been mutually beneficial and should be encouraged elsewhere in the training community.

APPENDIX A

ACRONYMS

ARI	Army Research Institute
ARTEP	Army Training and Evaluation Program
ATMT	Antitank Missile Test
CAC	Combined Arms Center
CACDA	Combined Arms Concept Development Activity
CATRADA	Combined Arms Training Development Activity
CDC	Combat Developments Command
CDEC	Combat Developments Experimentation Command
COM	Center of Mass
CTEA	Cost and Training Effectiveness Analysis
DDC	Defense Documentation Center
DRAGON-TEA	DRAGON-Training Effectiveness Analysis Experiment
EEA	Essential Elements of Analysis
HIMAG IIA	High Mobility/Agility, Part IIA
HONEST I	Helicopter Operations in a Night Environment Against a Simulated Threat, Phase I
IPR	In Progress Review
ITV	Improved TOW Vehicle
MAPPRO III	Test of New and Improved Maps and Map Products, Phase III
MECH ARTEP	Mechanized ARTEP Experiment
MILES	Multiple Integrated Laser Engagement System
MOE	Measures of Effectiveness
NTC	National Training Centers
PARFOX VII	Evaluation of the Frontal Parapet Foxhole, Part 7
TAHOE	TOW Against Helicopter Operation Experiment
TAMI	Tank and Mechanized Infantry Experiment
TCATA	TRADOC Combined Arms Test Activity
TDWG for ATES-FYTP	Training Development Working Group for Army Tactical Engagement Simulation Five-Year Test Program
TEA	Training Effectiveness Analysis
TEMAWS	Tactical Effectiveness of Minefields in the Anti-Armor Weapons System
TETAM	Tactical Effectiveness Testing of Anti-Tank Missiles
TSM-TES	TRADOC System Manager - Tactical Engagement Simulation
WSTEA	Weapon Systems Training Effectiveness Analysis

APPENDIX B

The Tank and Mechanized Infantry Test was formerly scheduled for execution during October-December of this year. The purpose of TAMI was to investigate current daylight offensive doctrine and tactics for defeating a Soviet motorized rifle unit in a hasty defense. This was a two-sided, force-on-force field experiment that matched a US tank heavy company team in the movement to contact against a reinforced motorized rifle company in a hasty defense. Although the test proponent was CACDA, USAIS and USAARMS, ARI, TSM-TES, CATRADA and DCST were very much involved in the design of the experiment. Specific objectives of this test were:

OBJECTIVE 1:	To gain insight into the dynamics of the tank and mechanized infantry company team conducting offensive operations.
OBJECTIVE 2:	To obtain information to compare the relative effectiveness of armor heavy company teams employing various offensive tactics against a threat motorized rifle unit.
OBJECTIVE 3:	To provide information to verify and/or provide recommended changes to current field manuals and training circulars.
OBJECTIVE 4:	To obtain information to investigate the effect of threat electronic warfare on a US tank/mechanized infantry company team.
OBJECTIVE 5:	To obtain data for use in tank and mechanized infantry play in high resolution combat models.
OBJECTIVE 6:	To provide information which can be used to investigate selected aspects of tank and mechanized infantry training programs.

TITLE: Formative Utilization of a Model for the Prediction of the Effectiveness of Training Devices

AUTHOR: Dr. Marshall A. Narva
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ABSTRACT: A model, which provides a framework for the application of judgmental data, has been under development for use in the prediction of the training effectiveness of training devices in the conceptual stage of development. During the utilization of the model for the assessment of two existing maintenance trainers, various aspects were uncovered which suggested a need for changes to enhance the practicality of its application. The revised model is presented with a discussion of the rationale for the changes. The model utilizes a task analysis of the operational situation to isolate the skills involved in the training objective. Judgments are then made concerning the coverage of the required skills, and the physical and functional characteristics of the device used for training in the required skills, in order to obtain an estimate of predicted training effectiveness.

Formative Utilization of a Model for the Prediction of the Effectiveness of Training Devices

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As a consequence of increasing budgetary and environmental restrictions associated with the conduct of military operations for training purposes there has developed an increased interest in the utilization of training devices, in lieu of or in conjunction with such operations. However, the development of the devices themselves may well entail the investment of substantial resources. Therefore, a need exists for a methodology for the systematic assessment of the characteristics of training devices under development. If such a methodology could be developed which would permit making valid predictions or recommendations concerning the selection or design of device concepts or prototypes, resources could be directed into the development of devices having the highest probability of leading to the best training results.

An auspicious attempt at such a model, called TRAINVICE, has been developed for the Army Research Institute (Wheaton, et al., 1976, a, b, c). This model is based on an extensive review of the literature and is the result of analytical work by a team of experienced behavioral scientists. This paper will outline the original TRAINVICE model, its applications, and present suggestions and rationale for a revised model based upon a formative utilization of TRAINVICE. The revision was undertaken with a view to enhancing the validity and practicality of application of the original model, based upon experience gained in its utilization. Further, the suggested revision aims to make the methodology more amenable to utilization by a wider spectrum of users.

The TRAINVICE Model

It would be well to review the TRAINVICE model at this time as it served as the foundation for the development of the revised model which will be discussed subsequently. A schematic representation of the TRAINVICE model is presented in Figure 1. This model is based on the assumption that certain attributes to be assessed in the training situation will lead to transfer of training to the operational situation. Therefore, the higher the rating on the assessment factors, the higher the transfer that will take place and the more effective the device. The model provides a framework for the making of these judgments concerning these attributes and combines the results of the judgments. The three variables entering into the assessment are (1) the transfer potential of the device, (2) the learning deficit to be overcome and (3) instructional effectiveness. As with any model, its effectiveness depends on the adequacy of the input data. Inputs into the model consist of descriptions of the tasks and subtasks represented in the

[1] The opinions expressed in this paper are not necessarily those of the Department of the Army.

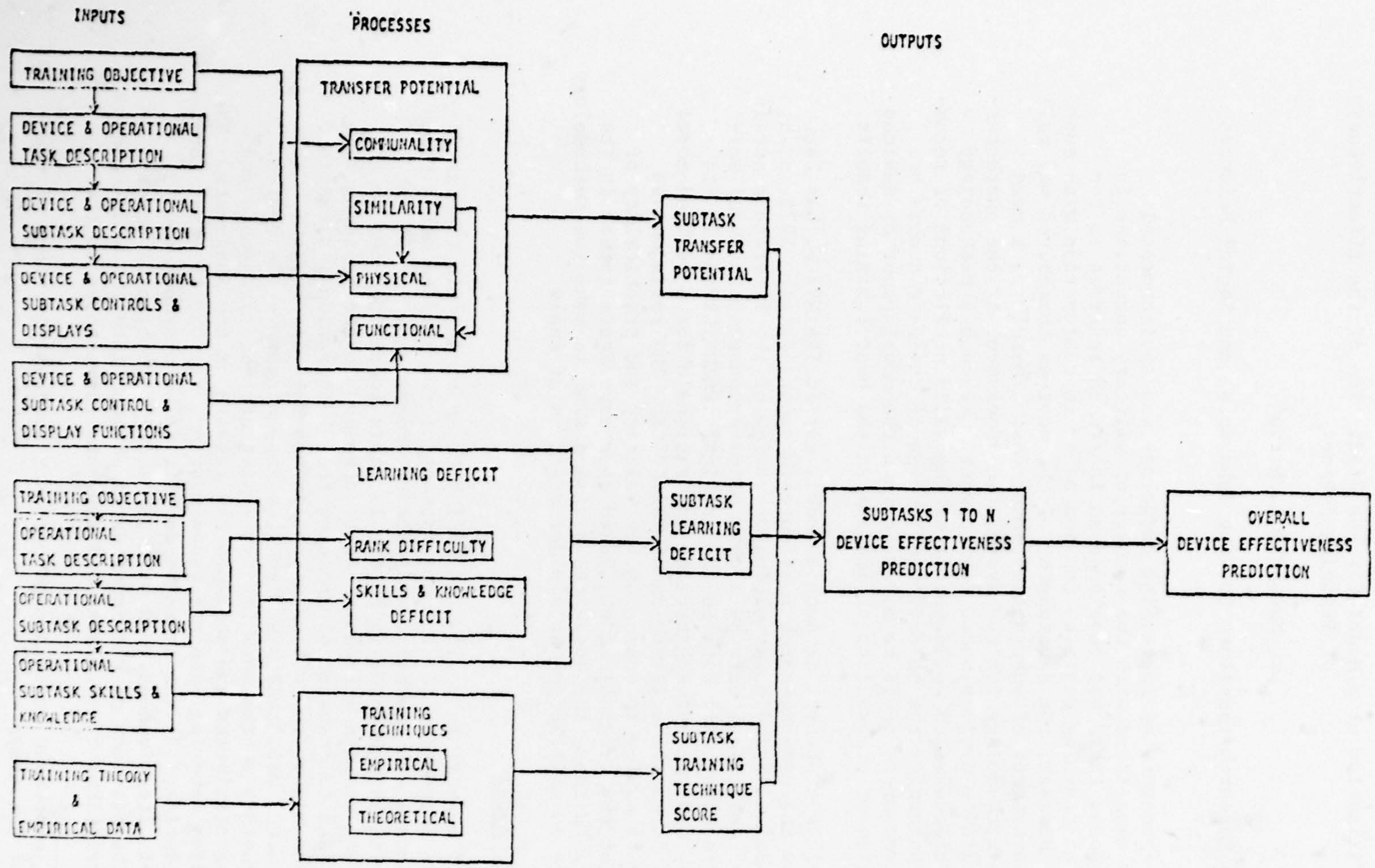

Figure 1. The TRAINVICE Model
(From Wheaton, et al, 1976 b)

operational situation, as circumscribed by the training objective, and those represented in the training device. The controls and displays and their functions for both situations are listed. In addition the skills and knowledge involved in each subtask in the operational situation are formulated for use in the model. Using these inputs, judgments are made using the rating scales given in Figure 2. The subtasks in the two situations, operational and training, are compared to ascertain if provision is made for representation of the subtasks in the training device, in the communality analysis. Next, the displays and the controls for both situations are compared, on physical and functional similarity. As may be seen from the rating scales, the more similar the display or control in the training device is to the operational situation, the higher the score. This is based on the premise that the greater the physical or functional similarity, the greater the transfer of training that will result. Physical similarity refers to the appearance and physical aspects of the displays and controls involved; i.e., their "fidelity"; functional similarity refers to the amount of information conveyed by the display or involved in the operation of a control, in information processing terms. The learning deficit analysis is based upon (1) the assessment of the level of proficiency in each skill or knowledge for the students upon entering the training situation, (2) the desired level of proficiency in each skill or knowledge for the students upon leaving the training situation, and (3) the difficulty (in terms of training time) of training in the skills or knowledges involved in a subtask. This analysis yields a weighted learning deficit for each subtask. The judgments concerning the level of each skill or knowledge are made using the scales shown in Figure 2, adapted from Demaree (1961). The last analysis involved in the TRAINVICE model is an assessment of how well the training device adheres to "good" training techniques. In order to perform this analysis, each of the subtasks is cast into one or more categories of behavior. As given in Figure 2, these categories are those of Braby et al (1972), which are derived from an earlier behavioral categorization by Willis and Peterson (1961). For each of the behavioral categories represented in the subtask, a list of guidelines, also those of Braby et al (1972), are consulted and judgments made, using the scale shown in Figure 2, of the degree to which the guidelines are followed, or not followed, relative to the manner in which the subtask is represented in the training device. The guidelines are broken up into those dealing with the stimulus, response, and feedback aspects of the training situation. For each subtask, the lowest obtained score on each of the three aspects is used to derive an average training technique score. All of the preceding ratings, after conversion to yield a score ranging from 0 to 1, are then fed into an equation to formulate an index of prediction of training effectiveness, ranging from 0 to 1. This equation is as follows:

$$\frac{\Sigma(Ci \times Si \times Ti \times Di)}{\Sigma\, Di}$$

where C is task communality, S similiarity, T training techniques, and D the training deficit scores for each subtask. The equation was derived from a transfer of training equation of Gagne, Foster and Crowley (1943),

TASK COMMONALITY

1 Present
0 Absent

PHYSICAL SIMILARITY

3 Identical
2 Similar
1 Dissimilar
0 Missing

FUNCTIONAL SIMILARITY

3 Identical
2 Similar
1 Dissimilar
0 Missing

LEARNING DEFICIT ANALYSIS

REPERTORY SCALE

0 No experience, training, familiarity, etc.
1 Has only a limited knowledge of subject or skill
2 Has received complete briefing on subject or skill
3 Understands subject or skill to be performed
4 Has complete understanding of subject or skill

CRITERION SCALE

0 Should have no experience or training
1 Should have limited knowledge of subject or skill
2 Should have received complete briefing on subject or task
3 Should have understanding of subject or skill to be performed
4 Should have complete understanding of subject, or be highly skilled

TRAINING TECHNIQUE ANALYSIS

Behavioral Categories:
Recalling facts & principles
Recalling procedures
Nonverbal identification
Nonverbal detection
Using principles, interpreting
Making decisions
Continuous movement
Verbal detection, identification
Positioning & Serial movement
Repetitive movement
Written verbalization
Oral verbalization
Other verbalization

3 Optimal implementation of this technique
2 Good implementation of this technique
1 Fair implementation of this technique
0 Principle inapplicable or device neither implemented nor violated this principle
-1 Mild violation of this training principle
-2 Serious violation of this principle
-3 Complete violation of this principle

Figure 2. Rating Scales Used in TRAINVICE Model.

which was for use with empirical data, while the TRAINVICE extrapolation deals with judgments made concerning aspects of a device assumed to bring about subsequent transfer of training.

A validation study has been performed on the model, utilizing data obtained during the course of two field studies as criteria against which to compare the predictions derived from use of the model (Wheaton, et al., 1976). The devices were tank gunnery trainers involved with burst-on-target techniques and tracking with the main gun of the M60A1 tank. In each case, the prediction of no differences between the training devices involved was found to be consistent with the equivalence in transfer actually found in utilization of the various devices. This was felt to be a promising but not definitive finding.

In order to obtain additional validation data on the model, but also to obtain experience in utilization of the model to determine if there were aspects that might be changed in order to enhance the practicality of utilization of the model, three Army Research Institute personnel applied the model to two maintenance trainers undergoing evaluation at the Army Ordnance Center and School.[1] This afforded the opportunity to obtain data within a different context than that dealt with by gunnery trainers. These trainers were concerned with automotive troubleshooting. No difference in training effectiveness was predicted for the two trainers, which agreed with the results of the empirical evaluation. Various aspects of the model which caused difficulty in its utilization were noted and influenced the development of the modified version. In addition, ARI conducted a three-day workshop, in which the developers of the original model and individuals who had utilized the model or had an interest in its utilization participated, and this furnished further ideas for possible modification.[2]

The Revised Model

A schematic representation of the main components of the revised model is given in Figure 3.[3]

Basically, the model considers three main aspects of the training device during the course of the assessment; what, why, and how. "What" involves an analysis of what skills (or knowledge) should be covered in the training situation and what skills are covered, in order to ascertain that the spectrum of skills covered neither exceeds that which is necessary nor leaves out any that should be covered. It determines that the skills covered are in keeping with the training objective. The "why" refers to that aspect which determines why these skills should be

[1] An ARI report summarizing this utilization of the model is in preparation.

[2] The revised model does not necessarily reflect the views of the participants in the workshop. Proceedings of the workshop are in preparation as an ARI report.

[3] The author would like to express special appreciation for the technical review and inputs provided by Ms Barbara Mroczkowski, TRADOC/TSC.

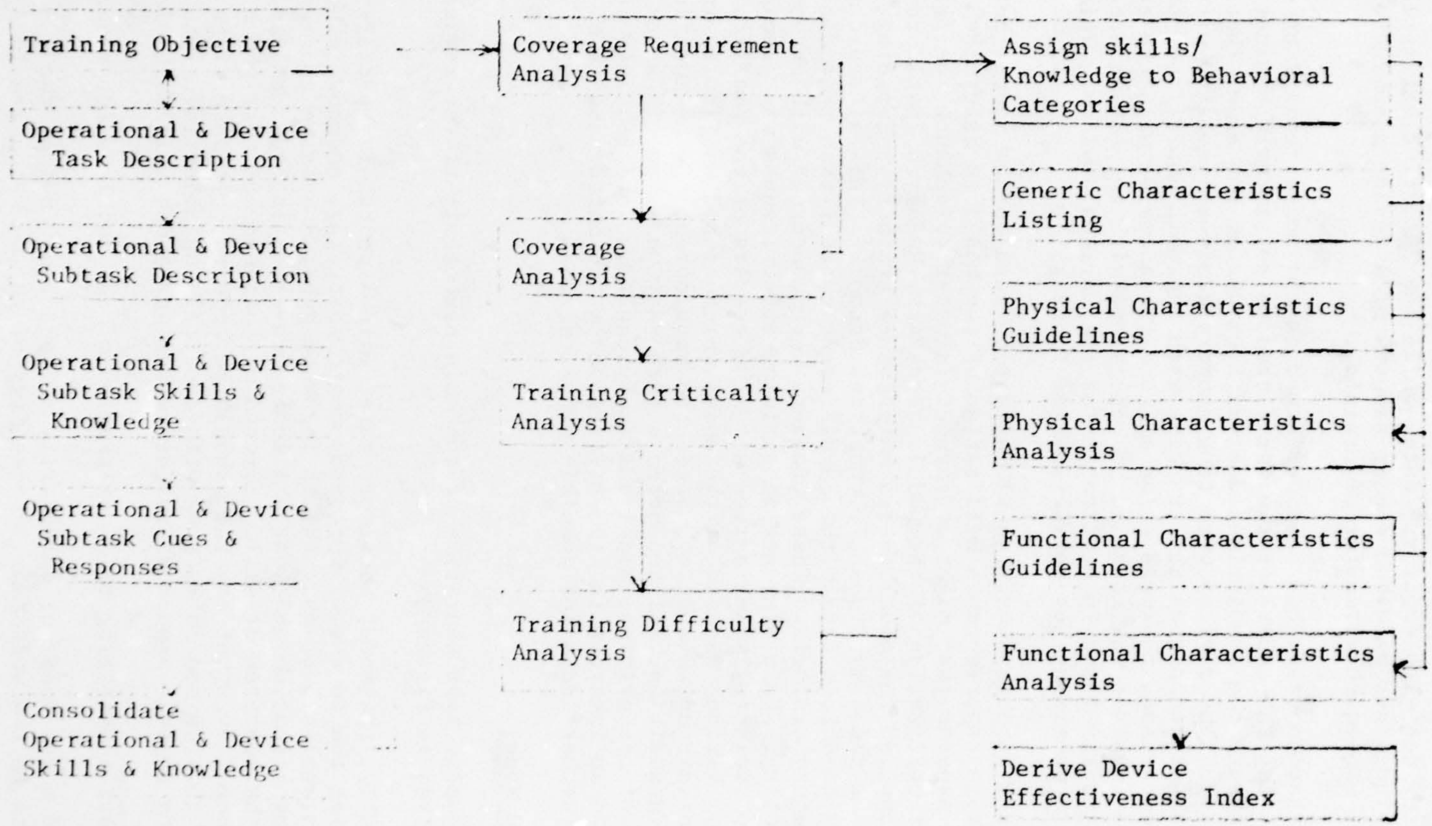

Figure 3. The Revised Model

covered, apart from their being included within the spectrum of skills subsumed under the training objective. This consists of two main aspects; training criticality, which relates to the degree of proficiency required in each skill at the end of the training, and training difficulty, which considers the degree of difficulty to be expected in training for each skill. These two analyses, in essence, give a weight to each skill. The last main analysis, refers to the "how" of the training device; that is, how is each skill taught, and compares the training situation to guidelines of "good" practice. These guidelines are applied to two aspects of the training device; the physical and the functional.

As with the TRAINVICE model, the assumption is being made that the potential for transfer of training will increase as a function of the degree to which the skills are represented, within the constraints of the training objective, and the degree to which the training situation follows guidelines for "good" practice. In addition each of the skills is appropriately weighted by degree of skill needed and degree of difficulty.

Input data requirements. The first requirement is the statement of the training objective. As presented in TRADOC Reg 350-100, Systems Engineering of Training (1973), or in the Interservice Procedures for Instructional Systems Development (1975), the training objective states the action, or task, that the student should be capable of performing, and the conditions and standards of performance he is to attain as a result of the training. The training objective carves out a piece of the operational situation that is to be subjected to training and determines to what level this segment is to be trained, which may vary with relationship to the level actually required in the operational situation. This training objective may vary as to level of specificity, with the subsumed task and associated subtasks also varying as to specificity. It should be kept in mind that the procedures given in such publications as TRADOC Reg 350-100-1 or TRADOC Pam 350-30 (ISD model) deal with the derivation of a program of instruction, of which a training device may be one small segment. The training device may be considered to be but one of many possible media or instructional delivery systems. Therefore, the procedures must be utilized at a level of specificity suitable for the assessment of a training device. Guidance to the conduct of a task analysis are given in such publications as those above and those of R. Miller (1953a), Rankin (1975) and Chenzoff and Folley (1965). However, no amount of guidance can substitute for the good judgment of the analyst in formulating meaningful segments of activity. Information must be derived concerning the task, component subtasks, required skills and knowledge, and the cues and responses involved in the execution of each of the skills, for both the operational and the training situations. Therefore, the same input requirements exist as for the original TRAINVICE model, with the additional requirement for the definition of skills and knowledge being exercised in the training situation. In TRAINVICE, it may be recalled, the subtasks are broken out for both the operational and the training situations, but the skills and knowledges are broken out only for the operational situation, in conjunction with the derivation of the learning deficit for each of the skills. This was concerned only with the characteristics of the student and did not consider what is

offered by the device itself. In the revised model, the unit of concern is at the skills/knowledge level rather than the subtask, although these two units of activity can be very similar. The rationale for going with skills and knowledges as the unit of concern is based on the assumption that the prime objective of a training device is to provide for the acquisition and practice of those skills and knowledge required to carry out the task subsumed in the training objective.

Within each skill or knowledge, the cue(s) and response(s) involved are to be extracted rather than the displays and controls, as in the TRAINVICE model, to provide for greater flexibility in applying the model.

Since the same skills or knowledge may occur over the course of more than one subtask, the skills and knowledges for the operational situation are consolidated into one list in order to avoid duplication. The same is done for the skills and knowledges from the training situation.

Coverage requirements analysis. The first analysis to be performed is that concerned with the requirement for training for each of the skills subsumed under the training objective, from the point of view of the operational situation. The judgment is made for each skill (or knowledge) as to whether it should be included or not be included in the training situation. Depending upon the complexity of the training objective, this judgment may or may not be straightforward. With a fairly constrained or simple training objective or task, most of the skills may be readily judged to be necessary for the training situation. In some cases, it may be necessary to postpone this analysis until the subsequent analyses dealing with training criticality and training difficulty have been made; in some cases judgment concerning training coverage requirement may be modified as a result of the subsequent criticality and difficulty judgments. It should be noted that this analysis does not deal with the mission criticality of the subtask, it is assumed that all the subtasks are necessary to mission success, but rather with the necessity for providing training for the skills subsumed within each subtask. This analysis is a "gate" only; it determines if the skill should be represented in the training. Depending upon the stage of development of the training device, this analysis may help to delineate the range of skills to be represented in a device as well as assessing the range of skills represented in a device. It may also help to define or modify the training objective, if the stage of development of the training program permits. If the skill is judged to require its presence in the training situation, it is given a "1"; if not it is given a "0". If, due to the stage of development, this analysis is not considered necessary, all skills would be rated "1." It should be noted that this judgment is similar to that made during the course of the learning deficit analysis in the TRAINVICE model. In assessing the level of the skill required at the close of training, provision was made for a "0" rating, which indicated that no training was required in the skill under consideration; it was the low end of the scale. This type of judgment has been broken out into this separate coverage requirement

analysis in the revised model to permit a more complete assessment of the range of skills in the training situation.

Coverage Analysis. In conjunction with the Coverage Requirement analysis, a Coverage Analysis is also performed. Comparing the operational and training lists of skills, an assessment is also made for each skill in the operational situation as to whether it is represented in the training situation. If it is, it is given a rating of "1," if it is no, it is given a "0." This analysis is the same as the Communality analysis performed in the TRAINVICE model. As in TRAINVICE, both actual and potential coverage may be determined depending upon how the device is utilized. However, this analysis is made concerning skills rather than subtasks, and the list of operational skills has been "adjusted" by the coverage requirement analysis performed previously.

If the coverage requirement rating is "1" and the coverage rating is "0," this indicates that training in this skill is lacking and steps should be taken to include it, if possible, or the device will suffer in the overall rating. On the other hand, if the coverage requirement rating is "0" and the coverage rating is "1," this would indicate that unnecessary training is being provided and should be eliminated from the device, if possible, or the overall rating of the device will suffer.

If a "0" rating is given for a skill either in the Coverage Requirement or Coverage Analysis, no further analysis for that skill need be done as the overall rating for that skill reduces to "0" due to the multiplicative nature of the derived index.

Training Criticality Analysis. For each of the skills that have earned a "1" rating on both the Coverage Requirement and the Coverage Analyses; that is, for those skills that have been judged to be necessary in the training situation and are indeed represented, a judgment is made as to the degree of proficiency required in that skill at the end of training. Ratings are made using the scale shown in Figure 4. This is the same scale as utilized in the TRAINVICE model for the criterion scale used in the Training Deficit Analysis, with the exception of the "0" point on the scale.

As noted previously, this scale is adapted from Demaree (1961). Once again, it is to be noted that this analysis is concerned with training criticality and not mission criticality, although mission criticality may enter into the determination of training criticality. As noted in Reg 350-100-1, the standards for skills are usually derived from the standard established for the task or subtask. However, this standard must be tempered by the nature of the skill and the degree to which the training on the skill will be merged with subsequent on-the-job training and the degree to which the training is embedded in other aspects of the training program than that involving exposure to the device.

Training Difficulty Analysis. In addition to assessing the level of proficiency required for each of the required skills, an assessment is made, in the Training Difficulty Analysis, of the degree of difficulty

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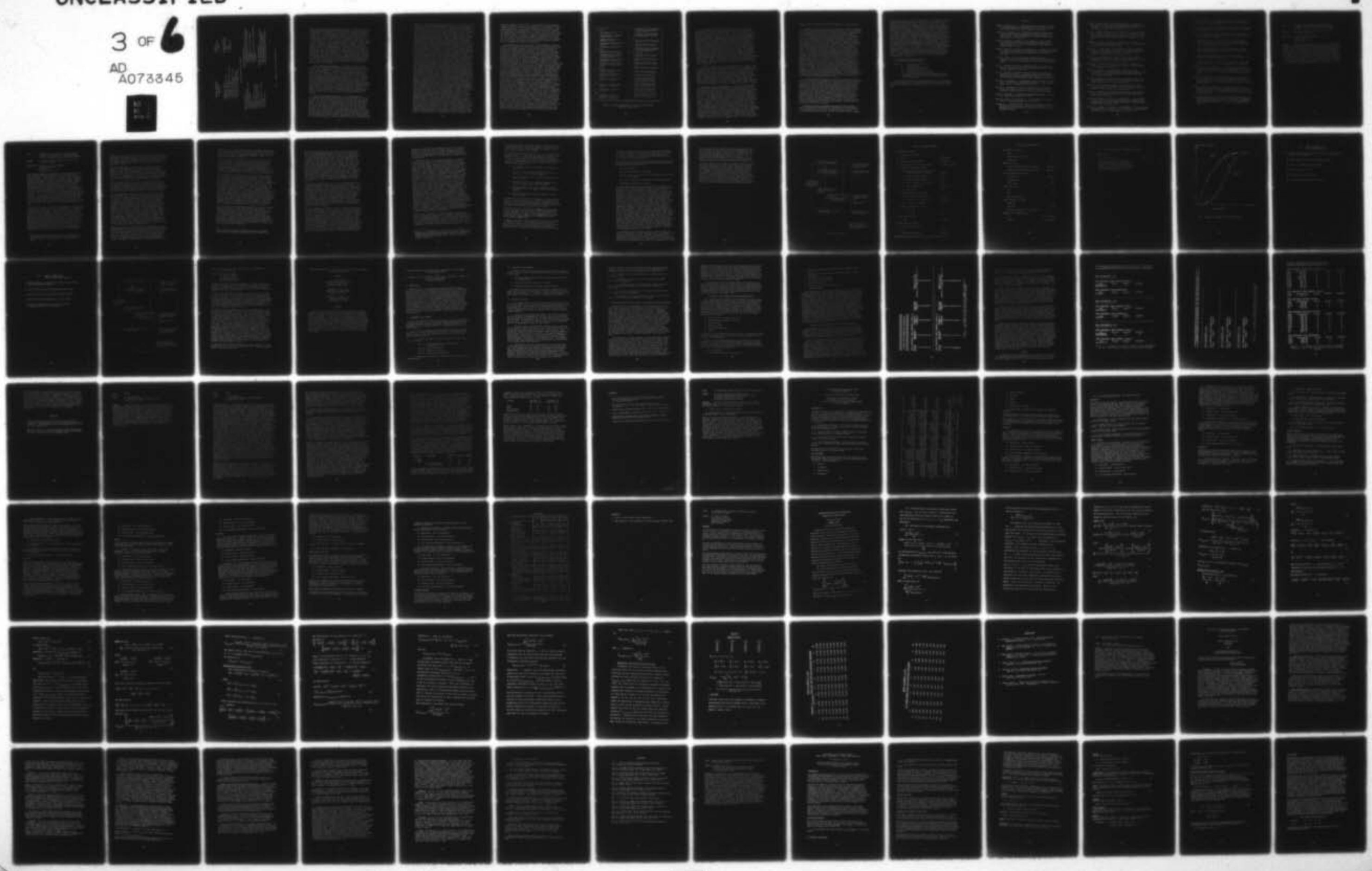

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MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

COVERAGE REQUIREMENT
- 0 Not required
- 1 Required

COVERAGE
- 0 Not covered
- 1 Covered

TRAINING CRITICALITY
1. Should have limited knowledge of subject or skill
2. Should have received complete briefing on subject or task
3. Should have understanding of subject or skill to be performed
4. Should have complete understanding of subject, or be highly skilled

TRAINING DIFFICULTY
1. Minimal or none
2. Some
3. Much
4. Substantial

DEVICE CHARACTERISTICS ANALYSIS

Behavioral categories:
- Rule learning and using
- Classifying-Recognizing Patterns
- Identifying symbols
- Detecting
- Making decisions
- Recalling bodies of knowledge
- Performing gross motor skills
- Steering & guiding-Continuous movement
- Positioning Movement & Recalling procedures
- Voice communicating

PHYSICAL CHARACTERISTICS
- 0 Not adequate for requirements/guidelines
- 1 Adequate implementation for requirements/guidelines
- 2 Good implementation for requirements/guidelines
- 3 Outstanding implementation for requirements/guidelines

FUNCTIONAL CHARACTERISTICS
- 0 Not adequate implementation of guideline
- 1 Adequate implementation of guideline
- 2 Good implementation of guideline
- 3 Outstanding implementation of guideline

Figure 4. Rating Scales Used in Revised Model

which is to be expected in attaining that level of proficiency, for the particular skill and trainee population involved. As seen in Figure 4, a scale with four points is used. This scale is modified from that of Rankin (1975). In determining this rating, the following factors must be taken into account, the inherent difficulty of the skill, the amount of proficiency in the skill by the trainee population due to prior training or experience, and the level of proficiency required at the end of the training, as reflected in the training criticality rating. While no formal procedures are recommended for the assessment of each of these factors, each must be taken into account by the analyst. This is a similar procedure to that followed in deriving the weighted learning deficit score in the TRAINVICE model. It may be recalled that for that derivation, a rating is given for each skill relative to the degree of proficiency held by the trainees at the beginning of the training and also at the end of the training and the two are substracted. Then the subtasks are rank ordered on difficulty. The ranks were then multiplied by the mean subtask deficit to give the weighted deficit score. It was felt during the application of the TRAINVICE model to the maintenance trainers that the rank ordering of the subtasks on difficulty was difficult. In the revised model, each of the skills is rated against an absolute scale of difficulty.

Device Characteristics Analyses. Up to this point we have been concerned with assessing what skills are covered by the device, their "fit" to the training objective, and why the skills have to be included in the training situation. We now turn our attention to how these skills are to be taught. In order to do this, attention must now be turned directly to the device and its characteristics. Up to now, the device has been considered only from the point of view of the coverage of the desired skills and knowledge that it offers. It may be recalled that the TRAINVICE model looked at the displays and controls in the Physical and Functional Similarity analyses and at the subtasks, translated into behavioral categories, in terms of how well certain principles were utilized by the device. In essence, the physical and the functional characteristics of the device were considered, and it is these two aspects that are broken out and considered in the Device Characteristics Analyses of the revised model. Therefore there are two analyses; the Physical Characteristics Analysis and the Functional Characteristics Analysis.

Physical Characteristics Analysis. In the TRAINVICE model, a Physical and a Functional Similarity analysis was performed. It may be recalled that the more similar the physical or functional aspect of the display or control on the training device relative to the corresponding display or control in the operational situation, the higher the rating given. This was based on the premise that the greater the similarity, the greater the transfer potential. However, this may be called into question. As R. Miller (1954, p. 29) has pointed out, "The design of training devices should be directed towards maximum transfer of training value, not physical realism." He further states, "Some stages of training and kinds of task trained require less physical realism than others," and "The kinds and extent of physical realism built into a given training device should be based upon careful examination of what is psychologically

important." Demaree (1961, p. 44) has said that "...the general rule is to represent the operational equipment so that realism is attained with regard to what is to be learned and not to the operational equipment." In other words, more specific critera are required for the evaluation of the transfer potential of the physical characteristics of the training device, in terms of the stage of learning and type of behavior involved than a simple correlation with the physical characteristics of the operational equipment. R. Miller (1953b, c), Demaree (1961), G. Miller (1974), Kinkade and Wheaton (1972), Lumsdaine (1960), Muckler (1959), Caro (1976), Klein (1976) and Smode (1971) are among those who have addressed the problem of the physical characteristics or "fidelity" of training devices. They do give recommendations; however, for the most part they consider classes of devices, or gross characteristics, and do not use the same classifications of devices or behaviors. It is difficult to extrapolate to the generic characteristics of devices and to a particular device. Much work is still needed to come up with guidelines to assist the analyst in assessing a specific display or cue presented by a device, or to compare one device within a particular class of devices with another device as to its transfer potential. However, the procedure followed in the original TRAINVICE model of correlating physical and functional similarity with transfer potential was rejected as being too rigid and indeed possibly misleading. While Caro (1970) had advocated a procedure in which the stimuli and responses in the operational and training situations are compared, in which positive transfer was assumed to occur when both stimuli and responses were similar, he was also concerned with the stimulus-response pairing, predicting negative transfer when the stimuli were similar but the responses to the similar stimuli were different, something not adequately considered in the TRAINVICE procedure. Also, the Caro analysis was performed within the context of instrument flight simulators, a complex perceptual-motor behavior that may indeed call for a high degree of realism in the training, but which may not represent a valid requirement for other types of behaviors. In casting about for guidance for a procedure to assess the physical characteristics of a device, it was decided to combine the procedure suggested by Braby, et al (1975) of the Navy Training Analysis and Evaluation Group in conjunction with their Training Effectiveness Cost Effectiveness Prediction (TECEP) technique with selected guidelines from the Interservice Procedures for Instructional Systems Development (the ISD model)(TRADOC 1975). These guidelines, it should be noted, are a simplified version of the learning guidelines developed by Aagard and Braby (1976) in conjunction with the TECEP model. Various sets of guidelines were considered; those formulated by Willis and Peterson (1961), Gagne and Briggs (1974), an earlier set by Braby et al (1972) (which was the set used in the Training Techniques Analysis of the TRAINVICE model), the Aagard and Braby (1976) guidelines, and the simplified version of their guidelines given in the ISD model (TRADOC 1975). In order for the revised model to have widest applicability, the guidelines from the ISD model were adopted. These guidelines deal for the most part with functional aspects of the training situation, such as the sequencing of learning events. However, selected guidelines were chosen as being applicable to the design of specific elements of the device or training situation. These specific elements, be they displays-controls, inputs-outputs, or cue-response pairs may be likened to the

simulation elements of Smode (1971). The adequacy of these simulation elements determine the perceptual equivalence of the training and operational environments. They are the elements in the total mosaic of the training device, which essentially is a spatial and temporal placement and sequencing of these elements and, as Matheny (1974) has pointed out, the assumption may be made that it is perceptual equivalence that results in positive transfer. To bring the number of possible specific forms that each element may assume into manageable proportions, it is necessary to translate the specific skills to be represented on the training device into behavioral categories and the specific simulation elements must be translated into generic characteristics.

The guidelines extracted from those in the ISD model give limited guidance to the analyst as to the specific physical characteristics that the cues and responses should take for maximum transfer potential for the type of behavior involved. Therefore, the analyst must make a judgment as to what generic characteristics are required. His judgment as to what is required is merged with his assessment of how well the generic characteristics of the cue or response do follow the available guidelines. In order to make these judgments, a list of generic characteristics dealing with the stimulus and response characteristics of the training device are used. This listing, given in Figure 5, is taken from the listing given in the TECEP technique of Braby, et al (1975). To make this judgment, the skill is first placed into one of the behavioral categories, shown in Figure 4, which was also taken from the TECEP technique, and which is also utilized in the ISD model. This permits access to a list of guidelines to the physical characteristics deemed desirable for each of the behavioral categories. Then the cues and associated responses subsumed under each skill are considered, utilizing the list of generic characteristics. The cue or response is categorized into each of the applicable generic characteristics and to each characteristic a rating is given, using the scale shown in Figure 4. This scale ranges from a "0" rating which means that that generic characteristic is not adequate from the point of view of the analyst's judgment of what is required and/or the implementation of the applicable guideline to a rating of "3" which represents an outstanding implementation. Therefore, each cue-response pair involved in each skill represented by the training device receives a cluster of ratings on the applicable generic characteristics. The pattern of these ratings will serve to "highlight" the various physical characteristics of the device, both the outstanding aspects and those that require change. They may serve as a profile of the characteristics, much as that proposed by Smode (1971). In order to derive the Physical Characteristics rating for the skill involved, the ratings given on each of the generic characteristics are added to give the total for that skill. Therefore, the presence of a "0" rating does not eliminate that skill from the total rating but does serve to downgrade the total rating for the skill. In order to derive a baseline against which the rating may be compared in the final device rating, the number of generic characteristics involved is multiplied by "3," the highest possible rating, to give the maximum possible rating for that skill as far as Physical Characteristics of the device are concerned.

STIMULUS CAPABILITIES

Visual Form

1. Visual Alphanumeric - words, numbers and other symbols presented graphically.

2. Visual Pictorial, Plane - a two-dimensional image, a representation in the form of a photograph or drawing.

3. Visual Line Construction, Plane - a two-dimensional figure made of lines, such as a mathematical curve or graph.

4. Visual Object, Solid - a three-dimensional image or reality that is viewed from exterior perspectives.

5. Visual Environment - A three-dimensional image or reality that is viewed from inside.

Visual Movement

6. Visual Still - a static visual field, as with a still photograph, drawing, or printed page.

7. Visual Limited Movement - a basically static visual field with elements that can be made to move, as with an animated transparency or simple panel with switches that move.

8. Visual Full Movement - a visual field in which all elements can move, as with a motion picture, flight simulator, or operational aircraft.

9. Visual Cyclic Movement - a visual field which moves through a fixed sequence and then repeats the sequence in a repetitive manner, as with a film loop.

Visual Spectrum

10. Black and White - a visual field composed of either black or white elements, as with the printed page or line drawings.

11. Gray Scale - a visual field composed of black, white and continuous gradations of gray, as with a black and white photograph or television picture.

12. Color - a visual field composed of various segments of the visual spectrum, as with color television or motion pictures.

Scale

13. Exact Scale - actual visual field or a one-to-one replication of that field as with a full-sized mock-up, simulator, or operational system.

14. Proportional Scale - a representation of reality in other than full scale, such as a scaled model map or photograph.

Audio

15. Voice Sound Range - a limited quality of sound which enables spoken words to be used as the medium of communications, but not suited to more demanding tasks, such as music or sound recognition exercises.

16. Full Sound Range - a quality of sound reproduction that contains all the significant elements of the sound and is suited to the demanding task of sound recognition exercises.

17. Ambient Sounds - a complex sound environment with sounds emanating from various sources and from various directions, including background noise and task significant sounds.

Other

18. Tactile Cues - signals received through the sense of touch, including sensations related to texture, size or shape.

19. Internal Stimulus Motion Cues - the sensations felt by a person when he moves his arm, leg, fingers, etc.

20. External Stimulus Motion Cues - the sensations felt by a person when he is moved by some outside force in such a way that his body experiences roll, pitch, yaw, heave, sway and/or surge.

TRAINEE RESPONSE MODES

21. Covert Response - a response which the trainee creates in his mind but does not express in an observable manner.

22. Multiple Choice - a response mode in which a trainee selects a response from a limited set of responses.

23. Pre-programmed Verbal Performance - a response mode in which a trainee creates a short answer to a question having a limited set of correct answers.

24. Free-Style Written Performance - a response mode in which a trainee writes a response in his own words.

25. Decision Indicator - a verbal or perceptual motor response in which the trainee indicates that he has made a divergent type decision.

26. Voice Performance - a response mode in which a trainee speaks, including conversation.

27. Fine Movement Manipulative Acts - a response mode in which a trainee makes discrete and small movements of dials, switches, keys or makes sensitive adjustments to instruments. Act may involve use of small instruments.

28. Broad Movement Manipulative Acts - a response mode in which a trainee makes large movements of levers or wheels on large pieces of equipment or by the use of hand held tools.

29. Tracking - a response mode in which a trainee continuously controls a constantly changing system, such as steering an automobile or holding a compass bearing in steering a ship.

30. Procedural Manipulative Acts - a response mode in which a trainee performs the sequence of steps in a procedure, such as in the carrying out of the items on the checklist for pre-flighting an aircraft or turning on a radar system.

Figure 5. Generic Characteristics List Used in Revised Model (From Braby et al, 1975)

The requirement imposed upon the analyst to make a judgment as to the generic characteristic requirements follows that utilized in the TECEP technique. That technique, however, is concerned with the selection of an instructional delivery system, of which a training device is both one alternative. Indeed, the TECEP technique gives recommendations as to which delivery systems permit the application of the learning guidelines for each of the behavioral categories; recommendations based on the pattern of matching of the generic characteristics inherent in the various delivery systems and those judged to be necessary by the analyst. Jorgensen (1976) has utilized a similar matrix approach in which the required generic characteristics are matched against various media, of which training devices are but one class, in order to select training concepts (or media) most suitable for training various tasks. Similar procedures may be found in the ISD model (TRADOC, 1975) (which is taken from the TECEP technique), the Air Force Handbook for Designers of Instructional Systems (1974), Parker and Downs (1961), Nunnelly (1966), Bennik and Hoyt (1977), Bretz (1971) and the review by Spangenberg, Riback and Moon (1973). However, these procedures are intended for the selection of or comparison of various media or instructional delivery systems rather than the scrutiny of a training device per se or comparison of training devices.

It will be recalled that in the TRAINVICE model, the analyst looks at the displays and controls and compares each with its counterpart in the operational situation and gives it a rating on physical and functional similarity. The physical similarity rating has been replaced in the revised model by the procedure given above for the reasons discussed above. The functional similarity analysis has been dropped as it was found in the application to the maintenance trainers to pose a difficult decision, not suitable for a procedure aimed at a wide spectrum of users, and was essentially tied to the physical characteristic, specifically being determined by the number of states (which in many cases were difficult to determine) which could be assumed by the display or control.

Functional Characteristics Analysis. The other analysis subsumed under the Device Characteristics Analysis is that concerned with the functional characteristics. While the Physical Characteristics analysis is concerned with the analysis of the elements of the training device per se the functional analysis is concerned with how these elements are utilized. This analysis may be compared to the Training Techniques Analysis of the TRAINVICE model. As in that analysis, the operations of the device are compared against guidelines to ascertain to what extent "good" training practices are followed. Certain changes have been made, however. Instead of using the subtask as the unit of concern, each skill is analyzed. It was found during the application of the model to the maintenance trainer, that there often was confusion and at times even conflicting guidelines to be considered, as the subtask was broken up into several behavioral categories, each with its own set of guidelines. In the revised model, each skill is translated into one of the behavioral categories (those used in the ISD model), as shown in Figure 4, and only that set of guidelines is considered. The guidelines have been changed to those used in the ISD model, as opposed to the earlier Braby guidelines which were used by the TRAINVICE model. These guidelines are

felt to be more straightforward and more suitable for a wider spectrum of users.

Therefore, to perform this analysis, each of the skills is translated into one of the behavioral categories given in Figure 4. The appropriate set of guidelines is consulted, as the functional, dynamic characteristics of the elements involved in training for that skill are considered. First a determination is made if the particular guideline is applicable. If it is, a rating, using the scale shown in Figure 4, is made. As in the Physical Characteristics analysis, the scale ranges from "0" which means that the guideline is not adequately followed in the training for that skill up to a rating of "3" which represents a judgment of outstanding implementation of the guideline. Therefore, each of the applicable guidelines receive a rating for that skill. In order to derive the Functional Characteristics rating for the skill, the rating given for each of the applicable guidelines are added to give the total for that skill. The guidelines not deemed applicable are not considered in the ratings. As the total is derived through addition rather than multiplication of the individual ratings, the presence of a "0" rating does not eliminate that skill from the total device rating, but does serve to downgrade the rating for the skill and also may serve as a "flag" for something that needs to be corrected. It will be recalled that in the TRAINVICE model the lowest rating was used and all other ratings discarded. It was felt that this was a waste of valuable information, and possibly misleading, as the one low rating could obscure the presence of other high ratings. The revised procedure takes all the ratings into account. In order to derive a baseline against which the rating may be compared in the final device rating, the number of applicable guidelines is multiplied by "3," the highest possible rating, to give the maximum rating for that skill for functional characteristics. This would represent a device with outstanding application of all the applicable guidelines for the behavior within which the skill falls.

In applying guidelines during the course of the Training Techniques Analysis in the use of the TRAINVICE model, some difficulty was encountered as the unit of activity emcompassed in a subtask did not lend itself to application of the guidelines, as many guidelines were concerned with a sequencing of events which covered a more extensive period of time than covered by the subtask. While this problem may be accentuated by the use of skills as the unit of activity, the possibility also exists that since skills exist over subtasks, the match of unit of activity and guidelines may be enhanced. It depends upon the particular task and subtasks involved. This problem tends to arise from the fact that the guidelines, both those used in the original TRAINVICE and the revised models, originate from those intended to be used for instructional system development and instructional delivery system or media selection, of which a training device may be one small segment. The development of more specific guidelines is required.

Derivation of Index of Predicted Training Device Effectiveness. It will be recalled that the derivation of the index of the effectiveness of the training device utilized in the TRAINVICE model was based on one of the formulas discussed by Gagne, Foster, and Crowley (1948) to express

transfer of training in terms of empirical data. The equation formulated for use with the revised model also follows a procedure discussed by Gagne, Foster, and Crowley (1948). While not based on one of their formulas directly, it is in keeping with their conclusion that the most useful and practical type of formulation is that based on percentage of maximum possible transfer. It assumes that if the device were to follow perfectly all of the guidelines, as judged necessary by the analyst, that maximum transfer, which could be attributed to the device, would be the result. This forms the baseline against which the device under evaluation is compared. Therefore, the maximum possible score for the Physical Characteristics Analysis and the maximum possible score for the Functional Characteristics Analysis added together would represent the "perfect" device for the training of that particular skill. This total is weighted by the Coverage Requirement, Coverage, Training Criticality and Training Difficulty scores derived for that skill. The derived score for each skill is then compared with the score representing maximum expected transfer. (If a "0" rating is given for either the Coverage Requirement or Coverage Analysis, the total score for that skill is reduced to "0" and makes no contribution to the derived index for the device.) To derive the score for the total device, each of the skill scores is added. Therefore, the index of predicted training device effectiveness is as follows:

$$\frac{\Sigma(CR \times C \times Ci \times D \times (PC + FC))i}{\Sigma(CR \times C \times Ci \times D \times (PC_{max} + FC_{max}))i}$$

where:
- CR Coverage Requirement Score
- C Coverage Score
- Ci Training Criticality Score
- D Training Difficulty Score
- PC Physical Characteristics Score
- FC Functional Characteristics Score
- PC_{max} Maximum Possible Physical Characteristics Score
- FC_{max} Maximum Possible Functional Characteristics Score for each skill.

This equation will yield an index ranging from 0 to 1. The larger the index, the higher were the ratings given on the Device Characteristics Analyses and presumably the greater the potential for transfer of training.

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TITLE: Definition of cost effective training programs: A summary of current research efforts with application to forecasting future training requirements

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ABSTRACT: This paper presents the results achieved and the analytic methods developed by Litton Mellonics in three recent research efforts for the USAIS to define effective training programs for the M16, TOW and DRAGON. Field experimentation was not possible and hence analytic procedures had to be developed to support this effort. Recommendations for training were based on analyses of five factors: threat, operator requirements; cost, safety and state of the art constraints on realism in training; operator capabilities; and knowledge of the effectiveness of certain basic training principles. Since projections can be made as to the status of these factors in a future time the authors suggest that their method and study logic can be applied to the problem of defining training resource requirements now for future time frames.

TITLE: Definition of cost effective training programs: A summary of current research efforts with application to forecasting future training requirements [1]

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Army Training managers and planners have been presented with a twofold challenge to improve the effectiveness of weapon system training programs. TRADOC Pamphlet 71-8 points out that in many instances soldiers can not achieve the accuracy or hit probability of which their weapon systems are theoretically capable. Fighting and winning outnumbered requires that such a performance gap between theoretical and actual capabilities be closed. The first challenge is, thus, to improve the effectiveness of current training programs. The training planner must also anticipate in the design of training programs. Hence there is a second challenge, a need to extend the current efforts and plan effective training strategies now for a future in which we can anticipate fewer training resources. As it is necessary to devise effective training programs for today, it is also necessary to look into the future and define effective training programs under projected resource constraints. This must be done both for our current weapons still in the inventory at that future date, and also for future weapon systems so that forecasts of the requirements for critical training resources can be made.

The problem of training effectiveness determination is not difficult when empirical data are available. The Army however, is attempting to forecast resource needs to implement effective training for a future training environment. Field tests cannot easily be run and methods are needed for predicting training program effectiveness that are not totally dependent on empirical data collection. Litton Mellonics has recently been involved in three research efforts for the USAIS designed to analyze the training effectiveness of three weapon systems: the M16, TOW and Dragon. This paper discusses our methods for those studies. These training effectiveness analyses (TEAs) are germane to the problem of future program design and training cost effectiveness prediction. The three weapons are by no means prototypes or futuristic weapons, but the methodology we developed for identifying the data needed for a TEA for current

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weapon systems has application for defining effective training programs for future weapon systems or for current weapons in some future time frame. In the three studies, time was not sufficient for field tests of M16 training recommendations nor dollars available for live TOW or Dragon firing. An analytical as opposed to an empirical approach was required.

Our general data analysis scheme is shown in this first slide. (SLIDE 1 - BASIC STUDY PLAN). It portrays both the sequence of analysis and our underlying study logic. Basically we sought to define, in terms of the weapon defeatable threat, the most realistic training scenarios possible. We then considered limitations imposed by cost (i.e., TOW ammunition) or safety (i.e., how to play suppression without live return fire) and defined those training trade-offs likely to retain effectiveness without exceeding constraints.

One basic premise guided our effort. We accepted as the measure of training effectiveness, the ability of the trainee, at the end of training to perform that task or job for which he was trained, according to prestated standards and conditions. The standards and conditions were determined from an analysis of the threat situations with which the weapon system's operators were likely to be faced.

The starting point for the analysis thus was the statement of the job environment to be encountered by the trainee after he was on the job. For a weapon system, this was an analysis of the threat to be faced by, and the performance required of, the individual in combat, to survive and win. The analysis of threat provides several essential ingredients for the TEA. First, standards of performance were thus rational and the performance gap of interest became not actual performance versus theoretical weapon system capabilities, but actual performance versus rationally derived performance requirements. Secondly, training conditions could be made realistic. The trainee could be trained to counter conditions similar to those he would most likely experience on the battlefield. This is consistent with the principles of training found to be important in the Army's research efforts in support of the REALTRAIN program. These principles state that for effective training: (1) the cues encountered by the trainee should resemble those that actually occur on the job which elicit critical behaviors; and (2) the trainees should be able to respond in training as they will be required to respond on the job.

Threat data analysis and combat performance determination for M16, TOW and Dragon started with a review of literature and consultation with experts to determine suitable parameters within which to define the combat-oriented threat situations for use in the analysis. Since cues and responses were essential, the parameters focused primarily on what the individual U.S. soldier or crew would be likely to see (what range, what numbers, what deployment, what target configurations) and what the threat would do (typical speed,

tactics, etc.) on the mid-intensity Northern European battlefield. All data were reviewed with intelligence community and USAIS experts and final threat parameter values were defined as shown in the next three slides. (SLIDE 2 - M16 THREAT PARAMETERS) (SLIDE 3 and 4 - TOW and DRAGON THREAT PARAMETERS).

If the intention of the studies had been range design, we might have stopped here, since these values define the parameters for inclusion in range configurations which are likely to lead to effective transfer of training from range to battlefield. Our intent, however, was realistic training criteria which means standards of performance as well as conditions under which to train and test. The standards had to be such that, if attained, the individual could be expected to survive and win on the battlefield.

In our approach to this problem, we chose two different strategies -- one for the M16 and another for TOW and Dragon. The use of two strategies was necessitated because combat data existed for rifle fire, but none for TOW and Dragon. We based our analysis for the M16 on Dorothy Clark's "breakpoint theory" in which it is hypothesized that at certain casualty levels, a unit will try to break off an engagement, or at least will change to a defensive position. The basic data for breakpoint analysis is shown in the following slide from Dorothy Clark's 1954 study.[2] (SLIDE 5 - BREAKPOINT GRAPH) A comment is important here. First we are aware of criticism of these data, with regard to certain assumptions about the probabilistic nature of commander's decision making and the role typically played by such factors as motivation, leadership and societal variables in modifying the behavior of units under pressure. We are also unaware, however, of any better study done to update Clark's. We did query commanders on their expectation of troop performance. The mean breakpoint chosen by commanders was a .60 casualty fraction which falls at the 90 percent probability of breaking on the curves.

The next task was to use the threat parameter data and the estimated number of casualties to deduce what an individual would have to contribute to casualties if he operated in a squad context. Using a 25 man attacking unit at least 15 casualties (60%) would have to be attained. Unit commanders with combat experience were given scenario descriptions and asked to state the desired performance levels of troops in terms of rounds per hit per range band. Over 250 unit commanders were questioned and that data used together with breakpoint data to establish minimum acceptable standards expressed in terms of number of hits to be achieved in training, given a specific allocation of ammunition.

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[2] Clark, Dorothy; <u>Casualties as a Measure of the Loss of Combat Effectiveness of an Infantry Battalion</u>; ORO-T-289, August 1954.

The TOW and Dragon represented a different situation than did the M16 for setting training standards. First of all, both weapons have, by design, high first round hit probability if properly used since both are wire-guided, optically tracked systems employing a sighting system that requires merely that the gunner keep the crosshairs on the sight aligned with the target during the missile flight time in order to achieve a hit. Whereas, the rifle is designed to be multishot weapon, antiarmor weapons must achieve high first round hit probabilities in order to allow crew survival because of the extreme firing situature of the weapon which makes detection of the weapon's position relatively easy. Second, very little live-fire data from combat or training exist for either the TOW or the Dragon. As was the case with the M16 study, commanders' expectations were determined, but here they were used to see if differences existed between doctrine and commanders' expectations. This was done as a guide to improving the management of training by correcting misconceptions about weapon performance or target behavior.

Since live fire data were at best very sketchy for TOW and Dragon, recommended training standards had to be deduced from three sources of information. First, since the weapon is a guided weapon, simulations could be run that supported the contention that high hit probabilities were possible with proficient crews. Next, training device scores did indicate that some trainees could achieve high training device scores which, (assuming relationships between training device scores and live fire proficiency) would support the simulations. Finally, an independent human factors analysis indicated that no required psychological or motor tasks were beyond the capabilities of the average trainee. Given these three pieces of convergent data it was concluded that high simulated hit probability values could probably be attained in training, and values were set accordingly.

Given the standards and conditions needed to demonstrate that training had been effective, it was necessary to determine the training setting to be recommended. That setting was to include the physical design of ranges, devices or equipment needed, the target scenarios necessary, and the nature and sequence of trainee tasks. In defining our recommended program we used the basic premise that, with respect to training setting, the most effective training results from actual hands-on experience in an operational setting. Such realism is often limited, however, by cost and safety considerations. It was necessary to identify trade-offs with full realism either in terms of training strategies or devices that offered high degrees of transfer of training. For the M16, transfer of training studies for two individual training devices, LASERTRAIN and WEAPONEER; and MILES for squad training are currently being conducted or planned.

In the case of the M16, we recommended a series of live fire and engagement simulation exercises on ranges that are as realistic as the state of the art allows. We then recommended that when transfer of training data are available, devices be substituted to the extent that high transfer of training and cost reduction could be demonstrated.

For the TOW and Dragon, we were again confronted by a situation in which empirical data on live fire effectiveness or device effectiveness neither exist nor could be expected to be obtained in the immediate future. Moreover, we could not realistically expect live fire during training because of the high cost of live ammunition. The problem was one of designing an effective alternative to live firing in a situation where live fire experimentation to document transfer of training could not be conducted. To do this we chose an analysis procedure called TRAINVICE. The TRAINVICE model was developed by Wheaton et. al (1976) for ARI.[3] It compares a training device with operational equipment on three dimensions, task communality, physical fidelity and functional fidelity. It also considers the difficulty of a task and the degree to which the device incorporates proven principles of effective training. These factors are treated in a quantitative expression that yields a figure of merit known as tau, which purports to be indicative of transfer of training potential. The metric properties of tau are not known. It should therefore, be used only as a relative value until future research is done to validate the model. In our study we used TRAINVICE to analyze the probable effectiveness of several alternative training device configurations. For the TOW we considered the M-70 trainer and the TVT. For Dragon we considered the Launch Effects Trainer (LET) and the TVT. In both cases alternatives incorporating the TVT were judged to be more effective. That is, the alternatives that provide the greater degree of feedback to the trainee were predicted to have a higher transfer of training value. This provided two useful pieces of information. It allowed us to suggest program improvements given current techniques as well as to suggest planned evolution of devices and techniques to incorporate features that would be expected to enhance training effectiveness.

At this point in the analysis we had considered training criteria, devices and techniques which could be expected to constitute an effective training environment. It now remained to put these together into a recommended program that stated what strategies should be used and what resources (time, ammunition, facilities) were necessary to increase the probability that trainees would attain the desired standards when asked to perform under realistic conditions. That is,

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[3] Weaton, G. R.; Fingerman, P. W.; Rose, A, M.; and Leonard, R. L. Jr; Evaluation of the Effectiveness of Training Devices: Elaboration and Application of the Predictive Model, Research Memorandum 76-16, The U.S. Army Research Institute for the Behavioral and Social Sciences; July 1976.

it is not sufficient to expose the trainee to repeated realistic events with live fire or effective simulation. Strategies must be developed that provide a high probability of learning. We used three steps for the M16, TOW and Dragon.

We first conducted a human factors analysis of each weapon system to determine the skills and knowledges required for weapon system operation. Next, we reviewed available diagnostic data in an attempt to determine what errors of performance were persisting in spite of current training procedures. Finally we applied a set of principles that REALTRAIN research has shown to be effective in training program design to wit:

(1) The individual must be a participant in training, not an observer.

(2) The cues in training should resemble those that occur on the job to elicit critical behaviors.

(3) The trainee should be able to respond in training as he would in combat.

(4) The trainee should receive immediate feedback in that situations change as the individual responds.

(5) Training should be structured to eliminate specific performance errors in a sequence that builds on previous training.

(6) Whole task performance should be evaluated in a post-training session.

In retrospect, I'll admit that the second step, the definition and analysis of common, recurrent errors, intuitively appealing as it sounded, was not useful since good diagnostic systems do not exist for the M16, and the value of the diagnostic data available for TOW and Dragon can't be determined without live fire data.

Given these analyses we were able to recommend training programs for M16, TOW, Dragon and outline those further tests necessary to validate standards, conditions and the actual effectiveness of recommended programs. A summary of our recommendations are shown on the next two slides (SLIDE 6 - M16 RECOMMENDATIONS, SLIDE 7 - TOW AND DRAGON RECOMMENDATIONS).

To summarize our effort, let me return to the first slide. (SLIDE 8 - BASIC STUDY LOGIC). Using threat data and operator task requirements we postulated the standards to be met by the trained weapons operator and the conditions under which he should be trained and tested. We then used constraints to a fully realistic

training setting to define training device requirements and empirical data or a predictive model to suggest which devices to employ. The training strategies to be used were recommended based on assessments of operator psychomotor capabilities and the effectiveness of some basic training strategies.

The implication of this general approach for future training resource prediction became apparent when we realized that our total analyses had been dependent on five factors.

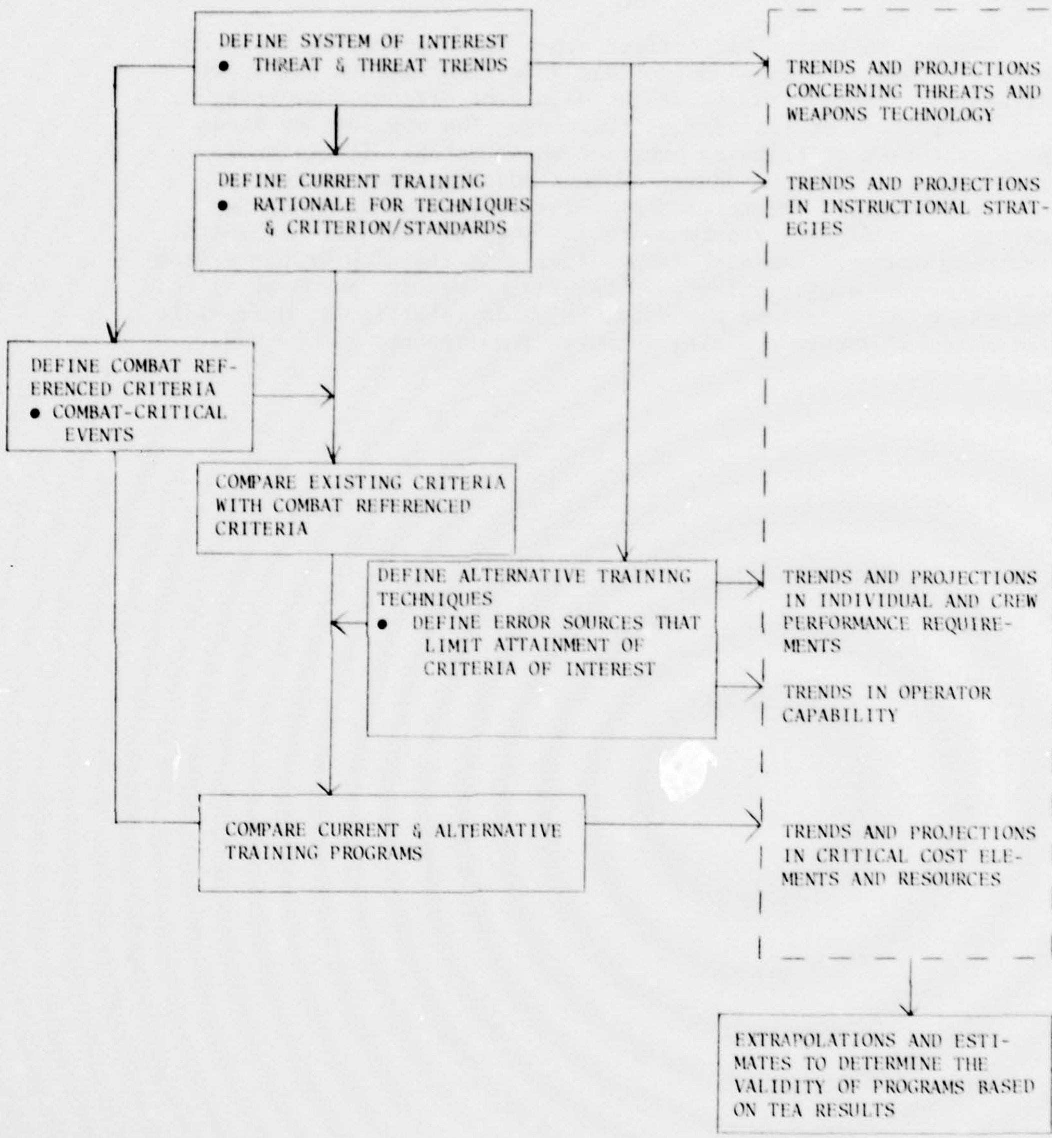
- Threat statements
- Weapon system operator requirements
- Cost, safety, or state of the art limitations on live firing or training realism.
- Proven or theoretical effectiveness of training strategies.
- Operator psychomotor capabilities.

Use of our general approach for any weapon system is dependent on knowledge of the status of those five factors for the weapon system in question at the time the weapon system of interest is to be deployed. For three of the factors intelligent guesses can be made for trends for the near time frame, say 1985-1995. Specifically, threat force capabilities can be projected given the current pattern of weapon system evolution, the state of technology and evolving threat force doctrine developments . Weapon system operator requirements can be estimated by noting trends in the family of weapons of which the future weapon system is a member and a review of the state of the art in our own weapon system technology. Future constraints are probably the easiest to postulate since they will undoubtedly include less land for maneuver, less live ammunition, more severe environmental impact constraints and no relaxation in safety considerations. Trends in operator psychomotor capabilities are less easily predicted than in the case of the first three factors, however, for the time frame of interest, namely now to 1995, I doubt we will see dramatic changes in unaided human capabilities although we may see technology replacing operator tasks. With respect to future training strategies and their effectiveness, trends cannot be projected. However, we can assume that those strategies currently held to be effective will not subsequently be proven ineffective but rather will be improved upon. In that case, projections about effective future training and the resources required to implement those programs will be conservative estimates subject to revision as new training strategies are developed.

I do acknowledge that our ability to forecast threat trends and resources constraints is limited and diminishes as a function of just how far into the future we look. Moreover, since human factors considerations and operator requirements statements for new weapon systems are typically post hoc, we seldom have a clear picture of the man side of the man-machine system until after prototypes are fielded.

These say only that the estimation of training program and device needs for future systems must be iterative and approximate. We can guesstimate new threats, operator tasks, resource restraints, and technological compensation for operator limitations for any system and thus approximate the training system needs. The documented guesses and assumptions we have to make at time zero form the basis for guiding data collection at subsequent steps during the evolution of the weapon from concept to fielded system.

In summary, Mellonics has defined a procedure for doing training effective analyses based on certain data requirements and a set of explicit assumptions about the nature of effective training and the process of transfer of training. The application leads to a statement of training programs whose actual effectiveness is verifiable. The extension of that model to the future, is however, the more interesting since, I believe we have defined certain sets of data requirements and logical processes by which training planners can work in parallel with training weapon system developers to evolve effective training programs. Moreover, I believe we have defined processes by which intelligent, defensible estimates of future training resource requirements can be made.



SLIDE 1 BASIC STUDY PLAN

SLIDE 2 M16 THREAT PARAMETERS

1. Threat Squad in Attack

a.	Force ratio (threat/US)	∿3/1
b.	Front	50-60 meters
C.	Distance between individuals	6-8 meters
d.	Average speed of individual movement	3.6 m/sec (8 mph)[1]
e.	Critical target ranges (meters):	
	(1) When forced to dismount by terrain or anti-armor fires - dismount	500-1000
	(2) Effective range of M16A1	460
	(3) Effective range of AKMS	300
	(4) Final Coordination Line (Final CL)	200
	(5) Final assault begins	80-100
	(6) Final charge	25-30
f.	Exposure times per man exposure:	
	(1) 200-400 m; 15-30 m advances;	5-10[1] secs
	(2) 100-200 m; 15-30 m advances;	5-10[1] secs
	(3) 0-100 m; one advance;	20-25[1] secs
g.	Number of men moving at a time:	
	100 - 400	2-3
	0 - 100	all

2. Threat Squad in Defense

Front	50-60 meters

3. Threat Platoon in Attack

a.	Force ratio (threat/US)	∿3/1
b.	Front	200 meters
c.	Distance between squads	20 meters

[1]Moving Man Target Methodology Study - CDEC experimentation data

SLIDE 3 TOW AND DRAGON TEA

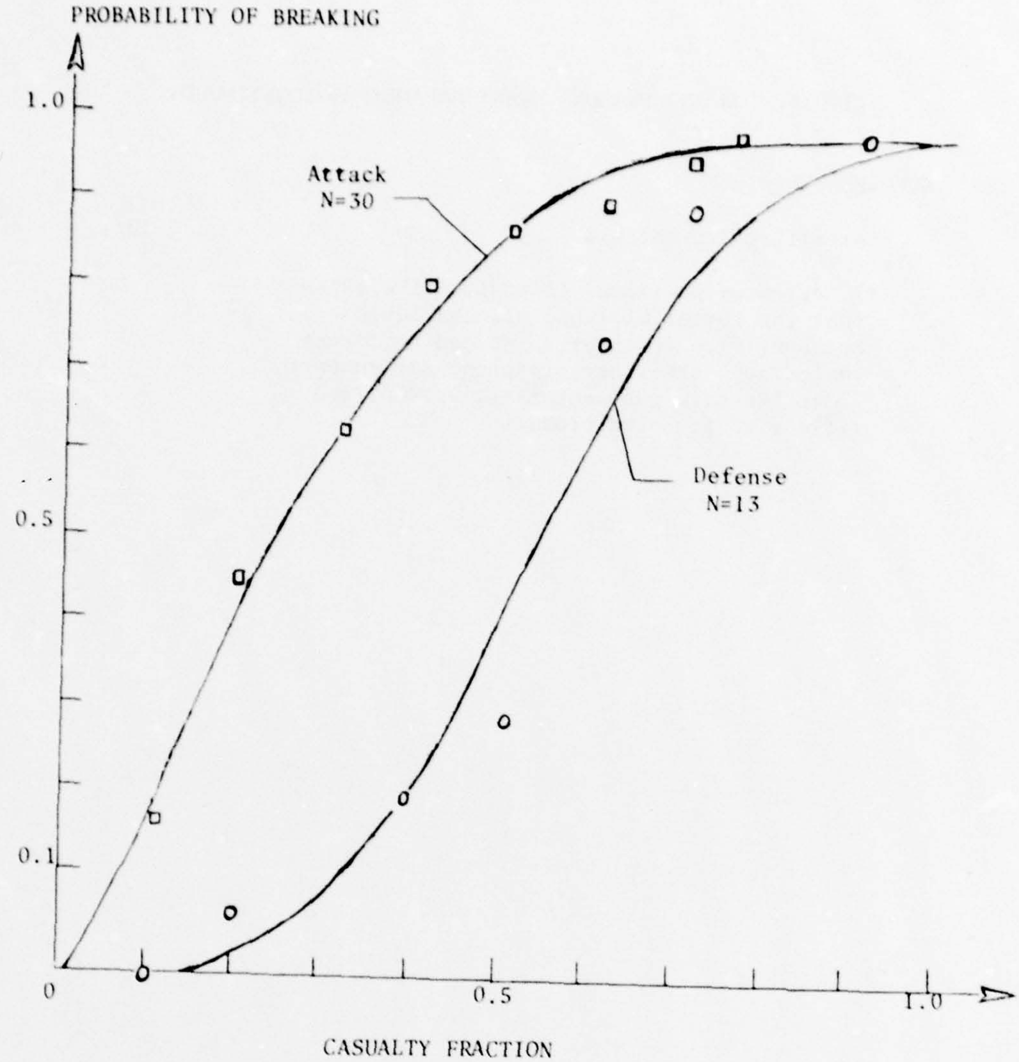
FORCE RATIO (threat/U.S.)	
Tanks	3/1
Armored Personnel Carriers	2/1
Combined Arms	3/1
FRONTAGES (meters)	
Combined Tank/Motorized Rifle Regiment	4000-5000
Combined Tank/Motorized Rifle Battalion	1000-1500
EFFECTIVE WEAPON RANGES (meters)	
T-62 Main Gun	1500-2000
SAGGER Missile	3000
BMP 73 mm Gun	1000
TARGET SPEED (KPH)*	
T-62	50
BMP	65
*Road Speed Capability	
TARGET EXPOSURE** (seconds)	
SHORT HALT	4-8
LONG HALT	60-90
**(Distance between halts, 50-150 meters)	
PROBABILITY OF TARGET KILL (P_k)	
T-62	$\leq$ .50 at 1500m
SAGGER	$\leq$ .60 at 3000m

SLIDE 4 TOW AND DRAGON THREAT PARAMETERS (CONTINUED)

SUPPRESSIVE FIRE*

Artillery (threat/U.S.)	3-10/1

*A review of doctrinal literature indicates that the threat will use all available means of fire to suppress defending forces including: artillery, tactical air support, 73 mm BMP main gun, automatic weapon, and rifle fire from individuals.



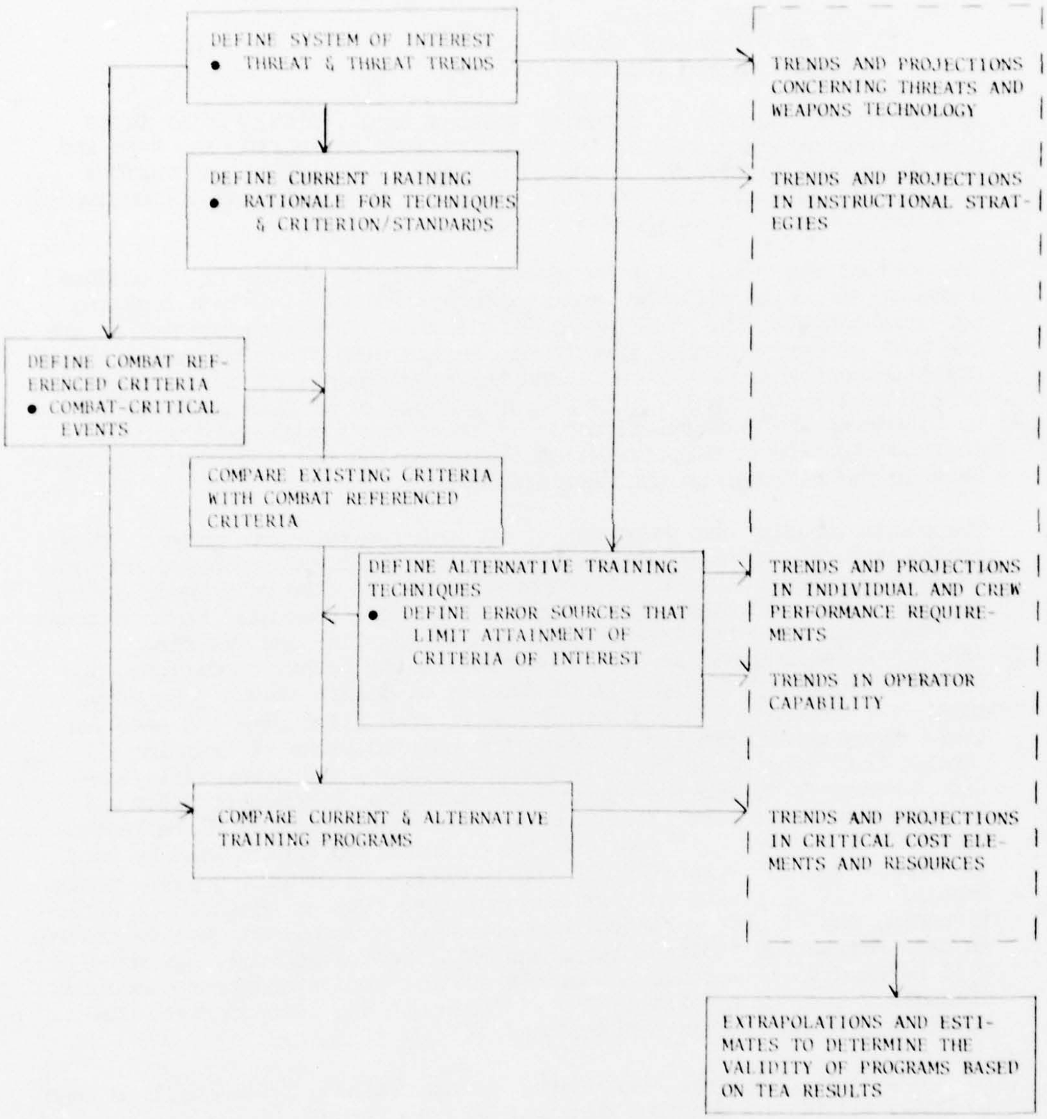
SLIDE 5 PROBABILITY OF BREAKING VS CASUALTY FRACTION*

SLIDE 6 SUMMARY RECOMMENDATIONS
M16A1 MARKSMANSHIP TRAINING

- REPLACE CURRENT NORM - REFERENCED STANDARDS WITH PROPOSED THREAT-BASED, GO/NO GO STANDARDS
- IMPROVE REALISM OF MOVING TARGET PORTRAYAL (TRAMPS)
- EXPEDITE DEVELOPMENT OF MILES AND MAGLAD
- IMPROVE M16A1 SIGHT DESIGN
- REINSTITUTE KD RANGES IN TRAINING
- HOLD BRM TRAINING UNTIL LATER IN BCT CYCLE

SLIDE 7 SUMMARY RECOMMENDATIONS
TOW AND DRAGON GUNNERY TRAINING

- REPLACE CURRENT 4 - CATEGORY TRAINEE QUALIFICATION STANDARDS WITH NEW DICHOTOMOUS STANDARD
- ADOPT NEW RANGE DESIGN WITH UNCOOPERATIVE MOVING TARGET
- INCREASE USE OF THE TVT AND IMPROVE FEEDBACK TO TRAINEE
- ADOPT NEW STRINGENT STANDARD FOR M-70 AND LET SCORES
- VALIDATE THE SIMULATOR (M-70 and LET) QUALIFICATION STANDARDS AGAINST A LIVE FIRE CRITERION



SLIDE 8 BASIC STUDY PLAN

TITLE: New Directions in Rifle Marksmanship: The Development of a Total Training System

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Dr. Thomas J. Tierney, Jr.
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ABSTRACT: The purpose of Infantry weapons (e.g., M16A1 Rifle) is to provide the Infantry soldier the capability of delivering the type and volume of fire necessary for the successful execution of the various individual and small unit combat actions which, considered collectively, comprise the mission of the Infantry.

The goal of the Total Rifle Marksmanship Training System is to produce a combat effective rifleman (man-weapon system) within given training resource constraints. The two driving forces of this system design are the cost effectiveness of investments in training (the Cost and Training Effectiveness Analysis process) and the requirements of the battle-field (the threat environment). The primary objective of this paper will be to blueprint the interrelationship of these two forces and their individual/collective contribution to increasing the potential effectiveness of the rifleman on the battlefield.

The weapon is only one component of the total man-weapon system. This system must be evaluated in relation to the total environmental context in which it is to be used. Therefore, the first step will be to define the conceptual and operational threat. The next step will be to discuss in some detail how the top-down process of the Cost and Training Effectiveness Analysis is used to establish the tasks, conditions, and standards at successively refined degrees of detail (e.g., job, duty, task, and element). This discussion will also illustrate the need for field tests to empirically validate the contributions of training innovations. The Basic Rifle Marksmanship Test, and Laser Rifle/Rimfire Adapter Test, and the Weaponeer/Lasertrain (Diagnostic Rifle Marksmanship Simulators) Test will be used as examples of field tests of training systems. In addition, these tests and others will be used to demonstrate the strengths and short-comings of designing field tests. Emphasis will be placed on test design issues such as Measures of Effectiveness, use of proper control groups, subject sampling, quality control in data collection and reduction, and experimental control. Examples will be used to illustrate how errors in test design and test execution frequently lead to results of "no differences" and inappropriate conclusions of "equal" treatment effects.

In conclusion, the Total Rifle Marksmanship Training System will be used to bring all of the previous discussions into focus - Operations Research Support of the Army in the 80's.

Operations Research at the Corps Level: An Analysis of V Corps Major Training Area Operations

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ABSTRACT

This paper presents the results of applying Operations Research methodology to V Corps' Major Training Area Operations in the Federal Republic of Germany. The study covers three aspects of these MTA operations: the movement of tracked vehicles to training sites, the prediction and allocation of supply costs associated with MTA's and the scheduling of MTA operations. The primary focus of this paper is on the computer-assisted scheduling system which has been developed to aid V Corps' G-3 Section in its development of the Corps' training program. The discussion includes a description of the scheduling system and a demonstration of its capabilities. The topics of rail movement and supply costs are addressed briefly, outlining the analysis in these areas. The complete analysis of the three topics is contained in a Naval Postgraduate School report.

Operations Research at the Corps Level: An Analysis of V Corps Major Training Area Operations

by

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I. INTRODUCTION

The Dictionary of the United States Army Terms (AR 320-5) defines operations research as: "The analytical study of military problems, undertaken to provide responsible commanders and staff agencies with a scientific basis for decision on action to improve military operations." In the spirit of this definition the authors undertook an analysis of Major Training Area Operations in V Corps, USAREUR and addressed problems in this area of concern to the Corps Commander and his staff. This application of operations research resulted in a computer assisted system of scheduling MTA operations. In addition, the study systematically sets forth MTA movement costs and aspects of MTA supply costs for consideration by the commander in his decision-making process. This paper briefly discusses the analysis performed with regard to movement and supply costs and concentrates on describing the computer assisted scheduling system and users guide which has been forwarded to V Corps for use by the G-3 training section.

II. STATEMENT OF THE PROBLEM

A. V Corps conducts battalion-sized training exercises at four major training areas (MTA's). All battalion-sized combat elements participate and perform gunnery, maneuvers, and ARTEP's during MTA operations. The management of these training activities is a monumental task for the following reasons:

1. V Corps competes with other USAREUR and allied units for the use of MTA. This fact, combined with the desires of V Corps' unit commanders, creates a year long scheduling exercise before the following year's program is finalized.

2. V Corps must budget for these activities in separate categories as follows:

Class II - Personal/Administrative Supplies and Organizational Tools

Class III - Petroleum Products

Class IV - Construction Materiel

Class IX - Repair Parts for Equipment.

3. Funds must be properly allocated to each unit based on anticipated training.

III. OBJECTIVES OF THE ANALYSIS

In view of the above situation the objective of the analysis was to provide V Corps with readily implementable methodologies for accomplishing the following:

A. Increasing combat training within the constraints imposed by the rail movement budget;

B. Accurately predicting supply costs of MTA's;

C. Reducing the burden of the manual scheduling system.

The realization of these objectives required the quantification of supply costs of specific training events in terms of Classes II, III, IV, IX and rail transportation. In addition, construction of a computer based model capable of handling the real world constraints was necessary.

IV. OVERVIEW OF ANALYSIS

A. The quantification of training costs in terms of Classes II, III, and IV were feasible and V Corps was predicting and allocating these costs by training event. This allocation was accomplished through battalion training matrices and cost factors. The accuracy of these costs was not suspect, assuming good estimates for expected vehicle mileage during the training exercise.

Class IX costs were much more elusive. The Corps was using cost factors provided by the Department of the Army. They were incorporated into the battalion training matrices and provided what was felt to be the best available cost predictions. However, the DA cost factors were highly prone to error. In June 1977 V Corps was in the process of collecting data for the development of their own cost factors.

Basically, these cost factors are averages and are in terms of dollars per mile. Due to their large variance, even accurate cost factors can grossly miscalculate the actual cost of a training event. Using results from a Research Analysis Corporation study [1] and the results of data collected by the authors in June 1977, an alternative methodology was developed. The new technique essentially entailed the management of selected vehicle components at a level higher than battalion, thereby eliminating some of the variance at the battalion level. The remaining Class IX items would be managed at the battalion level and costs for specific training events better predicted.

B. Rail Movement costs were quantifiable, but required extensive calculations before one could acquire a projection for a specific movement. This hindered effective management by commanders when they were evaluating alternative training sites, vehicle mixes, and substitute modes of transportation such as heavy equipment transporters or road marching. Using the rate scales and vehicle characteristics, the authors generated cost tables listing the rail shipment costs from garrison locations to training sites for individual vehicles and the 100% TOE

man side of the man-machine system until after prototypes are fielded.

complement of armor, artillery, mechanized infantry and armored cavalry battalions/squadrons. These tables provide a usable management tool for commanders attempting to select the optimal mode of transportation.

C. The computer based scheduling model was feasible, based on the following concepts:

1. Commanders determine their unit's training requirements based on military judgment;

2. Commanders determine a desirable unit schedule and in the process, minimize rail movement costs for their unit using the cost tables; and

3. The Corps training schedule is then generated with the help of a computer based scheduling model.

This approach leaves the determination of what is the best training mix up to the subjective judgment of the commanders.

V. THE COMPUTER ASSISTED MAJOR TRAINING AREA SCHEDULING SYSTEM BACKGROUND

The North Atlantic Treaty Organization has established a series of twenty Major Training Areas (MTA) throughout the Federal Republic of Germany. With few exceptions, the elements of V Corps, U.S. Army Europe, conduct their MTA operations at the following locations: Baumholder, Grafenwoehr, Hohenfels, and Wildflecken. Traditionally, a manual scheduling system has been employed to coordinate the use of these facilities. This system requires an extensive man-hour commitment which is dominated by the requirement to check and cross-check authorized allocations against existing schedule commitments and new requests. The actual extent of this effort might be better illustrated by the fact that V Corps must be capable of scheduling sixty-four individual battalions into these four MTA's each year, and that many of these battalions must be scheduled two or three times in order to participate in different types of training. Also, the scheduling process must be a dynamic system which is capable of preparing an initial schedule and of processing numerous changes during the entire year. In short, the manual scheduling process is a tedious, time consuming effort which is extremely vulnerable to oversight and error.

The requirement to develop and manage an annual MTA schedule combines the results of two types of decisions to produce a final product. The first of these decision processes drives the schedule to accomplish its ultimate goal of providing the necessary training to maintain the combat readiness of all V Corps elements. This type of decision includes providing adequate priorities to the proper units, insuring that an assigned training area satisfies the requirements generated by a particular type of training, and insuring that training is scheduled at a time which logically coincides with each battalion training program. These decisions are critical to management and depend on an understanding of the unique situation of each unit. The second type of decision involved in

managing the MTA schedule is the static decision generated by the physical constraints of each MTA and by the constraints imposed by higher headquarters. These decisions insure that training is scheduled only during authorized time periods, that density constraints for all training areas during all periods are satisfied, and that schedule information is disseminated to all appropriate elements. Density constraints designate the maximum number of battalion-sized elements that may occupy an MTA at any one time. These decisions are constrained and tedious, but are necessary to insure the development of a feasible schedule.

A situation such as the one described above can be effectively and efficiently managed by employing a computer-assisted scheduling system which separates the two types of decisions involved in developing the MTA schedule. The computer-assisted system requires the manager to make the critical management decisions and employs a computer to resolve the constrained decisions, to consolidate all schedule information, and to process this information in a useful format.

Description of the Computer-Assisted Scheduling System

The computer-assisted scheduling system designed for V Corps follows the basic methodology used by the V Corps G-3 Section to manually prepare the MTA schedule; however, specific modifications allow for computer assistance in the scheduling process. Once the initial MTA allocations are received from USAREUR, V Corps must prepare this information for dissemination to its subordinate major headquarters. The computer assisted system requires that this preparation include the classification of each allocation into one of six training categories and the division of all availability dates into specific training periods. The six training categories are as follows:

a. tank qualification gunnery, level one;

b. tank qualification gunnery, level two;

c. weapons firing;

d. artillery firing;

e. maneuver training; and,

f. support.

The length of time required to accomplish each type of tr^ining can be used to divide the aggregate allocation periods assigned by USAEUR into a list of specific training periods.

With the information provided by the V Corps training section, each headquarters can plan its MTA training and submit the necessary training requests. Each request must follow a format which will permit the transfer of this information to the scheduling program. Therefore each request must include the following information:

a. unit designation;

b. major headquarters (8th ID, 3rd AD, etc.);

c. the type of unit requesting training (armor, mechanized infantry, etc.);

d. the type of training requested (one of the six major training categories);

e. the priority of the training;

f. the training period requested;

g. the density requested;

h. the garrison location of the unit; and

i. the MTA requested.

Once the requests for training are submitted to V Corps, the task of compiling these requests into a feasible schedule begins. At this point the management decisions have been made, period numbers have been assigned to training dates, and appropriate priorities have been given to the most critical training. The next step is to prepare a schedule subject to the density constraints imposed by USAREUR, and to organize this schedule in a format suitable for dissemination. This entire process can be accomplished quickly and efficiently by the V Corps Program to schedule MTA Operations. The input procedures necessary for the proper execution of this program are outlined in a user's guide which is an appendix to reference [2]. The program will produce a feasible schedule or identify overscheduled periods and/or indicate errors found on input cards. Training managers can then request additional densities, adjust training priorities, or take whatever action is necessary to resolve the conflicts identified by the scheduling routine. Once the input information is adjusted, the program can be run again in an effort to gain a feasible solution. To establish an initial schedule this process can be repeated as often as necessary. Subsequent changes may be made by simply adding or deleting the appropriate input data.

Capabilities of the Computer Assisted Scheduling System

The scheduling program provides the scheduler with three separate management aids. First, the program scans each input constraint (and request) for errors which would cause the program to function improperly. When an error is found, a message is printed which describes the error and identifies the invalid input card. The execution of the program may or may not be terminated, depending on the type of error found.

Next, the program provides a summary of the pertinent statistics generated by the schedule. These figures are presented by major training category and period number. Figure 1 shows a portion of this summary generated using information from the FY-1978 USAREUR MTA schedule. This example presents the statistics only for the tank gunnery training categories. A complete printout will show the figures for all six training categories. The meaning of the numbers in each column is evident from the column headings. One feature of the system is demonstrated by the figures for period twelve under tank gunnery, level two. Note that column three shows no units scheduled; column four shows that one density is still available; and column five shows one unit not scheduled because of density

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V CORPS MTA SCHEDULING STATISTICS

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TYPE TRAINING: TANK GUNNERY, LEVEL 1.

PERIOD NUMBER	AUTHORIZED DENSITY	NUMBER OF UNITS SCHEDULED	REMAINING AVAILABLE DENSITY	NUMBER OF UNITS NOT SCHEDULED BECAUSE OF DENSITY CONSTRAINTS
1	3	3	0	4
2	3	2	1	0
3	3	1	2	0
4	3	3	0	0
5	3	2	0	1
6	3	0	3	0
7	3	2	0	0
8	3	0	3	0
9	3	0	3	0
10	3	0	3	0

TYPE TRAINING: TANK GUNNERY, LEVEL 2.

PERIOD NUMBER	AUTHORIZED DENSITY	NUMBER OF UNITS SCHEDULED	REMAINING AVAILABLE DENSITY	NUMBER OF UNITS NOT SCHEDULED BECAUSE OF DENSITY CONSTRAINTS
1	2	1	1	0
2	2	0	2	0
3	1	1	0	1
4	1	1	0	1
5	1	0	1	0
6	1	0	1	0
7	1	1	0	1
8	1	1	0	0
9	1	1	0	0
10	1	1	0	0
11	1	0	1	0
12	1	0	1	1
13	1	1	0	0
14	1	1	0	0

Figure 1 - AN EXAMPLE OF THE STATISTICS SUMMARY PRINTED BY THE V CORPS MAJOR TRAINING AREA SCHEDULING PROGRAM

constraints. In this case the one unit requesting training asked for a density of two; however, only one density was authorized during this period. As a result this unit was not scheduled.

This statistical summary is followed by a listing of the scheduled training and a listing of the training which was not able to be scheduled due to training area constraints. These schedules are available in three separate formats and the scheduler dictates which format or group of formats will be printed. The formats are designed to assist planners at varying levels of the chain of command.

Option one prints the schedule separated by major headquarters. The training is further grouped by requesting unit designator and type training. Figure 2 presents an example of the schedule format which is printed using option one. This figure lists the training scheduled for three units of the 3rd Armor Division. Notice that in this format, once a designator is printed, all of the training scheduled for that unit will be listed regardless of training area. For each approved training exercise the schedule identifies the training area, the period during which the training will take place, the authorized unit density for the training, and the priority assigned each training event.

Option two prints the schedule for each major headquarters by MTA. Figure 3 shows an extract from a schedule prepared using this format. The section of the schedule shown in Figure 3 lists three units from the 8th Infantry Division with training scheduled at the Baumholder MTA. In this format any training scheduled for areas other than Baumholder will be listed in a separate section under the appropriate training area heading. The schedule information given by this option is identical to the information obtained from option one. It is simply presented in a different format.

Option three gives the scheduled training by MTA and type training, regardless of major headquarters. Figure 4 gives an example of this format for the Grafenwoehr MTA. Due to space limitations only five of the six type training categories appear; in the actual printout the support category also appears. This option generates schedule information in this form for all four MTAs. If any type training category is not scheduled for a given MTA, then that heading will not appear on the schedule printout. Notice that each unit is identified with its major headquarters and that each training category is ordered by period. Notice also that within each period the training is ordered by priority. This feature is demonstrated by the training scheduled for the training category of weapons firing in Figure 4. The first four units all have training scheduled during period one and they are ordered by priority. The first two units listed, the 2-36 and the 3-36/3rd AD, have priority one; the third unit listed, the 1-36/3rd AD, has priority two; and the fourth unit, the 58/11th ACR, has priority three.

Summary

The computer assisted scheduling system is a feasible alternative to the completely manual system traditionally employed by V Corps. It provides the training officer with the capability to manage the training

SCHEDULED MTA TRAINING FOR: 3RD ARMORED DIVISION.

UNIT_DESIGNATOR:_3-32

TYPE_TRAINING:_TANK_GUNNERY,_LEVEL_1.

MTA	PERIOD	DENSITY	PRIORITY
GRAFENWOEHR	1	1	1
GRAFENWOEHR	7	2	2

TYPE_TRAINING:_FIELD_MANEUVERS._

MTA	PERIOD	DENSITY	PRIORITY
HOHENFELS	9	1	1

UNIT_DESIGNATOR:_2-32

TYPE_TRAINING:_TANK_GUNNERY,_LEVEL_1.

MTA	PERIOD	DENSITY	PRIORITY
GRAFENWOEHR	7	1	1

TYPE_TRAINING:_FIELD_MANEUVERS._

MTA	PERIOD	DENSITY	PRIORITY
HOHENFELS	9	2	2

UNIT_DESIGNATOR:_2-33

TYPE_TRAINING:_TANK_GUNNERY,_LEVEL_1.

MTA	PERIOD	DENSITY	PRIORITY
GRAFENWOEHR	1	1	1
GRAFENWOEHR	1	1	1

TYPE_TRAINING:_TANK_GUNNERY,_LEVEL_2.

MTA	PERIOD	DENSITY	PRIORITY
WILDFLECKEN	7	1	1

Figure 2 - AN EXAMPLE OF SCHEDULE FORMAT OPTION ONE PRINTED BY THE V CORPS MAJOR TRAINING AREA SCHEDULING PROGRAM

```
*****************************************************************************
     SCHEDULED TRAINING AT THE BAUMHOLDER  MTA FOR: 8TH INFANTRY DIVISION.
*****************************************************************************

------------------------------------------------------------------

UNIT_DESIGNATOR:_1-68
---------------------

TYPE_TRAINING:_FIELD_MANEUVERS._

PERIOD       DENSITY       PRIORITY
   8            1              1

------------------------------------------------------------------

UNIT_DESIGNATOR:_2-68
---------------------

TYPE_TRAINING:_FIELD_MANEUVERS._

PERIOD       DENSITY       PRIORITY
  15            1              2

------------------------------------------------------------------

UNIT_DESIGNATOR:_5-68
---------------------

TYPE_TRAINING:_FIELD_MANEUVERS._

PERIOD       DENSITY       PRIORITY
  19            1              2
```

Figure 3 - AN EXAMPLE OF SCHEDULE FORMAT OPTION TWO PRINTED BY THE V CORPS MAJOR TRAINING AREA SCHEDULING PROGRAM

SCHEDULED TRAINING AT THE GRAFENWOEHR MTA.

TYPE_TRAINING: TANK_GUNNERY, LEVEL_1.

UNIT	HEADQUARTERS	PERIOD	DENSITY	PRIORITY
2-33	3RD AD	1	1	1
3-32	3RD AD	1	1	1
2-33	3RD AD	1	1	1
1-11	11TH ACR	2	1	1
3-11	11TH ACR	2	1	1
2-11	11TH ACR	3	1	1
1-70	BDE 76	4	1	1
1-68	8TH ID	4	1	1
2-68	8TH ID	4	1	1
2-68	8TH ID	5	2	1
4-69	8TH ID	5	1	2
2-32	3RD AD	7	1	1
3-32	3RD AD	7	2	2

TYPE_TRAINING: TANK_GUNNERY, LEVEL_2.

UNIT	HEADQUARTERS	PERIOD	DENSITY	PRIORITY
3-68	8TH ID	1	1	1

TYPE_TRAINING: WEAPONS_FIRING.

UNIT	HEADQUARTERS	PERIOD	DENSITY	PRIORITY
2-36	3RD AD	1	1	1
3-36	3RD AD	1	1	1
1-36	3RD AD	1	1	2
~~58~~	~~11TH ACR~~	~~1~~	~~1~~	~~3~~
23	3RD AD	3	1	1

TYPE_TRAINING: ARTILLERY_FIRING.

UNIT	HEADQUARTERS	PERIOD	DENSITY	PRIORITY
2-92	42ND FA	1	1	1
~~3-16~~	~~8TH ID~~	~~1~~	~~1~~	~~1~~
2-83	41ST FA	2	1	1
1-40	3RD AD	2	1	1
2-75	41ST FA	2	1	3
1-83	8TH ID	3	1	1
2-81	8TH ID	3	3	11
1-2	8TH ID	4	1	1
2-27	3RD AD	5	2	1
~~2-20~~	~~BDE 76~~	~~5~~	~~1~~	~~1~~
2-6	3RD AD	6	1	1
2-3	3RD AD	6	1	1
6-9	42ND FA	7	1	1
2-5	41ST FA	8	1	1

TYPE_TRAINING: FIELD_MANEUVERS.

UNIT	HEADQUARTERS	PERIOD	DENSITY	PRIORITY
1-36	3RD AD	1	1	1
1-33	3RD AD	12	1	1
2-92	42ND FA	13	1	2
3-33	3RD AD	14	1	1

Figure 4 - AN EXAMPLE OF SCHEDULE FORMAT OPTION THREE PRINTED BY THE V CORPS MAJOR TRAINING AREA SCHEDULING PROGRAM

schedule according to existing priorities, while relieving him of the time consuming task of continually checking training area constraints. The computer program is written in FORTRAN IV programming language and requires a computer system with a 230,000 byte core storage capacity. The total Central Processing Unit time required for one run in less than three minutes. The computer hardware requirement necessary to support this system is satisfied by existing V Corps assets. Finally, the program output provides the V Corps training section with scheduling information in a form which can assist in the effective management of the MTA schedule and facilitate the dissemination of scheduling information.

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TITLE: CHOOSE

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ABSTRACT: CHOOSE is the name of a computerized system used to aid choosing the best candidate from a host of actively competing candidates. This system is used in conjunction with a user structured attribute system against which each of the candidates are judged. Traditionally these attributes are numerically scored and weighted into a mean overall score for each candidate. Upon receiving the mean overall scores and thus the corresponding ranking of the candidates, the decision maker is left wondering how much better the highest ranking candidate is than the next one on down the list, etc. CHOOSE directly addresses this problem by retaining the inherent variability residing in the attribute scores and weights. This nondeterministic characteristics of the weights and scores is used to determine how significantly better the top candidate is to the runner up, etc.

TITLE: CHOOSE

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CHOOSE is the name of a computerized system used to aid the Manager, the Board of Directors or the decision committee in determining what the risk is in selecting a single candidate vendor, contractor, design, etc., from a host of actively competing candidates. The usual structure set up for selecting the best candidate consists of establishing a set of attributes against which each candidate is judged. Each candidate is given a numerical score indicative of how well this candidate measures up to the ultimate quality level desired for each of the given attributes. Generally these quality levels for each attribute are reviewed by more than one expert. This introduces the possibility of creating some variation in the attribute scores (experts don't always agree). These attribute scores are then combined together into an overall score for each candidate. This task is generally accomplished by assigning numerical weights to each attribute, thus designating some relative importance or a ranking of importance to the various attributes. For convenience, these weights are usually structured to sum up to 100.0. However, managers are not always in 100% agreement on the relative importance of these various attributes. Thus, there also exists the possibility of variation in the attribute weights. The attribute weights and scores are lastly multiplied together and summed to yield an overall score for each of the candidates. The candidates are next ranked from the most desirable to the least desirable via the magnitude of the candidate's overall scores. Prior to making these computations, the attribute scores derived by each expert in his attribute area and the weights assigned to each attribute are averaged to remove the inherent variability residing in each of these factors. Upon receiving the mean overall scores and thus the corresponding ranking of the candidates, the decision maker is left wondering how much better the highest ranking candidate is than the next one on down the list, etc.. The decision maker has no way of knowing if the top candidate is significantly better than the second ranking candidate. It may be that the difference between the two top candidate's mean overall scores is in the noise level or it may be a very significant difference. Because the problem has been given a deterministic treatment, the decision maker cannot ascertain the significance of the candidates ranking.

CHOOSE directly addresses this problem by retaining the inherent variability residing in the attribute scores and weights. CHOOSE is a simulation approach which accommodates the variability of the scores and weights down at the attribute level by letting the individual attribute scores and the attribute weights be entered as historgrams or popularized distributions such as the normal, uniform,triangular, etc.. At this grass roots level, the statistical literature abounds with distribution estimating techniques. It is not the intent of this paper to discuss these techniques, but rather to illustrate the facility CHOOSE has for productively utilizing these techniques.

Upon starting each simulation iteration, the program randomly selects a set of attribute weights and attribute scores for each candidate from their respective distributions. The necessary multiplication of attribute weights times attribute scores for each candidate is made followed by the summing together of the weighted attribute scores for each candidate into an overall candidate score. The candidates are next ranked according to their overall score for this iteration. The candidate rankings for each iteration are progressively accumulated together to yield the number of times each candidate placed in each of the possible ranking positions.

Use of this system clearly demonstrates that in many selection problems the variance residing in the attribute scores and weights can dramatically effect the confidence level when making a candidate selection. For instance:

A 25 attribute selection problem, with the overall scores being able to range from 0 to 100, had mean overall scores of 88 and 84 respectively for the two top candidates. However, upon entering the level of variability expressed by the scoring experts and managers assigning the weights, the top candidate was the winner 91% of the time, while the runner up was the winner 9% of the time. In a similar 25 attribute selection problem, the top two candidates had mean overall scores of 81 and 69 respectively. However, when entering the level of variability expressed by the scoring experts and managers in the analysis, the top candidate was the winner only 57% of the time, while the runner up was the winner 40% of the time and the remaining 3% went to the other candidates. These two examples clearly indicate that the magnitude of the difference between the mean overall candidate's scores is not necessarily indicative of how much better or worse a given candidate is when compared to another.

When a low confidence selection occurs it is often advisable to critically review those scores and weights which tend to cause the candidates to be ranked in an order different from the mean ranking. The process of finding these critical scores and weights can be aided by the use of the sensitivity analysis features structured in CHOOSE. CHOOSE assigns each score and weights its mean value and then takes one given score and systematically increases it in value until the upper boundary of its distribution has been reached. Along the way up to the upper boundary, all the crossover points which caused a reranking of the candidates is noted, if any such crossover points exists. After running up to the upper boundary, CHOOSE returns this given score to its mean value and then systematically decreases it in value until the lower boundary of its distribution has been reached. Again, all the crossover points are noted. After all the scores are analyzed as previously described, the weights are also given the same analysis. Upon finding the critical scores and weights, it maybe advisable to spend some extra money to study the critical scores and weights. Perhaps some of the attributes were ill-structured or not inclusive enough or the scoring experts were not expert enough, etc.. At any rate, the sensitivity analysis should provide a starting point for augmenting the analysis made to date so that greater confidence can be achieved in choosing the winner.

Another feature CHOOSE has structured in it, is the ability to utilize weight constraints. It may be desirable to keep the magnitude of the weights in some specific rank ordering. Process wise this is accomplished by randomly selecting one distribution which will not be sampled this iteration. Next, a single random sample is selected from each of the remaining weight distributions. The value of the omitted distribution is computed by adding together all the randomly selected weights of the other distributions and then subtracting this total sum from 100. If the computed value for this lone unsampled distribution does not lie within its range, a new single random sample is again selected from the other weight distributions. Again if the value computed for the unsampled weight distribution does not lie within its range, the resampling process is repeated until an acceptable value for the unsampled distribution is found. Once a suitable combination of weights has been found, the weights are subjected to the weight constraints. If any of the constraints are violated, the whole process is started over from the beginning and repeated until a suitable randomly selected combination of weights has been found which will not violate any of the weight constraints.

Another feature CHOOSE has is the ability to accommodate selection problems which have been subfactored into attribute levels. For example, it may be advantageous to break a source selection problem up into three principle attribute areas of management, cost and technical ability. Under each of these three principal attribute areas, subattributes may be structured and under these subattributes other subattributes may be structured, etc..

The following very simplified actual problem illustrates how CHOOSE can aid in the process of choosing the winner. A candy manufacturer has two new candidate candies it is considering putting out on the market. However, there is only enough promotional money available for launching one of the two candidates. Historically, this company has used a four factor system to evaluate the market potential of candies. This system consists of (1) taste, (2) smell, and (3) looks. Looks was further subfactored into (3a) rich appearance and (3b) uniform texture. The Board of Director's preferences for weighting these factors into an overall score for each candidate maps into the following triangular distributions:

FACTOR	SUBFACTOR	MINIMUM	MOST LIKELY	MAXIMUM
1. Taste		37	42	47
2. Smell		28	33	38
3. Looks		20	25	30
	3a. Rich Appearance	45	50	55
	3b. Uniform Texture	45	50	55

Further, management is unanimously agreed that at the factor level all the weights should be constrained so that taste is always given greater emphasis than smell. The company conducted a market research study in which 5,000 people scored the two candies on a scale of 0 to 100 for each of the four

attributes. Plots of the responses for these four attributes fit the normal distribution and further passed the chi square of goodness of fit test at a high significance level. The resultant means (rounded) and standard deviation (rounded) are as follows:

ATTRIBUTE	CANDIDATE #1		CANDIDATE #2	
	Mean	Std. Dev.	Mean	Std. Dev.
Taste	87	2.7	73	2.8
Smell	67	2.3	72	2.9
Rich Appearance	62	3.1	77	3.4
Uniform Texture	81	3.6	84	3.3

Candidate No. 1 and No. 2's mean overall score was 76.5 and 74.5 respectively. The narrow spread between the mean overall scores may tend to lead the Board of Directors into thinking that it is a toss-up between the two candidates. However, inquiry into the significance of these scores via the use of CHOOSE clearly shows that candidate number 1 is a significantly better choice since it was the simulated winner 85% of the time.

CHOOSE has been coded in FORTRAN IV. The program is very flexible in being able to handle a wide range of problem sizes and structures. It will currently handle problems of up to 75 attributes and 1000 iterations in a 100K bytes region on IBM 360 gear. Large problems and more iterations are and can be accommodated, however, more core will be required. Users must code the attribute level structure, the weight constraints and the multiplication and addition of the weights and scores into overall scores for each candidate. There isn't any documentation for the program and this currently makes it difficult to use. However, CHOOSE has been well tested on many private industry problems and few military problems.

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TITLE: US Army Remotely Piloted Vehicle (RPV) Uncertainty Analysis

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ABSTRACT: The Army's present mini-RPV program is in the advanced development phase of its life cycle. At this point in the life cycle, any definite analysis is very difficult because:

a. Of the lack of a definite requirement(s).

b. The various type of equipment available and useful for RPVs, but in different stages of their development.

However, the Army needs from the developer, a better definition of programs, in terms of cost and schedule, earlier in the program. This paper is a result of the Army's need to define the cost and schedule of the RPV program. The process described was in fact used to develop the risk portion of the RPV outline development plan. The paper shows the analyst how to identify the areas of uncertainty within the program and the degree (level) of uncertainty for the requirements, program schedule, technical risk and cost estimating methodology. The paper also presents the results of the actual uncertainty analysis for the Army's RPV Program. Each subsystem risk level has been assessed with respect to the various contingencies. This paper also presents the results of the actual analysis for the Army's RPV program along with methods for other applications to other programs and equipment.

US ARMY REMOTELY PILOTED VEHICLE (RPV) UNCERTAINTY ANALYSIS

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INTRODUCTION

In making decisions in cost estimating, top management is faced with the difficulty of evaluating assumptions which involve uncertainty. In general, uncertainty and risk are used as synonymous terms. In decision theory, the distinction is made that the concept of risk deals with measurable probabilities while the concept of uncertainty does not. To perform a complete uncertainty analysis, the following elements must be considered:

a. Requirements uncertainty. This is due to the fact that the system configurations are subject to a variety of changes introduced during the development program.

b. Program schedule. Schedule slippage depends on requirement changes and unforseen major technical problems.

c. Technical risk. This can be caused by unforseen technical problems or technical changes.

d. Cost estimating methodology. Uncertainty about the point estimate due to variability of data and the techniques used to analyze the data.

The uncertainty level assessed for each subsystem is based upon definitions shown in the following table.

RISK ASSESSMENT

The RPV subsystems considered feasible have been reviewed for the requirement, schedule, technical risk and cost estimating technology uncertainty. These subsystems are:

a. Airframe.

b. Propulsion.

c. Flight Control.

d. Navigation.

UNCERTAINTY ANALYSIS GUIDE	VERY LOW UNCERTAINTY	LOW UNCERTAINTY	MEDIUM UNCERTAINTY	HIGH UNCERTAINTY	VERY HIGH UNCERTAINTY
REQUIREMENTS	Established. Requirement would not change.	Established. Possibility of requirement change.	Established, but full test program required to demonstrate.	Tests must be carried out to verify level established. Requirement could change.	Tests must be completed before achievable level can be set.
PROGRAM SCHEDULE	Schedule previously demonstrated achievable.	Must be adapted for the specific application. Schedule is achievable.	An achievable schedule can be set up. Slippages may occur.	Definite back-up programs must be planned to protect schedule.	Time can not be estimated; testing must continue until successful demonstration is achieved.
TECHNICAL RISK	System developed and tested satisfactorily. Required modifications will be minor.	Some engineering required, but well within existing technology.	Team work is needed to design. Tests required to demonstrate.	Team work required to design. More than one approach needs to be followed to assure success.	New design concept. Feasibility has to be demonstrated.
COST ESTIMATING METHODOLOGY	Have cost data to base estimate. Estimating error can be calculated.	Have similar items to compare cost.	Have similar items but costs not directly relatable.	Great deal of effort must be expended to get data base for cost estimate.	Development effort required to obtain cost estimating data base.

e. Mission Equipment.

f. Data Link.

g. Launch.

h. Recovery.

i. Ground Control Station.

j. Ground Support Equipment/Test Measurement and Diagnostic Equipment.

The uncertainty levels assessed to each subsystem are based upon the definitions shown in the following table--Uncertainty Analysis Guide.

The following is a brief description of the functions and characteristics of each subsystem with the uncertainty level assessed. This can be used as a guide to provide information to improve the rationality of the trade-offs.

Airframe:

a. Fixed Wing RPV. The fixed wing and rotary wing RPV airframes are made of plastics and fiberglass. The primary objective in the structure area is to make it lightweight, low cost, and with low radar cross section. The airframe for both types of vehicles should be designed to protect the electronics and sensors and be able to recover them in usable condition in case of crash.

(1) Requirements. Low uncertainty.

(2) Program Schedule. Very low uncertainty.

(3) Technical Risk. Very low uncertainty.

(4) Cost Estimating Methodology. Low uncertainty.

b. Rotary Wing RPV. There has been very little work done on rotary wing RPVs. They present a problem with high vibration, stability, and difficulty in achieving a hover, but they have an advantage in the launch and recovery operation.

(1) Requirements. Very high uncertainty.

(2) Program Schedule. Very high uncertainty.

(3) Technical Risk. Very high uncertainty.

(4) Cost Estimating Methodology. Very high uncertainty.

Propulsion:

Air-Cooled Reciprocating Engines. At present, the engine in use is an off-the-shelf adaptation from lawn mowers, chain saws, or go-carts. There is limited development effort being expended on new engines. Technical risks fall into three areas: (1) future horsepower requirements require development of a whole new engine (very low risk with the existing engines); (2) endurance--the present engines have not been able to stand up to the demands of longer endurance missions; (3) quality of the present engines is not high enough to allow purchase and direct installation of the engine without rework.

(1) Requirements. Very low uncertainty with the existing engines. High with a new engine program.

(2) Program Schedule. Very low uncertainty with the existing engines. High with a new engine program.

(3) Technical Risk. Very low uncertainty with the existing engines. Medium with a new engine design.

(4) Cost Estimating Methodology. Very low uncertainty with the present engine. Medium uncertainty with a new engine program.

Flight Control:

a. The Electrostatic Stabilizer. To reduce the skill level needed for the operator, it is felt that an autopilot is required. The electrostatic stabilizer does not provide a complete autopilot capability, since only the roll and pitch angles are held constant. An altimeter, magnetometer, and pitot tube sensor would have to be added to provide an altitude, heading, and velocity hold capability. The electrostatic equipotential planes are known to deviate from the horizontal near thunderstorms, and use of electrostatic sensors would pose a definite limitation on flights made during thunderstorms. Additional basic R&D work is required before it can be used successfully in RPVs. This stabilizer is radioactive and loses sensitivity due to radioactive decay requiring periodic replacement.

(1) Requirement. Low uncertainty.

(2) Program Schedule. Very low uncertainty.

(3) Technical Risk. Low uncertainty.

(4) Cost Estimating Methodology. Low uncertainty.

b. Environmental Reference Autopilot. This autopilot provides flight corrections based upon external environmental parameters such as static pressure, angle of attack, and geometric flux. The system is inexpensive and lightweight. The autopilot has a magnetometer for heading indication that can be in error while the aircraft is banked, due to the large dip of the geomagnetic vector in the temperate zones. Solar storm conditions affect the earth's magnetic field and would provide some limitation on the usefulness of a magnetometer. This system also lacks vertical reference that makes it less suitable for RPV missions requiring target location and artillery adjustment, without the addition of a position gyro to provide an accurate aircraft orientation capability. Technical risk is considered very low.

(1) Requirement. Low uncertainty.

(2) Program Schedule. Very low uncertainty.

(3) Technical Risk. Very low uncertainty.

(4) Cost Estimating Methodology. Very low uncertainty.

c. Position Gyro Autopilots. This autopilot system, in addition to the vertical gyro, has a directional gyro with a flux gate (magnetometer) and an automatic altitude control. This autopilot is heavier and more expensive than the other two. However, it provides an accurate directional reference during short periods of banked or maneuvering flight.

(1) Requirement. Low uncertainty.

(2) Program Schedule. Very low uncertainty.

(3) Technical Risk. Very low uncertainty.

(4) Cost Estimating Methodology. Low uncertainty.

Navigation:

There are two categories of operational mode for RPV navigation: First, the determination of the position of the RPV made at the ground control station (ground computation of position). Second, the determination made on board the RPV (airborne computation of position).

a. Ground Computation of Position. With this system, the RPV flight must be directed by ground command, via a data link. This data link can be vulnerable to enemy electromagnetic interference unless anti-jam techniques are incorporated in the system.

(1) Requirement. Medium uncertainty.

(2) Program Schedule. Medium uncertainty, depending on development results.

(3) Technical Risk. Medium uncertainty, considering the anti-jam protection that has to be incorporated in the data link.

(4) Cost Estimating. Medium uncertainty. The anti-jam technique selected has to be supported by a demonstration test.

b. Airborne Computation of Position. The RPV can be internally preprogrammed to fly to a sequence of selected waypoints without contact with the ground control station. Programmers are basically clocks which activate many functions at predetermined time intervals.

(1) Requirement. Very low uncertainty.

(2) Program Schedule. Very low uncertainty.

(3) Technical Risk. Very low uncertainty. A similar system has been used in other applications.

(4) Cost Estimating Methodology. Very low uncertainty.

Mission Equipment:

Perhaps the most progress to date has been made in the payload area. Small, low-cost television, laser, and flir systems have been developed. Some work is still necessary. Interchangeable payloads are being designed to perform unmanned airborne missions for real time surveillance, reconnaissance, and target acquisition, location, and designation.

a. Real-Time TV Surveillance. A TV camera will be used to detect tanks.

b. Real-Time TV and Photo Reconnaissance. A 35mm or 70mm panoramic camera will be added to the TV system.

c. Target Acquisition. A stabilized TV system with automatic tracking will be used to extend the RPV's daytime detection range.

d. Target Location and Artillery Adjustment. A laser range finder will be added to the stabilized TV system and boresighted through the camera to provide tracking accuracies.

e. Target Designation. A laser designator will be added to provide a designation capability for cannon-launched guided projectiles and "smart" bombs as well as laser guided missiles.

All of the above mission equipment require real time data. This is the primary benefit from RPVs, and it is obvious that jamming of this data link would negate the military usefulness. Therefore, appropriate anti-jam protection is essential. The technology is available; several techniques have been developed, and there is no particular method clearly superior for all applications. Rather, the optimum selection must result from a complex trade-off analysis. At this time, the risk assessment is considered the same for all payloads.

(1) Requirement. Very high uncertainty.

(2) Program Schedule. Very high uncertainty. Flight demonstration has to be conducted.

(3) Technical Risk. Very high uncertainty. Several techniques have been developed, and there is no particular method clearly superior for all applications.

(4) Cost Estimating Methodology. Very high uncertainty.

Data Link:

The general requirements for a tactical RPV are the command link, a sensor data (video) link, RPV position determination, and countermeasures protection. Real time sensor data is the primary benefit from the RPV, and it is obvious that jamming of this link would negate the military usefulness. Therefore, appropriate anti-jam protection is essential to maintain a useful RPV capability. The technology to provide these benefits is available. There are three distinct RF links, command, telemetry, and sensor required, and must be considered for anti-jam protection. Each of these three links has different requirements and, hence, requires a separate analysis. There is no particular method clearly superior for all applications. Rather, the optimum selection must result from a complex trade-off analysis.

Data Link Frequency Band Requirement. The majority of RPV data link systems demonstrations have used off-the-shelf RF components in the G-Band. However, DA has indicated that G-Band operation will not be available for RPVs and, therefore, recommended that J-Band be considered for future world-wide RPV application. A substantial effort is required to develop the J-Band RF components. The development is considered very high cost and schedule risk. This risk is applicable to any data link chosen for RPVs.

(1) Requirement. Very high uncertainty.

(2) Program Schedule. Very high uncertainty.

(3) Technical Risk. Very high uncertainty.

(4) Cost Estimating Methodology. Very high uncertainty.

Launch:

There is no great difficulty with the launch of mini-RPVs with a number of alternatives, including catapults, stored energy launchers, and the vehicle powered launch.

a. Catapult. The launcher for the Aquila uses a 300 psi air compressor. There are other possible power sources, including a mechanical spring or gas generator cartridge.

(1) Requirement. Very low uncertainty.

(2) Program Schedule. Very low uncertainty.

(3) Technical Risk. Very low uncertainty.

(4) Cost Estimating Methodology. Very low uncertainty.

b. Stored Energy Launcher. Energy is stored in a flywheel or spring which is dissipated to launch the RPV. The flywheel has a complex mechanism compared to the bungee system that is very simple, less costly, and more reliable. Any bungee system presents a hazard in the bungee cords either breaking or coming loose from the cart or other secure point. Technical risks are considered very low.

(1) Requirement. Very low uncertainty.

(2) Program Schedule. Very low uncertainty.

(3) Technical Risk. Very low uncertainty.

(4) Cost Estimating Methodology. Very low uncertainty.

c. Vehicle Powered Launch. This is accomplished with the vehicle powered or vehicle plus booster take-off in various configurations. The truck launch would be the most applicable configuration and does not add additional weight to the RPV. The truck launch would invoke some delay between launches as a new vehicle is mounted and prepared for flight.

(1) Requirement. Very low uncertainty.

(2) Program Schedule. Very low uncertainty.

(3) Technical Risk. Very low uncertainty.

(4) Cost Estimating Methodology. Very low uncertainty.

Recovery:

a. Net. This is the system now being used. The net catches the RPV when it is flown into it. Damage to the RPV from the abrupt stop and difficulty of removing the vehicle from the net are the major drawbacks to the net systems. The removal of an RPV from a net is a difficult task; it could be tangled in lines or suspended five or more feet above the ground. Nets, either vertical or horizontal, will require skill to land with undue damage to the RPV or develop an automatic landing system.

(1) Requirement. Medium uncertainty.

(2) Program Schedule. Medium uncertainty.

(3) Technical Risk. Medium uncertainty.

(4) Cost Estimating Methodology. Medium uncertainty.

b. Parachute. This system provides limited control of the descent. It could land in trees, rough terrain, or strike the ground in the wrong attitude, and it is at the mercy of the winds. The chute is folded on board the aircraft, and the mechanism needed to activate the chute will add a weight penalty to the RPV. The chute has to be repacked and inspected, and any aircraft parts destroyed with each landing have to be replaced. The advantage that the parachute offers is the operational flexibility. The system is not limited to any certain terrain and involves no set up or tear down to hamper tactical movements.

(1) Requirement. Low uncertainty.

(2) Program Schedule. Low uncertainty.

(3) Technical Risk. Low uncertainty.

(4) Cost Estimating Methodology. Low uncertainty.

c. Helium Bag with Vertical Straps. This system consists of two bags, one filled with helium and the other with air. Vertical straps are attached to the floating bag and to the air bag, which remains on the ground to form a vertical net about 30 feet high. The RPV flys into the

vertical straps and catches one on each wing tip. The vehicle then settles onto the airbag which is resting on the ground. Either bottled helium or equipment to locally generate the gas would be required. The bags (30' x 16' x 6') will be large and heavy, and they have to be deflated for transport. The bags will require the repair of any leaks that occur. The system will be susceptible to problems in a windy environment due to possible drifting and shifting. The bags have to remain inflated even when no recovery operations are underway, due to the time, cost, and difficulty involved in deflating and inflating the bags. This system is not operationally flexible; it requires set up and tear down time before the unit may be relocated. It is difficult to conceal.

(1) Requirement. Low uncertainty.

(2) Program Schedule. Low uncertainty.

(3) Technical Risk. Low uncertainty; does require development effort.

(4) Cost Estimating Methodology. Low uncertainty.

Ground Control Station:

The ground control station consists of a shelter that will house all the data processing controls and displays necessary for the planning, launch/recovery, enroute and terminal phases of RPV flight. The station has to be able to control several RPVs in various phases of a mission simultaneously, and it is required to be almost invulnerable to enemy detection, to be mobile, and to have a minimal tear down and set up time.

(1) Requirement. *High uncertainty.

(2) Program Schedule. *High uncertainty.

(3) Technical Risk. *High uncertainty.

(4) Cost Estimating Methodology. *High uncertainty.

* The risk is considered to be high because of the station required capability to control several RPVs in various phases of a mission simultaneously, and the technique used to minimize the station's vulnerability to enemy detection.

Ground Support Equipment/Test Measurement and Diagnostic Equipment:

The simplicity of the airframe and engine will not dictate requirements for extensive Ground Support Equipment. Test Measurement and Diagnostic

Equipment in support of the mission equipment appears to be the dominating requirement.

a. Ground Support Equipment. The RPVs ground starting provision would be incorporated into the launcher.

(1) Requirement. Very low uncertainty.

(2) Program Schedule. Very low uncertainty.

(3) Technical Risk. Very low uncertainty.

(4) Cost Estimating Methodology. Very low uncertainty.

b. Test Measurement and Diagnostic Equipment. The deployment of the RPVs indicates simple pre-flight checkout equipment. Fault isolating capability would be only to the major component. A complete diagnostic test capability would be appropriate at the maintenance level.

(1) Requirement. Very low uncertainty.

(2) Program Schedule. Very low uncertainty.

(3) Technical Risk. Very low uncertainty.

(4) Cost Estimating Methodology. Very low uncertainty.

c. Maintenance Facilities. The use of vans or shelters which are operational on their mobilizers appears to be the requirement for RPVs. Another significant need for shelters is to allow night operations or maintenance under blackout conditions.

(1) Requirement. Very low uncertainty.

(2) Program Schedule. Very low uncertainty.

(3) Technical Risk. Very low uncertainty.

(4) Cost Estimating Methodology. Very low uncertainty.

SUMMARY/CONCLUSION

This analysis has provided information for trade-offs rationality. The analysis placed in perspective each subsystem risk value with respect to various contingencies, and it is intended to give the decision maker the information to improve the rationality of his decision and a more accurate program cost estimating. The summary table shows the risk uncertainties levels for the RPV subsystems.

SUMMARY

REMOTE PILOTED VEHICLE (RPV) RISK UNCERTAINTIES LEVEL

	REQUIREMENTS	PROGRAM SCHEDULE	TECHNICAL RISK	COST ESTIMATE METHODOLOGY
AIRFRAME				
FIXED WING	LOW	VERY LOW	VERY LOW	LOW
ROTARY WING	VERY HIGH	VERY HIGH	VERY HIGH	VERY HIGH
PROPULSION				
AIR COOLED ENGINE	VERY LOW	VERY LOW	VERY LOW	VERY LOW
NEW ENGINE	HIGH	HIGH	MEDIUM	MEDIUM
FLIGHT CONTROL				
ELECTROSTATIC STABILIZER	LOW	VERY LOW	LOW	LOW
ENVIRONMENTAL REF AUTOPILOT	LOW	VERY LOW	VERY LOW	VERY LOW
POSITION GYRO AUTOPILOTS	LOW	VERY LOW	VERY LOW	LOW
NAVIGATION				
GROUND COMPUTATION OF POSITION	MEDIUM	MEDIUM	MEDIUM	MEDIUM
AIRBORNE COMPUTATION OF POSITION	VERY LOW	VERY LOW	VERY LOW	VERY LOW
MISSION EQUIPMENT				
REAL-TIME TV SURVEILLANCE	VERY HIGH	VERY HIGH	VERY HIGH	VERY HIGH
REAL-TIME TV & PHOTO RECONNAISS.	VERY HIGH	VERY HIGH	VERY HIGH	VERY HIGH
TARGET ACQUISITION	VERY HIGH	VERY HIGH	VERY HIGH	VERY HIGH
TARGET LOCATION	VERY HIGH	VERY HIGH	VERY HIGH	VERY HIGH
TARGET DESIGNATION	VERY HIGH	VERY HIGH	VERY HIGH	VERY HIGH
DATA LINKS				
REAL-TIME SENSOR DATA LINK	VERY HIGH	VERY HIGH	VERY HIGH	VERY HIGH
COMMAND LINK	VERY HIGH	VERY HIGH	VERY HIGH	VERY HIGH
RPV POSITION DETERMINATION	VERY HIGH	VERY HIGH	VERY HIGH	VERY HIGH
COUNTERMEASURES PROTECTION	VERY HIGH	VERY HIGH	VERY HIGH	VERY HIGH
LAUNCH				
CATAPULT	VERY LOW	VERY LOW	VERY LOW	VERY LOW
STORED ENERGY LAUNCH	VERY LOW	VERY LOW	VERY LOW	VERY LOW
VEHICLE POWER LAUNCH	VERY LOW	VERY LOW	VERY LOW	VERY LOW
RECOVERY				
NET	MEDIUM	MEDIUM	MEDIUM	MEDIUM
PARACHUTE	LOW	LOW	LOW	LOW
HELIUM BAG WITH VERT. STRAPS	LOW	LOW	LOW	LOW
GROUND CONTROL STATION	HIGH	HIGH	HIGH	HIGH
[illegible]				
GROUND SUPPORT EQUIPMENT	VERY LOW	VERY LOW	VERY LOW	VERY LOW
TEST MEAS. & DIAGNOSTIC EQT.	VERY LOW	VERY LOW	VERY LOW	VERY LOW
MAINTENANCE FACILITIES.	VERY LOW	VERY LOW	VERY LOW	VERY LOW

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1. Remotely Piloted Vehicle Concept Formulation.

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TITLE: A Confidence Region Procedure for Testing the Equality of More Than Two Means

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ABSTRACT:

A test procedure for the equality of morethan two means involving the intersection of $\mu_1=\mu_2=\ldots=\mu_k$ with the k-dimensional confidence ellipsoid on $\mu_1, \mu_2, \ldots, \mu_k$ is defined in this dissertation as a Confidence Region Procedure For Testing the Equality of More Than Two Means. The need for this procedure is shown in Chapter II. The procedure is developed in Chapter IV and practical applications of of the procedure are presented in Chapter V.

A History of Simultaneous Inference and Hypothesis Testing (Chapter III) includes both the philosophy of traditional null-hypothesis significance tests and the philosophy of the confidence-region procedure. A brief discussion summarizes the primary differences between inference and decision theory.

Both the generalized on-way Analysis of Variance (ANOVA) and the generalized confidence-region procedure are developed in this work. The distribution of the variables is restricted to the normal distribution with known and unknown variance. The sampling distribution is taken to be normal with a mean and standard deviation equal, respectively, to the unknown location and scale parameters of the related random variable distribution.

After developing the confidence region procedure to test the equality of more than two means, the author includes tables and calculation details which apply these procedures to two actual examples. The random variable selected for the first example is the fragment velocity in a detonator test while for the second example, the random variable is base overhead (O/H) rates for the various military installations in the United States.

A Confidence Region Procedure For Testing The Equality of More Than Two Means

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Jones and Karson (3) equated the elliptical confidence region test for $k=2$ populations directly to the standard Z test with known common σ. It would seem that a natural and logical analogy could be drawn between the elliptical procedure for $k > 2$ populations and the one-way ANOVA test. The development which follows is an algebraic generalization of the analogy between the ellipsoidal procedure for $k > 2$ populations and the one-way ANOVA test. Ultimately a relationship between their respective "F" values is derived. The theoretical development includes $k=2$, as a special case for completeness and clarity.

In general, the following approach is taken:

(1) For a given value of k, the one-way ANOVA test statistic for H_0 vs. H_1 is given algebraically, and the corresponding level α test procedure formulated. This test statistic is

$$F_{(k-1,n-k)} = \frac{\left[\sum_{i=1}^{k} n_i \bar{X}_i^2 - \frac{\left(\sum_{i=1}^{k} n_i \bar{X}_i\right)^2}{n}\right] \Big/ (k-1)}{S_p^2} \tag{1}$$

The test procedure is to reject H_0: $\mu_1 = \mu_2 = \ldots = \mu_k$ at level α if $F_{(k-1,n-k)} > f_{(k-1,n-k,1-\alpha)}$.

(2) A simultaneous ellipsoidal confidence region for $(\mu_1, \mu_2, \ldots \mu_k)$ at level $100(1-\gamma)$ is determined for the hypothesis testing problem which is at level α; H_0 is not rejected if $\mu_1 = \mu_2 = \ldots. = \mu_k$ intersects the ellipsoid.

(3) The form of an arbitrary ellipsoid for $\mu_1, \mu_2, \ldots \mu_k$ is

$$\sum_{i=1}^{k} \frac{(\overline{X}_i - \mu_i)^2}{a_i^2} = 1 \qquad (2)$$

Based on the fact that

$$\frac{n_1(\overline{X} - \mu_1)^2 + n_2(\overline{X}_2 - \mu_2)^2 + \ldots. + n_k(\overline{X}_k - \mu_k)^2}{kS_p^2} \qquad (3)$$

is distributed as $F_{(k,n-k)}$, the $100(1-\gamma)$ % simultaneous ellipsoidal confidence region for $\mu_1, \mu_2, \ldots \mu_k$ is given by

$$\left\{(\mu_1, \mu_2, \ldots \mu_k) \;\middle|\; \sum_{i=1}^{k} n_i(\overline{X}_i - \mu_i)^2 \le kS_p^2 \; f_{(k,n-k,1-\gamma)}\right\} \qquad (4)$$

Consider the expression from (12) given by

$$\sum_{i=1}^{k} n_i(\overline{X}_i - \mu_i)^2 = kS_p^2 \; f_{(k,n-k,1-\gamma)}.$$

This is equivalent to

$$\frac{\sum_{i=1}^{k} n_i(\overline{X}_i - \mu_i)^2}{kS_p^2 \; f_{(k,n-k,1-\gamma)}} = 1 .$$

It is now easy to see that the expression for a_i^2 in (2) can be written as

$$a_i^2 = \frac{kS_p^2 \, f_{(k,n-k,1-\gamma)}}{n_i} \qquad (5)$$

In summary then, the procedure developed in this paper labelled the ellipsoidal procedure is an alternate method to the ANOVA for testing the equality of more than two means. It is based on the simultaneous confidence region for $(\mu_1, \mu_2, \ldots, \mu_k)$ which is an ellipsoid in $(\mu_1, \mu_2, \ldots \mu_k)$ space as in 2 . Basically, the ellipsoidal procedure is to determine those combinations of $(\overline{X}_1, \overline{X}_2, \ldots \overline{X}_k)$ for which the H_0: $\mu_1 = \mu_2 = \ldots = \mu_k$ intersects the boundary of the ellipsoidal region given by 4 . Since in the case where H_0 is true, the expression for the ellipsoid can be reduced to a quadratic in the common mean, say μ, it is known that H_0: $\mu_1 = \mu_2 = \ldots = \mu_k$ and the ellipsoidal region will intersect if and only if the discriminant of a quadratic is non-negative, indicating real roots. It should be emphasized here that the assumptions underlying the ellipsoidal procedure are the same as those associated with the ANOVA. These are additivity, linearity, normality, independence and homogeneous variances. That is, the ellipsoidal procedure will be used for testing the

hypothesis H_0: $\mu_1 = \mu_2 = \mu_k$ based on independent random samples from normal populations with the usual assumption of homoscedasticity. The development begins with the symbolic generalization of the ANOVA, k=2.

ANOVA, k=2

Let $$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$ = pooled sample variance (6)

where $$S_1^2 = \frac{n_1 \Sigma X_1^2 - (\Sigma X_1)^2}{n_1(n_1-1)}$$ and $$S_2^2 = \frac{n_2 \Sigma X_2^2 - (\Sigma X_2)^2}{n_2(n_2-1)}$$ (7)

then

$$S_p^2 = \frac{(n_1-1)\left[\dfrac{n_1\Sigma X_1^2 - (\Sigma X_1)^2}{n_1(n_1-1)}\right] + (n_2-1)\left[\dfrac{n_2\Sigma X_2^2 - (\Sigma X_2)^2}{n_2(n_2-1)}\right]}{n_1 + n_2 - 2} \quad (8)$$

$$= \frac{n_1 n_2(\Sigma X_1^2 + \Sigma X_2^2) - n_2(\Sigma X_1)^2 - n_1(\Sigma X_2)^2}{(n_1 n_2)(n_1 + n_2 - 2)} \quad (9)$$

Let $\Sigma X_1 = n_1\overline{X}_1$; $\Sigma X_2 = n_2\overline{X}_2$; $\Sigma X_1^2 + \Sigma X_2^2 = \Sigma X_T^2$

Then

$$S_p^2 = \frac{n_1 n_2(\Sigma X_T^2) - n_2(n_1\overline{X}_1)^2 - n_1(n_2\overline{X}_2)^2}{(n_1 n_2)(n_1 + n_2 - 2)}$$

Therefore: $\Sigma X_T^2 = S_p^2(n_1 + n_2 - 2) + n_1\bar{X}_1^2 + n_2\bar{X}_2^2$ (11)

For the ANOVA test then:

$$F_{(1,n-2)} = \frac{\left[(\Sigma X_1)^2/n_1 + (\Sigma X_2)^2/n_2 - (\Sigma X_T)^2/(n_1+n_2)\right]\Big/1}{\left[\Sigma X_T^2 - \left\{(\Sigma X_1^2)/n_1 + (\Sigma X_2)^2/n_2 - (\Sigma X_T)^2/(n_1+n_2)\right\} - (\Sigma X_T)^2\right]\Big/(n_1+n_2)\Big/n_1+n_2-2} \qquad (12)$$

or equivalently

$$F_{(1,n-2)} = \frac{n_1\bar{X}_1^2 + n_2\bar{X}_2^2 - (n_1\bar{X}_1 + n_2\bar{X}_2)^2 \;/(n_1+n_2)}{S_p^2 \;(n_1+n_2 - 2)/(n_1+n_2 - 2)} \qquad (13)$$

Expanding and simplifying (13) results in

$$F_{(1,n-2)} = \frac{n_1n_2 \;(\bar{X}_1 - \bar{X}_2)^2}{(n_1+n_2) \;S_p^2} \qquad (14)$$

and H_0: $\mu_1 = \mu_2$ is rejected at level α if $F_{(1,n-2)}$ $f_{(1,n-2,1-\alpha)}$

<u>ELLIPSOIDAL PROCEDURE, k=2</u>

The ellipse for μ_1, μ_2 is given by

$$\frac{(\bar{X}_1 - \mu_1)^2}{a_1^2} + \frac{(\bar{X}_2 - \mu_2)^2}{a_2^2} = 1 \qquad (15)$$

where

$$a_1^2 = \frac{2S_p^2\, f_{(2,n-2,1-\gamma)}}{n_1}$$

$$a_2^2 = \frac{2S_p^2\, f_{(2,n-2,1-\gamma)}}{n_2} \tag{16}$$

Expanding (15) results in

$$a^2(\overline{X}_1^2 - 2\overline{X}_1\mu_1 + \mu_1^2) + a_1^2(\overline{X}_2^2 - 2\overline{X}_2\mu_2 + \mu_2^2) - a_1^2 a_2^2 = 0 \tag{17}$$

Letting $\mu_1 = \mu_2 = \mu$ say, (17) then becomes

$$\overline{X}_1^2 a_2^2 - 2\overline{X}_1\mu a_2^2 + \mu^2 a_2^2 + \overline{X}_2^2 a_1^2 - 2\overline{X}_1\mu a_1^2 + \mu^2 a_1^2 - a_1^2 a_2^2 = 0 \tag{18}$$

or

$$(a_1^2 + a_2^2)\mu^2 - (2\overline{X}_1 a_2^2 + 2\overline{X}_2 a_1^2)\mu + \overline{X}_1^2 a_2^2 + \overline{X}_2^2 a_1^2 - a_1^2 a_2^2 = 0 \tag{19}$$

The ellipse given by (15) and the line $\mu_1 = \mu_2$ will intersect if and only if the discriminant of (19) is non-negative.

The discriminant of (19) is given by

$$(-2\overline{X}_1 a_2^2 - 2\overline{X}_2 a_1^2)^2 - 4(a_1^2 + a_2^2)(\overline{X}_1^2 a_2^2)(\overline{X}_1^2 a_2^2 + \overline{X}_2^2 a_1^2 - a_1^2 a_2^2) \geq 0 \tag{20}$$

which reduces to

$$(\bar{X}_1 - \bar{X}_2)^2 \leq (a_1^2 + a_2^2) \tag{21}$$

or equivalently

$$(\bar{X}_1 - \bar{X}_2)^2 \leq 2\, S_p^2\, f_{(2,n-2,1-\gamma)} (1/n_1 + 1/n_2) \tag{22}$$

Therefore, solving for $f_{(2,n-2,1-\gamma)}$ results in

$$f_{(2,n-2,1-\gamma)} = (n_1 n_2)(\bar{X}_1 - \bar{X}_2)^2 / 2\, S_p^2 (n_1+n_2) \tag{23}$$

Equating (14) and (23) results in

$$F_{(1,n-2)} (n_1 + n_2)\, S_p^2 = f_{(2,n-2,1-\gamma)} (2)\, S_p^2\, (n_1 + n_2) \tag{24}$$

or

$$f_{(2,n-2,1-\gamma)} = F_{(1,n-2)}/2 \tag{25}$$

From the form of (14) and (23), the ellipsoidal confidence region test will be of level α, and hence the same test as (14) if $f_{(2,n-2,1-\gamma)}$ is chosen equal to $1/2 f_{(1,n-2,1-\alpha)}$. For this choice, the ellipsoidal confidence region on (μ_1, μ_2) has confidence coefficient $1-\gamma = G(f_{(1,n-2,1-\alpha)}/2)$ where G is the distribution function of the F distribution with 2 and n-2 degrees of freedom. Thus, the confidence coefficient for the ellipsoidal confidence region is the probability that an F distributed random variable with 2 and n-2 degrees of freedom is less than or equal to 1/2 the 100(1-α)th percentile point of the F distribution with one and n-2 degrees of freedom.

ANOVA (k = 3)

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + (n_3 - 1)s_3^2}{n_1 + n_2 + n_3 - 3} \tag{26}$$

where

$$s_1^2 = \frac{n_1(\Sigma X_1^2) - (\Sigma X_1)^2}{n_1(n_1 - 1)} \qquad s_2^2 = \frac{n_2(\Sigma X_2^2) - (\Sigma X_2)^2}{n_2(n_2 - 1)}$$

$$s_3^2 = \frac{n_3(\Sigma X_3^2) - (\Sigma X_3)^2}{n_3(n_3 - 1)} \tag{27}$$

using the identical algebraic argument as for k=2 case:

$$(\Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2) = s_p^2\ (n_1 + n_2 + n_3 - 3) + n_1\overline{X}_1^2 + n_2\overline{X}_2^2 + n_3\overline{X}_3^2 \tag{28}$$

or equivalently

$$\Sigma X_T^2 = s_p^2\ (n_1 + n_2 + n_3 - 3) + n_1\overline{X}_2^2 + n_2\overline{X}_2^2 + n_3\overline{X}_3^2 \tag{29}$$

The generalized ANOVA test statistic for k=3 can now be written as

$$F_{(2,n-3)} = \frac{\left[n_1\overline{X}_1^2 + n_2\overline{X}_2^2 + n_3\overline{X}_3^2 - \frac{(n_1\overline{X}_1 + n_2\overline{X}_2 + n_3\overline{X}_3)^2}{n_1 + n_2 + n_3}\right] / 2}{s_p^2\ (n_1 + n_2 + n_3 - 3)} \tag{30}$$

After simplification, (30) reduces to:

$$F_{(2,n-3)} = \frac{n_1 n_2(\bar{X}_1 - \bar{X}_2)^2 + n_1 n_3(\bar{X}_1 - \bar{X}_3)^2 + (n_2 n_3)(\bar{X}_2 - \bar{X}_3)^2}{(2)(n_1 + n_2 + n_3)S_p^2} \tag{31}$$

The ANOVA level-α test then is to reject H_0: $\mu_1 = \mu_2 = \mu_3$ if (31) is greater than $f_{(2,n-3,1-\alpha)}$;

or reject H_0 if

$$F_{(2,n-3)} > f_{(2,n-3,1-\alpha)} \tag{32}$$

<u>Ellipsoidal Procedure (k=3)</u>

The 3-dimensional ellipsoid is given by

$$(\bar{X}_1 - \mu_1)^2/a_1^2 + (\bar{X}_2 - \mu_2)^2/a_2^2 + (\bar{X}_3 - \mu_3)^2/a_3^2 = 1 \tag{33}$$

where

$$\begin{aligned} a_1^2 &= 3\ S_p^2\ f_{(3,n-3,1-\gamma)}/n_1 \\ a_2^2 &= 3\ S_p^2\ f_{(3,n-3,1-\gamma)}/n_2 \\ a_3^2 &= 3\ S_p^2\ f_{(3,n-3,1-\gamma)}/n_3 \end{aligned} \tag{34}$$

After expanding and substituting $\mu_1 = \mu_2 = \mu_3 = \mu$ say, (33) becomes:

$$\left[a_2^2a_3^2 + a_1^2a_3^2 + a_1^2a_2^2\right]\mu^2 - \left[2a_2^2a_3^2\bar{X}_1 + 2a_1^2a_3^2\bar{X}_2 + 2a_1^2a_2^2\bar{X}_3\right]\mu + \left[a_2^2a_3^2\bar{X}_1^2 + a_1^2a_3^2\bar{X}_2^2 + a_1^2a_2^2\bar{X}_3^2 - a_1^2a_2^2a_3^2\right] = 0 \tag{35}$$

The discriminant of the quadratic in μ given by (35) is equal to

$$\left[- 2a_2^2a_3^2\bar{X}_1 - 2a_1^2a_3^2\bar{X}_2 - 2a_1^2a_2^2\bar{X}_3\right]^2 - 4\left[a_2^2a_3^2 + a_1^2a_3^2 + a_1^2a_2^2\right]$$

$$\left[a_2^2a_3^2\bar{X}_1^2 + a_1^2a_3^2\bar{X}_2^2 + a_1^2a_2^2\bar{X}_3^2 - a_1^2a_2^2a_3^2\right] \geq 0 \tag{36}$$

Again, the ellipsoid given by (33) and H_0: $\mu_1 = \mu_2 = \mu_3$ will intersect if and only if (36) is non-negative. After expanding and simplifying, (36) can be reduced to

$$-(4a_1^2a_2^2a_3^4)(\bar{X}_1^2 - 2\bar{X}_1\bar{X}_2 + \bar{X}_2^2) - (4a_1^2a_2^4a_3^2)(\bar{X}_1^2 - 2\bar{X}_1\bar{X}_3 + \bar{X}_3^2) - (4a_1^4a_2^2a_3^2)(\bar{X}_2^2 - 2\bar{X}_2\bar{X}_3 - \bar{X}_3^2) + 4\,(a_1^2a_2^4a_3^4 + a_1^4a_2^2a_3^4 + a_1^4a_2^4a_3^2) \tag{37}$$

or equivalently

$$\cdot\, n_1n_2(\bar{X}_1 - \bar{X}_2)^2 + n_1n_3(\bar{X}_1 - \bar{X}_3)^2 + n_2n_3(\bar{X}_2 - \bar{X}_3)^2 \leq 3f_{(3,n-3,1-\gamma)}S_p^2(n_1+n_2+n_3-3) \tag{38}$$

Solving for $f_{(3,n-3,1-\gamma)}$ results in

$$f_{(3,n-3,1-\gamma)} = \frac{n_1n_2(\bar{X}_1 - \bar{X}_2)^2 + n_1n_3(\bar{X}_1 - \bar{X}_3)^2 + n_2n_3(\bar{X}_2 - \bar{X}_3)^2}{3\,S_p^2\,(n_1 + n_2 + n_3)} \tag{39}$$

Equating (31) and (39) we obtain

$$f_{(3,n-3,1-\gamma)}(3)\ S_p^2\ (n_1 + n_2 + n_3) = F_{(2,n-3)}(2)\ S_p^2\ (n_1 + n_2 + n_3) \qquad (40)$$

So that

$$f_{(3,n-3,1-\gamma)} = 2/3\ F_{(2,n-3)} \qquad (41)$$

As before, from the forms of (31) and (40), the ellipsoidal confidence region test will be of level c and hence the same test as (32) if $f_{(3,n-3,1-\gamma)}$ is chosen equal to $2/3\ f_{(2,n-3,1-\alpha)}$. For this choice, the ellipsoidal confidence region on (μ_1, μ_2, μ_3) has confidence coefficient $1-\gamma = G(2/3f_{(2,n-3,1-\alpha)})$, where G is the distribution function of the F distribution with 3 and n-3 degrees of freedom. Thus, the confidence coefficient for the ellipsoidal confidence region is the probability that an F distributed random variable with 3 and n-3 degrees of freedom is less than or equal to 2/3rds the $100(1-\alpha)$th percentile of the F distribution with two and n-3 degrees of freedom.

For arbitrary k, the ANOVA test can be written

$$F_{(k-1,n-k)} = \frac{\sum_{i \neq j\ 1}^{k} n_i n_j (\bar{X}_i - \bar{X}_j)^2}{(k-1) S_p^2 (n)} \qquad (42)$$

and the ellipsoidal procedure can be written

$$f_{(k,n-k,1-\gamma)} = \frac{\sum_{i \neq j}^{k} 1\ n_i n_j (\bar{X}_i - \bar{X}_j)^2}{(k)(n) S_p^2} \tag{43}$$

It can be seen by equating (42) and (43) that a general expression for the relationship between the computed F statistic for the ANOVA test and the statistic for the ellipsoidal procedure becomes

$$f_{(k,n-k,1-\gamma)} = (k-1)/(k)\ F_{(k-1,n-k)} \tag{44}$$

Expression (44) suggests that the ellipsoidal confidence region test will be of level α, and hence the same test as the ANOVA level α test if $f_{(k,n-k,1-\gamma)}$ is chosen equal to $(k-1)/k\ f_{(k-1,n-k,1-\alpha)}$. For this choice, the ellipsoidal confidence region on $(\mu_1 \mu_2, \ldots, \mu_k)$ has confidence coefficient $1-\gamma = G(\frac{k-1}{k}\ f_{(k-1,n-k,1-\alpha)})$ where G is the distribution function of the F distribution with k and n-k degrees of freedom. Thus, the confidence coefficient of the ellipsoidal confidence region is the probability that an F distributed random variable with k and n-k degrees of freedom is less than or equal to (k-1)/k times the 100(1-α)th percentile of the F distribution with k-1 and n-k degrees of freedom.

Note that when $n_1 = n_2 = \ldots = n_k = N$, (42) reduces to

$$F_{(k-1,n-k)} = \frac{N \sum_{i \neq j}^{k} 1 (\overline{X}_i - \overline{X}_j)^2}{(k)(k-1)S_p^2} \qquad (45)$$

and (43) reduces to

$$f_{(k,n-k,1-\gamma)} = \frac{N \sum_{i \neq j}^{k} 1 (\overline{X}_i - \overline{X}_j)^2}{k^2 S_p^2} \qquad (46)$$

Confidence Coefficient Determination

A computer was used to calculate tables of the critical values for F and the confidence coefficients. Picatinny Arsenal Mathematics/Scientific Library (M/SL) subroutines were utilized initially to generate an $F_{(\alpha,k-1,\ n-k)}$ table for six levels of α (.25, .10, .05, .025, .01, .005) with k-1 (numerator degrees of freedom) ranging from 2 through 10 in increments of 1, and the n-k (demoninator degrees of freedom) selected to correspond to those found in Table A-7c of Dixon and Massey (6), namely 5, 10, 15, 20, 24, 30, 40, 60, 120. Once this table was generated, a subroutine was prepared to adjust each F_α value by the appropriate factor (k-1)/k. These results are given in the six tables which are labelled "Modified $F_{(\gamma,k,\ n-k)}$ Table for Use With the Ellipsoidal Procedure". Another M/SL subroutine was then employed to develop the remaining six tables labelled "1-γ Values When Using the Ellipsoidal Procedure", by computing the

EXAMPLE

GROUP NUMBER

1	2	3	4
56	64	45	42
55	61	46	39
62	50	45	45
59	55	39	43
60	56	43	41

$H_0 = \mu_1 = \mu_2 = \mu_3 = \mu_4$

$\bar{x}_1 = 58.4 \quad \bar{x}_2 = 57.2 \quad \bar{x}_3 = 43.6 \quad \bar{x}_4 = 42.0$

$s_1^2 = 8.30 \quad s_2^2 = 29.7 \quad s_3^2 = 7.8 \quad s_4^2 = 5.00$

$n_1 = n_2 = n_3 = n_4 = N = 5; \quad s_p^2 = 12.70; \quad k = 4$

$$F_{(3,16)} = N \sum_{i \neq j}^{k} (\bar{x}_i - \bar{x}_j)^2 \quad k^2 s_p^2$$

$$= \frac{5\left[(58.4-57.2)^2 + (58.4-43.6)^2 + (58.4-42)^2 + (57.2-43.6)^2 + (57.2-42)^2 + (43.6-42)^2\right]}{(4)^2(12.70)}$$

$= \underline{22.3425}$

Critical value with 4,16 degrees of freedom, to equate ellipsoidal test with an ANOVA $(1-\alpha) = 0.95$ test $= 2.4$.

Because $22.34 > 2.4$; reject $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

Actual γ level $= 0.90$

ANOVA (1-ALPHA) = 0.95

MODIFIED $f_{(\gamma,k,n-k)}$ TABLE FOR USE WITH THE ELLIPSOID PROCEDURE

	2	3	4	5	6	7	8	9	10
5	3.305	3.862	4.058	4.152	4.192	4.242	4.270	4.285	4.293
10	2.480	2.735	2.783	2.784	2.714	2.759	2.748	2.729	2.718
15	2.270	2.454	2.468	2.448	2.416	2.391	2.371	2.347	2.331
20	2.175	2.327	2.325	2.296	2.249	2.228	2.196	2.178	2.151
24	2.130	2.267	2.257	2.224	2.182	2.151	2.118	2.098	2.070
30	2.085	2.214	2.190	2.152	2.107	2.074	2.039	2.018	1.989
40	2.040	2.154	2.130	2.088	2.041	2.005	1.969	1.938	1.908
60	2.000	2.101	2.070	2.024	1.914	1.928	1.899	1.867	1.836
120	1.960	2.048	2.010	1.960	1.908	1.868	1.829	1.796	1.764

ANOVA (1-ALPHA) = 0.95

1-Y VALUES WHEN USING THE ELLIPSOID PROCEDURE

	2	3	4	5	6	7	8	9	10
5	.878	.910	.922	.928	.931	.935	.937	.938	.939
10	.867	.901	.914	.921	.921	.929	.932	.933	.935
15	.862	.897	.910	.918	.922	.926	.929	.931	.933
20	.860	.895	.908	.916	.920	.924	.926	.929	.930
24	.859	.894	.907	.915	.919	.923	.926	.929	.930
30	.858	.893	.906	.914	.918	.922	.924	.928	.929
40	.857	.891	.905	.913	.918	.922	.924	.926	.927
60	.856	.890	.904	.912	.907	.919	.923	.925	.927
120	.855	.889	.903	.911	.915	.191	.922	.924	.926

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TITLE: Applications of Functional Analysis to Diophantine Optimization-Theory

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ABSTRACT: There does not exist an algorithm for solving arbitrary diophantine equations, integer-programming problems, or discrete optimizations. This paper shows that fixed-point theory is applicable to existence problems for solutions to non-linear diophantine equations, diophantine optimizations, and diophatine approximations. By considering common fixed *lattice* points for commuting families of continuous mappings in topological spaces, an *arithmetical* element is introduced into fixed-point theory and functional analysis. Then *general* techniques from functional analysis, which are supplementary to the usual theories of entire functions and meromorphic transcendental functions, are available for the solutions of number-theoretic problems (diophantine equations, integer-programming problems, and diophantine optimizations). Conversion methods from non-linear diophantine problems to fixed-lattice-point problems are given.

New formal relations are developed between (1) fixed-point theory and extreme-point theory and (2) fixed-point theory and Helly-type theory.

Applications of Functional Analysis to Diophantine Optimization-Theory

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"Everyone feels that there are analogies which cannot be expressed by a formula, and that they are the most valuable."

Henri Poincaré
Science and Method

1. Introduction. In 1970, it was established conclusively that there does *not* exist an algorithm for solving *arbitrary* diophantine equations, integer-programming problems or discrete optimizations. Indeed, this basic result quickly yields a *strengthened* form of Gödel's *second* incompleteness theorem: for any consistent axiomatic number theory, there exists a diophantine equation which has *no* positive integer solution, but such that this statement cannot be *proved* within the framework of the given axiomatization. This situation adds to the interest and importance of *special* algorithms for dealing with *special* classes of diophantine problems. But another basic question comes to mind; to what extent are *general* algorithmic methods still available for the solution of broad classes of diophantine problems in view of the non-existence of an algorithm for *all* diophantine problems? Indeed, such methods would be most valuable since it can be shown that there does not exist an algorithm for deciding of an arbitrarily given diophantine equation whether or not it has any prescribed number of solutions (e.g., one, even, odd or prime number of solutions).

This paper shows that fixed-point theory is applicable to *existence* problems for solutions to diophantine equations, diophantine approximations and diophantine optimizations. It is well known that fixed-point theory is applicable to problems for the existence of saddle points in general game-theoretic contexts and existence problems for the solutions of

certain classes of differential equations and integral equations. By considering common fixed *lattice* points for commuting families of continuous mapping in topological spaces, an *arithmetical* element is introduced into functional analysis and fixed-point theory. Then *general* techniques from (functional) analysis which are supplementary to the usual theories of entire functions or meromorphic transcendental functions (analytic number theory) are available for the solution of number-theoretic problems (e.g., diophantine equations, integer-programming problems and discrete optimizations). Conversion methods from diophantine problems to fixed-*lattice*-point problems are given. The technique for the determination of common fixed *lattice* points for families of continuous mappings is applicable to the problem of the existence of solutions of *specific* non-linear diophantine equations of Fermat type, Thue type, Skolem type, and Mordell type. Only the simplest finite-dimensional cases are considered. Other relations between fixed-point theory and lattice-packing theory will be developed elsewhere. New formal relations are developed between (1) fixed-point theory and extreme-point theory and (2) fixed-point theory and Helly-type theory.

2. Fixed-Point Preliminaries. Briefly, we discussed two of the simplest fixed-point theorems for convex sets. These are used and extended to diophantine systems. They can be generalized or extended in several directions. First, the so-called Brouwer-Schauder-Tychonov theorem states that if C is a compact convex set in a locally convex space and f is a continuous mapping of C into C then there exists $p \in C$ such that $f(p) = p$. As noted above, this theorem has many applications; another: Suppose D is a compact subset of Hilbert space H. Then D is convex iff for each point of H there exists a *unique* nearest point of D. Further, this important fixed-point theorem has *constructive* versions thereby facilitating the construction of solutions of various operator equations.

Secondly, the so-called Markov-Kakutani theorem states that if C is a compact convex subset of a linear topological space (*not* necessarily locally convex!) and if F is a non-empty commuting family of affine continuous mappings of C into C then there exists $p \in C$ such that $f(p) = p$ for all $f \in F$. It can be used in functional analysis to establish, among other results, the existence of invariant means on commutative groups.

3. New Fixed-Point Results. We extend an important case of results by K. Iséki [1] on common fixed points for contractive mappings to common fixed *lattice* points for *non-expansive* mappings [2]. Recall that non-expansive mappings include contractive mappings, orthogonal projections, and isometries. Note that a fixed point of a non-expansive mapping need *not* be unique (consider either the reflection of a closed disc about one of its diameters or the identity mapping). It will be basic to the present inquiry to investigate the mathematical *structure*

of the set of all common fixed points for communting families of various continuous functions. For simplicity, in theorem 1, we consider only finite-dimensional inner-product spaces and study the set of all common fixed points looking for a common fixed *lattice* point. It is here that we combined the geometry of numbers and fixed-point theory [3] [4].

Theorem 1. Let E^n be an n-dimensional Euclidean space with origin θ. Let B be a bounded closed convex subset of E^n. Let C be a non-empty commuting family of non-expansive mappings of B into B. If the (convex) body F of all common fixed points over C is disjoint with θ and contains a ball B^1 with Lebesgue measure $\mu(B^1) \geq 2^{n/2}\ (n/2 + 1)$ then F contains a common fixed *lattice* point $p \neq \theta$.

Note 1: If the body F is centrally symmetric about θ (hence $\theta \in F$) and with Lebesgue measure $\mu(F) \geq 2^n$ then, *trivially*, F contains a common fixed lattice point $p \in B$ with relatively prime coordinates; thus, $p \neq \theta$. Note 2: Theorem 1 can be generalized to Minkowski spaces but without guarantee that F is convex.

Proof: For each non-expansive mapping M in C, the set of all fixed points is non-empty (F. E. Browder), convex (since B is convex, E^n is strictly convex, and M is non-expansive), closed (continuity of M); thus, it is even compact. Hence, by commutativity of C and induction, finite intersections over C of sets of fixed points is non-empty, convex and compact. Since B is compact, the intersection over C of *all* sets of fixed points is non-empty, convex, and compact. Now apply Blichfeldt's theorem (or in the case of Note 1, Minkowski's theorem) on the existence of lattice points in *convex* bodies [5]. QED

Next, we give a common-fixed-Lattice-point extension of a result by A. A. Markov [6] and S. Kakutani [7] for topological vector spaces. Once again, for simplicity, we consider only the finite dimensional case and study the set of all common fixed points looking for a common fixed *lattice* point.

Theorem 2: Let L^n be an n-dimensional linear topological space with origin θ. Let S be a compact convex subset of L^n. Let C be a non-empty commuting family of affine continuous mappings of S into S. If the (convex) body F of all common fixed points over C is disjoint with θ and contains a ball B with Lebesgue measure $\mu(B) \geq 2^{n/2}(n/2 + 1)$ then F contains a common fixed *lattice* point $q \neq \theta$.
Note 1: If the body F is centrally symmetric about θ and with Lebesgue measure $\mu(F) \geq 2^n$ then, trivially, F contains a common fixed lattice point $q \in S$ with relatively prime coordinates; thus $q \neq \theta$.

Proof: For each affine continuous mapping M in C, the set of all fixed points is non-empty (Markov-Kakutani), convex (since M is affine), and closed (continuity); thus, it is even compact. Hence, by commutativity of C and induction, any finite intersection over C of sets of fixed points is non-empty,, convex, and compact. Since S is compact, the intersection over C of all sets of fixed point is non-empty, convex, and compact. As before, apply Blichfeldt's theorem or for Note 1, Minkowski's lattice point theorem. QED

Before proceeding further, we make five observations. First, the results so far weld together four basic kinds of notions: geometric (convexity, symmetry, affineness, and Lebesgue measure), arithmetical (lattice points and integers), algebraic (linear space), topological (compactness, continuity, and topological space). Secondly, techniques which establish the *constructive* existence of fixed points entail techniques for the constructive existence of solutions to non-linear diophantine equations. Thirdly, it should be clear that a whole *species* of theorems like theorems 1 and 2 exist in which one "mixes" the types of mappings used or allows one of the continuous mappings to be non-affine in Theorem 2 or non-non-expansive in Theorem 1. Of course, then, it need not follow that the non-empty set of all common fixed points is convex. Fourthly, Theorems 1 and 2 are identical for Euclidean spaces when all mapping under consideration are isometries (an isometry is a special case of both an affine mapping and a non-expansive mapping). Lastly, Theorems 1 and 2 provide a precise sense in which there are *qualitative* methods in number theory or topological methods in number theory.

4. Conversion. Fixed-point methods are already known to be useful for solving non-linear differential equations and integral equations. The following is a *general* technique for reducing an existence problem for a non-linear *diophantine* equation to a fixed-point problem: Suppose $F(x,y)$ is any real algebraic form linear in x. Consider the linear relation $F(x,y) = 0$. Suppose it has a unique solution $x = Gy$ for each y in some set S of a real vector space. Then a lattice-point solution in S of $F(x, x) = 0$ is logically equivalent to the existence of a fixed-lattice-point in S for the mapping G. I. e, the existence of solutions of a *specific* non-linear diophantine equation can be determined by means of the existence of a fixed-lattice-point for a transformation associated with a more general linear diophantine equation. E. g., given the non-linear diophantine equation

$1+x_1+x_1^2+x_1^3+x_1^4-x_2^2 = 0$. Convert to $1+x_1+x_1y_1+x_1y_1^2+x_1y_1^3-x_2y_2 = 0$.

Define continuous $G : E^2 \to E^2$ so that $\vec{x} = G(\vec{y})$. Then two fixed-lattice points of G are

$$(0, 1) \text{ and } (3, 11)$$

which are solutions of the given diophantine equation. Note that the second solution consists of distinct prime numbers alone.

We point out, again, that the solution of the original non-linear problem was obtained by solving a more general *linear* problem. Hence we have a clear sense in which it is incorrect to depreciate the value of linear problems by extolling the apparent flexibility and range of non-linear problems. Rather we need to understand even more general linear problems in arbitrarily many variables and arbitrarily high degree in *dummy* variables. It would seem that progress in the study of general systems has been more on the side of the introduction of new fundamental invariance relations than on investigations of the *negations* of well known ones.

In closing we note that this technique applied to the *quadratic* case of Fermat's Theorem, $x^2+y^2 - z^2 = 0$, results in a purely *linear* problem with infinitely many solutions.

5. Common Fixed Extreme Points and Exposed Points. Instead of checking fixed-point sets for the existence of lattice points, we turn to the problem of checking fixed-point sets for the existence of extreme points [8] [9] or exposed points [8] [10]. Recall that if f is a concave function and C is compact then f attains a minimum at an extreme point of C. Further, if C is convex as well, the extreme points of C form the *smallest* set by means of which, using only convex combinations, all points of C can be approximated. The relation of extreme points to linear programming is clear: in looking for an optimal solution we need only search the extreme points. More specifically, a feasible solution of a LP corresponds to an extreme point iff it is basic.

By way of review, we point out that extreme points depend on the linear structure of the underlying space but not on its topology. On the other hand, exposed points depend on the linear structure of the underlying space and on the topology of the space (the topology of the space determines the size of the conjugate space).

Theorem 3. Let S be a compact convex subset of locally convex topological vector space. Let C be a non-empty commuting family of affine continuous mapping of S into S. Let F be the (non-empty) set of all common fixed points over C. Then F contains *all* extreme points of S iff $C = \{I_S\}$, where I_S is the identity mapping on S.

Proof: By Markov-Kakutani F is non-empty, convex and compact. The "if" direction is trivial since S is compact and convex. The "only if" direction follows from the important theorem of Kreĭn-Mil'man by using the convexity of F: if S is a compact convex subset of a locally convex space and if $F \subset S$ then S is equal to the closed convex hull of F iff the closure of F contains all extreme points of S. QED

Of course, another basic question here is: determine necessary and sufficient conditions on C so that F contains at least one, but *not* all, extreme points of S. Such conditions require some kind of restriction since F may not contain any extreme point of S although S must necessarily contain an extreme point.

The next result is notable since it does not require compactness. It cannot be generalized from Hilbert spaces to Banach spaces for at least *two* reasons: although the fixed-point set F is non-empty and convex in Hilbert space, (1) F may be empty in Banach space and (2) F may not be convex Banach space.

Theorem 4: Let B be a bounded closed convex subset of Hilbert space. Let C be a non-empty commuting family of non-expansive mappings of B into B. Let F be the (non-empty) set of all common fixed points over C. Then F contains *all* extreme points of B iff $C = \{I_B\}$.

Proof: By F. E. Browder and others, F is non-empty, convex, and weakly compact. As before, the "if" direction is trivial. The "only if" direction follows from an extension of the Kreĭn-Mil'man theorem for certain non-compact sets. QED

As before, another basic question is: determine N&S conditions on C so that F contains at least one, but not all, extreme points of B. Of course, F will always have its own extreme points, but they may not be extreme points of B; similarly in Theorem 3 with "B" replaced with "S".

In closing this section, we note that Theorem 4 is nearly trivial when "bounded closed" is replaced with "compact". Even so, a satisfactory generalization to Banach spaces isn't available, since then F may not be convex although F is non-empty. Further, Theorems 3 and 4 remain valid, if one replaces "extreme points" with "exposed points". We point out that the ideas of this section are applicable to modern potential theory and barycentric measure-theory as well as to the problem of the extremal structure of fixed-point sets. Finally, we point out the possibility of an exceptionally strong type of fixed point, *viz.*, a common fixed extreme lattice point for a family of mappings. Clearly, there is a non-empty class of Integer Linear Programs for which the integer constraints turn out to be superflouous. This class is characterized by the property that *all* extreme points of the corresponding Linear Program are lattice points. However, if the corresponding LP has non-lattice extreme points then the integer constraints entail additional linear constraints. Although it is sufficient to generate the convex hull associated with the feasible lattice points, it is not necessary; indeed, it is often inefficient. It suffices to make the optimal integer solution an *extreme* point and *remove* all non-lattice extreme points with a more suitable objective-function value.

6. A New Kind of Fixed-Point Theorems. It is well known that fixed-point theory can be used to provide quick, if not simple, proofs of Helly's Theorem. On the other hand, there exist several simple proofs of Helly's Theorem [8] that make no use of fixed-point theory. Thus, one is led to the tantalizing question: can Helly's Theorem (whose proof does *not* depend on fixed-point theory) be used to establish new kinds of fixed-point results? Of course, it is hoped that a few of the new kinds of fixed-point results would be dimension-free. We give proofs for two new results on relations between combinatorial geometry and fixed-point theory discussed elsewhere without proof [11] [12]. These discussions treat what can be called fixed-point-extension theorems. I.e., common fixed points x_i ($i \varepsilon I$) over certain families F_i ($i \varepsilon I$) of continuous mapping can be used to establish the existence of a common fixed point y over a *broader* family B of continuous mappings. Here, "broader" is used in the strong substantive sense that $B \supsetneq \cup_{i \varepsilon I} F_i$ and "common fixed point" is used in the sense of a fixed point *common* to several mappings.

Theorem 5: Let $\{S_f : f \varepsilon F\}$ be a non-empty family of compact convex subsets of E^n. Let $\{C_f : f \varepsilon F\}$ be a class of non-empty commuting families of non-expansive mappings from S_f into S_f, $f \varepsilon F$. If each $(n + 1)$ or fewer members of $\{S_f\}$ have some fixed point over some members of $\{C_f\}$ in common then *all* members of $\{S_f\}$ have some fixed point over *all* members of $\{C_f\}$ in common.

Proof: For each pair S_f, C_f ($f \varepsilon F$), the set K_f of all common fixed points over C_f is non-empty (by hypothesis which is consistent with Browder's Theorem), convex (since E^n is strictly convex and acts through a non-expansive mapping), and compact (by continuity and convexity). Now, apply Helly's Theorem to these compact convex subsets $K_f \subsetneq S_f$ of common fixed points over C_f. Hence all members of $\{K_f\}$ have some point $p \varepsilon \cap_{f \varepsilon F} K_f$ in common; a *universal* common point. But the universal point p is a common fixed point over the family $\cup_{f \varepsilon F} C_f$. QED

Theorem 6: Let $\{T_f : f \varepsilon F\}$ be a non-empty family of compact convex subsets of an n-dimensional linear topological space.
Let $\{D_f : f \varepsilon F\}$ be a class of non-empty commuting families of affine continuous mappings of T_f into T_f, $f \varepsilon F$. If each $(n + 1)$ or fewer members of $\{T_f\}$ have some fixed point over some members of $\{D_f\}$ in common then *all* members of $\{T_f\}$ have some fixed point over all members of $\{D_f\}$ in common.

Proof: For each pair T_f, D_f ($f \varepsilon F$), the set L_f of all common fixed points over D_f is non-empty (by hypothesis which is consistent with the Markov-Kakutani Theorem), convex (since the mappings are affine), and compact (by continuity and convexity). As in Theorem 5, apply Helly's Theorem to the compact convex subsets $L_f \subsetneq T_f$. Hence there exists a *universal* point $q \varepsilon \cap_{f \varepsilon F} L_f$ such that q is a common fixed point over the family $\cup_{f \varepsilon F} D_f$. QED

In closing this section, we note:

(1) Each of the Theorems 5 and 6 can be used any place Helly's Theorem is used since each reduces to Helly's Theorem when the commuting families are identity mappings.

(2) Each of the Theorems extends a common point result for a family of sets to a common fixed point result over a family of mappings.

(3) It is known that Helly's Theorem holds in any Minkowski space. However, Theorem 5 does *not* generalize to arbitrary Minkowski space (the fixed-point sets needn't be convex), but Theorem 6 is most useful in any Minkowski space.

(4) Other relations between fixed-point theory and combinatorial geometry are given in [13]. Note that Lemma 2 of [13] is a double generalization of Helly's Theorem.

7. Stochastic Fixed-point Analogues. In closing this paper, we turn to a brief discussion of relations between stochastic-operator theory and fixed-point theory, i.e., between operator valued measurable functions and fixed-point theory. The purpose of this approach is two-fold:

(1) greater apparent generality and (2) to include probabilistic features as a preliminary step to establishing relations between fixed-point theory and fuzzy-sets theory. Theorem 7 is a stochastic analogue of the Markov-Kakutani Theorem in Banach spaces. It depends on:

Lemma. Let C be a compact convex subset of a Banach space. Let $T(\omega)$ be a continuous random operator from C into C. Then there exists a C-valued measurable function $f(\omega)$ such that $T(\omega)\, f(\omega) = f(\omega)$ a.s..

Proof: Clear. (The only difficult part is establishing the measurability of the fixed-point f(.)).

This Lemma, which can be generalized, can be used to establish the existence of solutions of *random* non-linear differential equations and integral equations.

Theorem 7. Let S be a compact convex subset of a Banach space. Let C be a non-empty commuting family of affine continuous random operators $T(\omega)$ from S into S. Then there exists a common fixed S-valued random variable over C; i.e., there exists an S-valued random variable $f(\omega)$ such that $T(\omega)\, f(\omega) = f(\omega)$ a.s. for every $T(\omega) \in C$.

Proof: Apply the Lemma and follow the proof for the *deterministic* Markov-Kakutani Theorem; as before measurability of f is difficult to show.

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TITLE: Rank Ordering of Laboratory Projects under Elastic Constraints: A Fuzzy Subsets Approach

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ABSTRACT: One important role in the management of a research and development organization is the selection and support of research and development projects under limited resources to effect a balanced program and to achieve the desired objectives. Two major constraints in the selection process considered in this study are the project cost growth and schedule slippage. There may be additional constraints such as performance, etc. Often such constraints are not well-defined and are elastic, reflecting the decision maker's preference and the need at the time. As part of the selection process, the constraints are compared and the projects ranked to provide a scoring for selection. In this paper, an approach through the theory of fuzzy subsets is presented to provide an orderly and consistent procedure to handle the elastic (fuzzy) nature of the constraints and to rank order the research and development projects under a fuzzy environment. The result of such rank ordering also provides a rational basis for a constraint trade-offs study, which is an important exercise in a search for viable alternatives and in budget allocation. It is hoped that continued effort in the study and application of fuzzy subsets will lead to useful support of military resource allocations in the years to come.

RANK ORDERING OF LABORATORY PROJECTS UNDER ELASTIC CONSTRAINTS: A FUZZY SUBSETS APPROACH

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INTRODUCTION

The mathematical research efforts in the US Army Aviation Research and Development Command (AVRADCOM) are directed primarily to the general domain of aerodynamics, propulsion, structures and decision analysis. The end results of these efforts contribute to fill the technological needs and requirements of advanced airmobile systems.

Decision risk analysis, a portion of the basic mathematical research efforts, is a subject which has been under continual development and refinement, and it has applied to various systems and programs within AVRADCOM and her Laboratories [1]. A significant accomplishment is the development of two algorithms [2]: cost impact and schedule variance. These algorithms are technically based in that the data to exercise the algorithms are collected from a technical risk assessment. The methodology not only allows the proper integration of technical aspects to cost and schedule, but also presents outputs accommodating "what if" types of questions. In particular, potential cost growth and schedule slippage consideration are exhibited, which provide a balanced appraisal for decision making.

These algorithms and risk approach were also applied to the project selection process [3] by highlighting the probable cost and schedule impacts. Based on the cost and schedule impact information, the decision maker is to compare and select the projects that are considered viable to achieve the set objectives. This paper mainly addresses to the concepts and techniques of making such selection.

DECISION ENVIRONMENT

Much of the decision making in project selection takes place in an environment in which the goals, the constraints, and the consequences of possible actions are not known precisely. For example, the condition to select a particular project may be that:

a. The acceptable cost growth should be in the range of 1 to 2 million dollars.

[ ] indicates references.

b. The maximum allowable schedule delay should not be significantly longer than 12 months.

c. The probability of success of the project should be about 0.8 or better.

In the above examples, the sources of fuzziness and imprecision are the underlined words of phrases. In addition, there is always the "what if" type of question that the decision makers can ask and should ask. The imprecised goals and constraints and the "what if" type of questions are not random or statistical in nature. They are the result of fuzziness and imprecision of the criteria of selection. In this study, the fuzziness arises from the elastic nature of cost growth and/or schedule delay as preferrenced by the decision maker and the need at the time.

Since the imprecisions are not random and statistical in nature, the problems should not be handled by the probabilistic approach and treated as if they were stochastic. The nature of this problem can be more properly handled by the postulate of an approach [4] which revolves around the theory of Fuzzy Sets [5].

PURPOSE AND SCOPE

The purpose of this paper is to describe briefly the concept of Fuzzy Subsets, to illustrate the usefulness of this concept in the context of project selection, and to suggest a technique to help quantify explicitly the imprecised goals and constraints. This quantification leads to rank in order the cost, schedule and perhaps performance and other impacts in a systematic, consistent and automatic manner in order to provide a rational and orderly basis for selection of projects.

ELEMENTS OF THE THEORY OF FUZZY SETS

The concept of fuzzy sets was first introduced by Professor L. A. Zadeh [5] in 1965 to deal with classes of objects encountered in the real physical world, which do not have precisely defined criteria of membership.

The theory of fuzzy sets is a body of concepts and techniques for dealing in a systematic way with classes whose boundaries are not sharply defined and in which an object may have a grade of membership intermediate between full membership and nonmembership. The principal motivation for this theory rests on the premise that much of human thinking involves the manipulation of fuzzy rather than nonfuzzy sets and is approximate rather than precise in nature.

The definition of fuzzy sets and their operations within the realm of decision making will be summarized in this section. However, before we go on to do so, it is necessary to distinguish between randomness

and fuzziness. Essentially, randomness has to do with uncertainty concerning membership or nonmembership of an object in a nonfuzzy set. Fuzziness, on the other hand, has to do with classes in which there may be grades of membership intermediate between full membership and nonmembership. The notion of a fuzzy set is nonstatistical in nature. To illustrate the point, the fuzzy assertion "Today's weather is fine" is imprecise by virture of the fuzziness of the term "fine." On the other hand, "The probability that it will rain today is 0.7" is a probabilistic statement concerning the uncertainty of the occurrence of a nonfuzzy event (rain).

Informally, a fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. A more precise definition may be stated as follows:

Definition: Let $X = \{x\}$ denote a collection of objects denoted generically by x. Then a fuzzy subset A in X is a set of ordered pairs:

$$A = \{(x, U_A(x))\}, \text{ for all } x \text{ in } X, \tag{1}$$

where $U_A(x)$ is termed the grade of membership of x in A, and $U_A: X \to M$ is a function from X to a space M called the membership space. When M contains only two points, 0 and 1, A is nonfuzzy and its membership function becomes identical with the characteristic function of a nonfuzzy set.

Example:
Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of costs (in million dollars) of research projects, such that

$$X = \{2, 3, 4, 5\}.$$

Then a fuzzy subset A, such that X is approximately 3.6 million dollars, could subjectively be given by

$$A = \{(0.5/2), (0.8/3), (0.9/4), (0.6/5)\}.$$

Equality: Two fuzzy sets are equal, A = B, if and only if

$$U_A(x) = U_B(x) \text{ for all } x \text{ in } X.$$

Empty: Fuzzy set A is said to be empty if and only if $U_A(x) = 0$ for all x in X.

Containment: A fuzzy set A is contained in or is a subset of a fuzzy set B, written as $A \subset B$, if and only if $U_A(x) \leq U_B(x)$ for all x in X.

Example:
Let the fuzzy subsets be

$$A = \{(0.5/2), (0.8/3), (0.9/4), (0.6/5)\},$$

$$B = \{(0.6/2), (0.9/3), (0.9/4), (0.8/5)\}.$$

By definition, $A \subset B$ implies:

$$U_A(x) \leq U_B(x) \text{ for all x in X.}$$

Intersection: The intersection of A and B is denoted by $A \cap B$ and is defined as the largest fuzzy set contained in both A and B. The membership function of $A \cap B$ is given by:

$$U_{A \cap B}(x) = \mathrm{Min}(U_A(x), U_B(x)), \text{ for all x in X,} \tag{2}$$

where Min $(a,b) = a$ if $a \leq b$ and Min $(a,b) = b$ if $a > b$.

Union: The union of A and B, denoted as AUB, is defined as the smallest fuzzy set containing both A and B. The membership function of AUB is given by:

$$U_{AUB}(x) = \mathrm{Max}\ (U_A(x), U_B(x)), \text{ for all x in X,} \tag{3}$$

where Max $(a,b) = a$ if $a \geq b$ and Max $(a,b) = b$ if $a < b$.

Example:
Using the same fuzzy subsets A and B,

$$AUB = \{(0.6/2), (0.9/3), (0.9/4), (0.8/5)\}.$$

Fuzzy Relations
A fuzzy relation R from a set $E = \{x\}$ to a set $F = \{y\}$ is EXF with membership function $U_R(x,y)$. EXF is a collection of ordered pairs (x,y), where x is in E and y is in F.

Example:
Let E = { Tom, Dick }, and F = { John, Jim }. Then a fuzzy relation of resemblance between members of E and F might be expressed as:

resemblance = { 0.8/(Tom, John), 0.6(Tom, Jim),
0.2/(Dick, John), 0.9/(Dick, Jim)} .

Alternatively, this relation may be represented as a relation matrix:

$$\begin{array}{ll} & \begin{array}{ll}\text{John} & \text{Jim}\end{array} \\ \begin{array}{l}\text{Tom} \\ \text{Dick}\end{array} & \begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.9 \end{bmatrix} \end{array}$$

in which the $(i,j)^{th}$ element is the value of $U_R(x,y)$ for the i^{th} value of x and the j^{th} value of y.

SIMPLE DECISION MAKING UNDER FUZZY SUBSETS

Let $X = \{x\}$ be a given set of alternatives. Also let $G = \{G_1, G_2, \ldots, G_n\}$ be a set of n fuzzy goals in the space of alternatives X, and $C = \{C_1, C_2, \ldots, C_m\}$ be a set of m fuzzy constraints also in the space of alternatives X. Then a decision D is the intersection of the goals $G_1, G_2, \ldots, G_n$ and the constraints $C_1, C_2, \ldots, C_m$. The decision membership function is:

$$U_D(x) = \text{Min}\ (U_{G_1}(x), \ldots, U_{G_n}(x), U_{C_1}(x), \ldots, U_{C_m}(x)). \tag{4}$$

In the above fuzzy decision D, we are assuming that all of the goals and constraints are, in a sense, of equal importance. There are some situations in which some of the goals and perhaps some of the constraints are of greater importance than others. In such cases, D might be expressed as a convex combination of the goals and the constraints, with the weighting coefficients reflecting the relative importance of the constituent terms. More explicitly, we may express $U_D(x)$ as:

$$U_D(x) = \sum_{i=1}^{n} \alpha_i U_{G_i}(x) + \sum_{i=1}^{m} \beta_i U_{C_i}(x) \tag{5}$$

where α_i and β_i are constants such that:

$$\sum_{i=1}^{n} \alpha_i + \sum_{i=1}^{m} \beta_i = 1$$

The optimal decision is defined as a maximizing decision among the alternatives, and its grade of membership can be expressed as

$$U_{DO} = \max U_D(x) \text{ for all } x \text{ in } X. \tag{6}$$

APPLICATIONS

The problem we are dealing with here is one of project selection. One aspect of the selection decision is to select the project that has the best probability of success. The constraints under consideration are the cost growth and schedule delay. According to the algorithms [2], the probability of success of project is a monotonic increasing function of cost and time. In other words, the more money and time given to the project, the more likely the project will have a higher probability of success. The decision maker then is to find a procedure to compare and to rank order the projects in terms of the probability of success under the imposed constraints of cost and time.

The additional cost and time allowable to complete the project is usually not well defined and is elastic, reflecting the decision maker's preference and the need at the time. These constraints are imposed by the decision maker and are not statistical in nature, and there is no randomness involved (in the sense of probability). They are qualitative statements reflecting the degree of preference. For example, the decision maker may want to impose, according to his experience and with respect to his understanding of the nature of the projects and his organization, that "I can tolerate the cost growth in the range of 1 to 2 million dollars," or "I do not want to see the cost growth to be too much beyond 1 million dollars," or "The schedule delay should not be significantly longer than 6 months." These types of propositions can be properly translated into quantitative measures through fuzzy subsets.

Let us now look at one fuzzy proposition "I can tolerate the cost growth in the range of 1 to 2 million dollars." In order to quantify the constraint, we may want to ask or help the decision maker to assign membership function to this fuzzy constraint. Assume that the result turns out to be that when the cost growth is at 0.5 million dollars, the decision maker is very satisfied and assigns a membership function of 0.9 to this cost growth. He then continues to assign a membership function of 0.6 to 1 million, 0.4 to 1.5 million, and 0.2 to 2 million, indicating that at 2 million dollars of cost growth, he is not satisfied at all. This membership function is summarized in the following table:

	COST GROWTH IN MILLION DOLLARS			
Constraint	0.5	1.0	1.5	2.0
U_C	0.9	0.6	0.4	0.2

The same procedure is applied in assigning membership function to schedule delay constraints.

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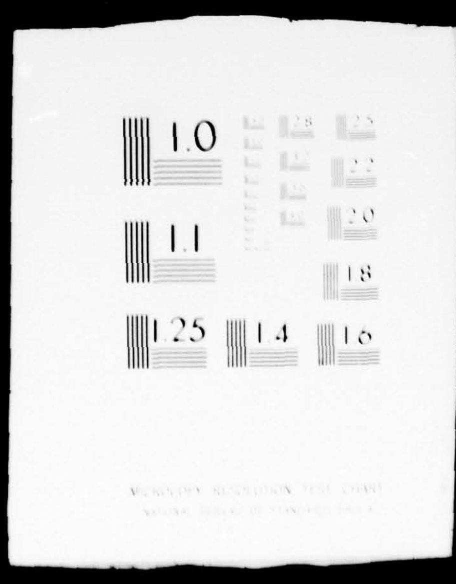

According to Bellman and Zadeh [4], a fuzzy decision is the confluence of fuzzy goals and fuzzy constraints as in equation (4). An optimal decision is a maximin procedure as in equation (6), and it can also be expressed as a convex combination of the goals and constraints, with weighting coefficients. Under the context of project selection, the objective is to select the project that has the highest probability of success. Under this context, the optimal decision is defined separately for each constraint as:

$$D_C = \max_j \max_i U_C(c_i)\, P_X(x_j | c_i), \text{ and} \tag{7}$$

$$D_S = \max_j \max_i U_S(s_i)\, P_X(x_j | s_i) \tag{8}$$

where D_C and D_S are optimal decisions respectively for cost and schedule constraints, $U_C(c)$ and $U_S(s)$ are the grades of membership of cost growth and schedule delay respectively, and $P_X(x|\ c)$ and $P_X(x|\ s)$ are the conditional probabilities of success of projects, with respect to cost growth and schedule delay respectively.

The optimal decision D for the combined consideration of both constraints is defined as:

$$D = \max_j \{\alpha \max_i U_C(c_i)\, P_X(x_j | c_i) + \beta \max_i U_S(s_i)\, P_X(x_j | s_i)\} \tag{9}$$

where constraints α and β are weights assigned to the constraints reflecting the relative importance of each of them.

In general terms, for n constraints y_i, i = 1,2,...,n, the decision policy is expressed as:

$$D = \max_j \sum \alpha_i \max_i U_Y(y_i)\ P_X(x_j | y_i) \qquad (10)$$

$$\sum \alpha_i = 1$$

where α_i are the assigned weights.

These decision policies will be applied to an example in the following section to illustrate the procedure of project ranking.

Example:
An example is taken from Reference [3] to illustrate the application of fuzzy sets to rank order the projects. Assume a set of five projects $X = \{x_1, x_2, x_3, x_4, x_5\}$. The cost growth and schedule delay, obtained from the two algorithms [2], are presented in Figures 1 and 2 respectively. The decision maker imposes fuzzy constraints to be "the allowable cost growth should be in the range from 8 to 9 million dollars," and "the schedule delay acceptable is around 14 months." Tables I and II summarize the grades of membership associated with the fuzzy constraints.

TABLE I

Cost Growth Constraint in 10^6\$	Grade of Membership $U_C(c)$	Project Probability of Success $P_X(x \mid c)$				
		x_1	x_2	x_3	x_4	x_5
8.0	0.7	0.3	0	0.15	0	0
8.5	0.5	0.31	0.67	0.46	0.19	0.03
9.0	0.1	0.31	0.7	0.51	0.57	0.57

TABLE II

Schedule Delay in Months	Grade of Membership $U_S(s)$	Project Probability of Success $P_X(x \mid s)$				
		x_1	x_2	x_3	x_4	x_5
13	0.8	0.35	0.74	0.04	0.13	0.07
14	0.5	0.35	0.74	0.13	0.16	0.45
15	0.3	0.35	0.74	0.22	0.22	0.64

Applying the decision policy presented in equations (7) and (8), the grades of membership associated with each project and the selection decision are summarized in Tables III and IV.

TABLE III

Cost Growth Constraints in 10^6 \$	Grade of Membership of Constraint	Grade of Membership of Project $U_C(c_i)P_X(x_j \mid c_i)$				
		x_1	x_2	x_3	x_4	x_5
8.0	0.7	0.21	0	0.11	0	0
8.5	0.5	0.16	0.34	0.23	0.10	0.02
9.0	0.1	0.03	0.07	0.05	0.06	0.06
U_D		0.21	(0.34)	0.23	0.10	0.06

TABLE IV

Schedule Delay Constraint in Months	Grade of Membership of Constraint	Grade of Membership of Project $U_S(s_i)P_X(x_j \mid s_i)$				
		x_1	x_2	x_3	x_4	x_5
13	0.8	0.28	0.59	0.03	0.10	0.06
14	0.5	0.18	0.37	0.07	0.08	0.23
15	0.3	0.11	0.22	0.07	0.07	0.19
U_D		0.28	(0.59)	0.07	0.10	0.19

If cost growth constraint is considered alone, project x_2 is selected. Similarly, if schedule delay is considered alone, again project x_2 is selected.

Now combining the cost and schedule and assigning weight to each constraint, with 0.7 assigned to cost and 0.3 to schedule, the decision policy is summarized below in Table V.

TABLE V

	Grade of Membership of Decision				
	x_1	x_2	x_3	x_4	x_5
U_D	0.23	0.42	0.18	0.10	0.11

In this case, project x_2 is selected.

Suppose project x_2 withdrew from competition. It is then easily seen from Tables III, IV and V that project x_3 is selected, if cost is the only constraint, and project x_1 is selected, if schedule is the only constraint. When both constraints are considered, project x_1 should be selected.

It is observed that decision is sensitive to the grade of membership assigned to the constraints. This reflects the subjectivity of the decision maker on the decision.

CONCLUSION

In this paper, we have briefly described an application of fuzzy subsets in ranking of projects, which leads to decision of selection. This ranking may be considered as a measure of effectiveness of the successful completion of laboratory projects within the cost and time constraints. The concept and approach presented here enable the decision maker to quantify the elastic constraints which are in the form of fuzzy propositions.

This type of imprecised constraints (fuzzy but not random) are often encountered in realistic environment under which alternatives are considered and decisions are made. The procedure presented in this paper provides a rational, systematic, and consistent method to handle such kind of constraints. It is not difficult to see that when the constraints become numerous, the situation can become complicated and confusing. This approach helps lay out before the decision maker a portfolio of his own utility and perference and perhaps his bias, and the effect of each constraint on the success of each project. With this portfolio available, the job of sensitivity analysis and trade-off study can be more easily handled. This exercise of sensitivity and trade-off study is important in the search of viable alternatives and in budget allocation.

The theory of fuzzy subsets is still at the stage of development. Research effort is needed and applications are still to be drawn from such concept. Fuzzy subsets relate well to the framework of human information processing and natural language comprehension, and the author believes that they describe more adequately than the probabilistic approach the environment under which many decisions are made.

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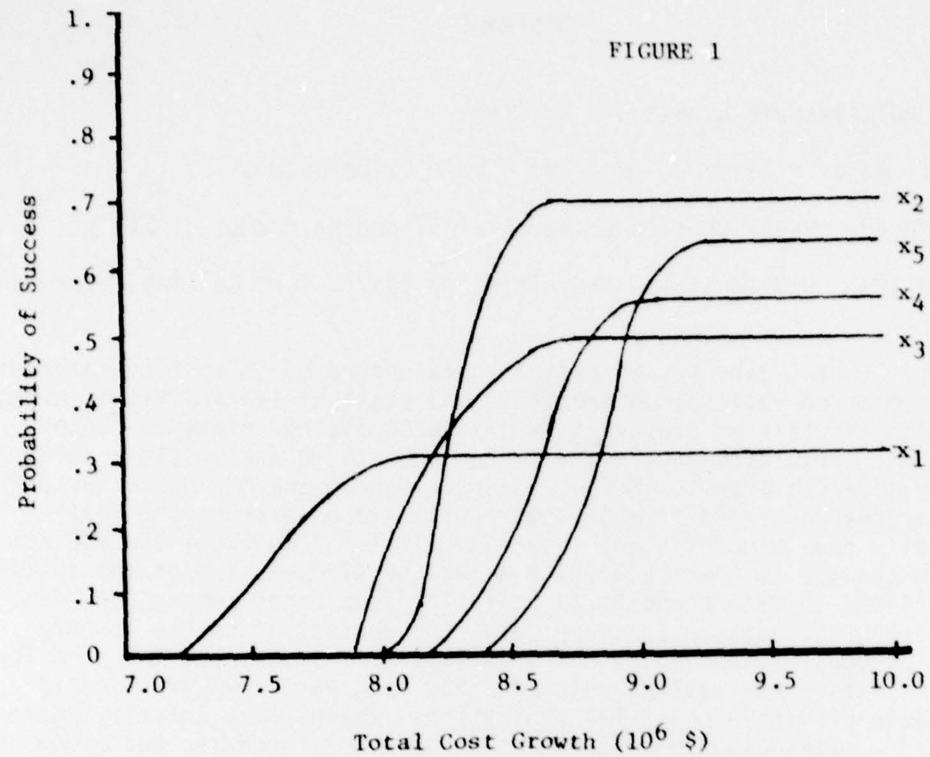
FIGURE 1
Probability of Success
1.
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
7.0
7.5
8.0
8.5
9.0
9.5
10.0
Total Cost Growth (10^6 \$)
x_2
x_5
x_4
x_3
x_1

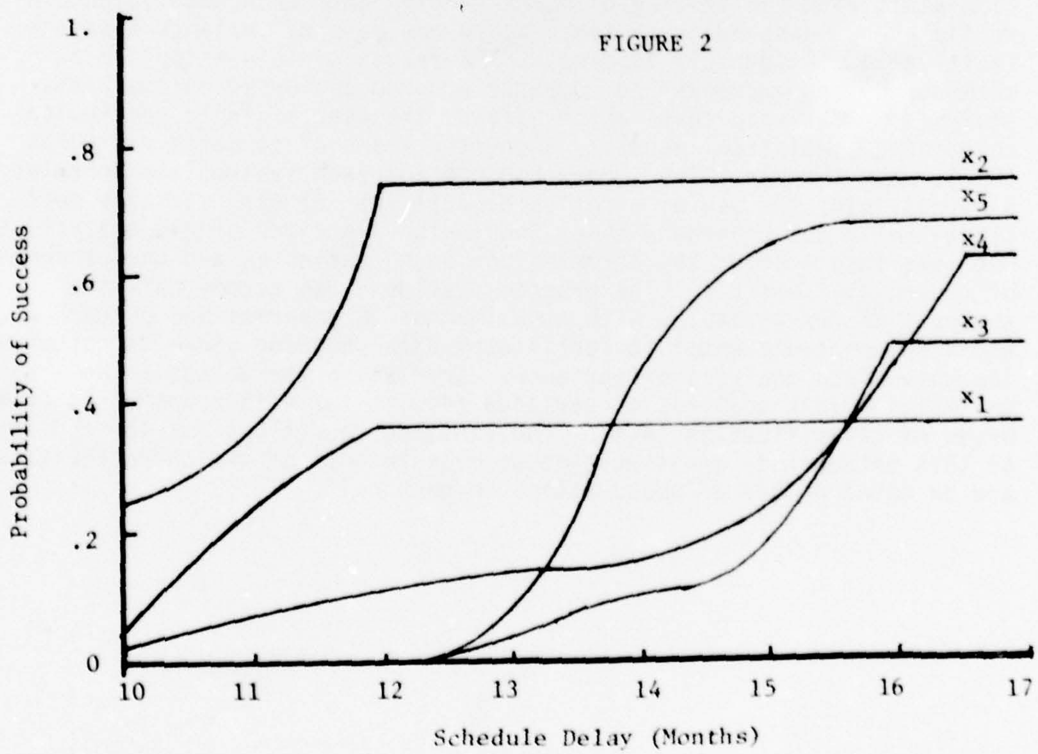
FIGURE 2
Probability of Success
1.
.8
.6
.4
.2
0
10
11
12
13
14
15
16
17
Schedule Delay (Months)
x_2
x_5
x_4
x_3
x_1

ABSTRACT

TITLE: Multivariate Hypothesis Testing

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PRESENTED BY: Major Charles R. Macchiaroli and Mr. James B. Widby

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ABSTRACT: In the analysis of ordinal data generated in testing materiel systems in an operational environment, the statistician/analyst is often faced with the task of producing an all encompassing inference (assessing the net worth/effect) about a system when there are multiple related measures of effectiveness (MOE). For cases where the MOE or characteristics are weighted and/or when the results are unanimous, the analyst can readily draw a conclusion. However, all too frequently the MOE are not weighted nor is there unanimity among the findings and/or the analyst is interested in data snooping (a posteriori unplanned comparisons aimed at exploring the data). In these cases the analyst often has to rely heavily on subjective evaluations or analysis. To minimize the need for these subjective evaluations which for the most part are performed in the absence of viable objective evaluations, the US Army Infantry Board conducted a methodology investigation to develop procedures and accompanying computer programs to facilitate objective analyses. This investigation dealt with the testing of means (equivalence or nonequivalence of multiple MOE means among systems) using analysis of variance approach (multivariate hypothesis testing). The result of this effort is a methodology and accompanying computer program employing Multivariate Analysis of Variance tests which affords the user a single conclusion concerning statistical equality among the means of compared variables (MOE), when the variables (more than one for each system) are correlated. Alternatively, the use of a series of analyses for each variable most likely would not present a clear conclusion since the single analysis does not take into account the correlations among variables and the presence of synergistic effects. The program developed can accommodate the analysis of ten variables with a maximum of 30 observations on each 5 row and 4 column treatments; it facilitates data snooping since it calculates the univariate analysis of variance, correlation variables, as well as the multivariate analysis of variance results for either one basis or two bases of classification tests. The limiting conditions for the application of this methodology are simultaneous measurements of the characteristics and an equal number of observations in each cell.

MULTIVARIATE HYPOTHESIS TESTING

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Major Charles R. Macchiaroli (Editor)
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Test and evaluation of equipment at the US Army Infantry Board (USAIB) consists of a series of subtests. Each subtest is designed to obtain data to evaluate the test item's performance against one or more Issues for Test and Test Criteria. These issues and criteria are developed by the test item's proponent (combat developer) and concern functions of the military system in an operational environment, such as firepower, mobility, survivability, and reliability. The issues and criteria are derived from the proponent's Independent Evaluation Plan and from the Coordinated Test Program.

The test requirements documents normally include both qualitative and quantitative criteria. The qualitative issues and criteria are addressed based on typical user reactions to the test item, man-equipment interface problems in employing the item in a field environment, and military utility judgments by USAIB test supervisory personnel. The quantitative issues and criteria are addressed based on visual or electronic counting or measuring procedures.

The test item's performance is also compared to the standard equipment that the test item is designed to replace. These comparisons are based on typical user employment of both items in situations representative of their prospective use. On occasion, multiple interrelated test items are under evaluation and are each being compared to a control (or standard) item.

The operational mode summary (OMS) and/or mission profiles (MP) included in the ROC or DP describe the anticipated mix of operational modes and their expected percentages of use in different operational and environmental conditions. The OMS and/or MP impose constraint on USAIB testing. However, their inclusion in the test assures realistic control of the situational variables.

For each subtest for both test and control items under specific expected operating conditions, more than one characteristic can be measured simultaneously with current USAIB instrumentation and Automatic Data Processing Equipment (ADPE). Selected measurements of the test item can be compared to the testing requirement or to the measurements of the control item. The wording of the requirement dictates the type comparison to be used. Normally, the comparisons made concern the equality of several means, whether or not there is a significant difference among the means.

In the analysis of ordinal data generated in testing materiel systems in an operational environment, the statistician/analyst is often faced with

the task of producing an all encompassing inference (assessing the net worth/effect) about a system when there are multiple related measures of effectiveness (MOE). For cases where the MOE or characteristics are weighted and/or when the results are unanimous, the analyst can readily draw a conclusion. However, all too frequently the MOE are not weighted nor is there unanimity among the findings and/or the analyst is interested in data snooping (a posteriori unplanned comparisons aimed at exploring the data).

Previously, the testing of the means was accomplished by the analysis of variance approach in which a simple analysis is made for each characteristic (variable). The result of the individual analyses are then combined in order to arrive at an overall conclusion with respect to all the characteristics taken together. This approach is subjective and fails to consider either the presence of synergetic effects or the relationship among the variables. To minimize the need for these subjective evaluations, Major William J. Owen and Mr. Jimmie C. Deloach, formerly of the US Army Infantry Board, conducted a methodology investigation to develop procedures and accompanying computer programs to facilitate objective analyses. Although there are several MVA techniques available to the analyst, this investigation is limited to multivariate analysis of variance (MANOVA) and deals with the testing of means (equivalence or nonequivalence of multiple MOE means among systems) using the analysis of variance approach (multivariate hypothesis testing).

Although the MANOVA application of MVA is recognized as being viable and offers much potential, the application is not widespread. Some reasons which account for the limited application are: it is a supplementary tool of analysis of variance (it is not the panacea); the description and procedures are for the most part fragmented among various references; and, the procedures are considerably more complex than the common univariate analysis of variance tests.

The result of this effort is a methodology and accompanying computer program employing Multivariate Analysis of Variance tests which affords the user a single conclusion concerning statistical equality among the means of compared variables (MOE), when the variables (more than one for each system) are correlated. Alternatively, the use of a series of analyses for each variable most likely would not present a clear conclusion since the single analysis does not take into account the correlations among variables and the presence of synergistic effects. The program developed can accommodate the analysis of ten variables with a maximum of 30 observations on each 5 row and 4 column treatments; it facilitates data snooping since it calculates the univariate analysis of variance, correlation variables, as well as the multivariate analysis of variance results for either one basis or two bases of classification tests. The limiting conditions for the application of this methodology are simultaneous measurements of the characteristics and an equal number of observations in each cell.

The required design characteristics for the MANOVA to be developed were that the procedures be logical, simple, and adaptable to ADPE.

Accordingly, the normal procedures ascribed for MVA were followed. Concurrently, with this process all methods were verified through the use of example problems from various references.

To exemplify the program's application, data from the Accuracy and Zero Correlation subtest of the Development Test II (Service Phase)/ Operational Test II of the Rimfire Adapter (RFA) for the M16A1 rifle were analyzed. Specifically, the four measures of dispersion were evaluated concurrently by treatment and range using a two-way MANOVA. The results of the application of this program agree with the RFA test report. The MANOVA utilized the correlation between variables whereas in the univariate analysis this information was disregarded. In the univariate analysis, four comparisons (one for each measure of dispersion) among the three RFA's had to be analyzed for each range while in the multivariate analysis, conclusions are provided from a single set of comparisons at each range. Accordingly, when correlation exists among variables, the MVA is superior to the univariate analsis.

During the test of the RFA, the following measures were employed:

a. Mean Offset ($\bar{O}$). The mean distance from the point of aim to the shot group centers of impact.

b. Mean Radius ($\bar{R}$). The mean distance from the center of impact of the shot groups to each of the impact points.

c. Average Extreme Spread ($\overline{ES}$). The mean of the distance between the two most extreme impact points for each of the shot groups.

d. Mean Drop ($\bar{D}$). The mean of the vertical components of the offsets.

The data for these four measures were collected in accordance with the approved test plan. Three groups of soldiers used one of the three RFA devices to engage a Modified Canadian Bull Target at 25, 42, 75, 100, and 125 meters from the prone supported firing position. Each of the soldiers fired three 5-round shot groups at each of the targets. Based on the x, y coordinates of each round of each shot group for each soldier for each RFA device, the means and standard deviations of each of the four characteristics were computed.

Univariate two-way analyses of variance were performed on each of the characteristics using range and device as factors. In this test there were three treatments (i.e., devices RFA-A, RFA-B, and RFA-C) and four measures of dispersion ($\bar{O}$, $\bar{R}$, $\overline{ES}$, and $\bar{D}$) for five ranges. For each range, each measure, and each treatment there are ten replications (i.e., 10 soldiers).

A two-way MANOVA was executed using range and device factors and the four measures as variables. The results of this MANOVA are shown in this table (Table 1).

Because of the significant interaction between device and range, comparisons based on the outcome of the two-way MANOVA were not made between the individual devices and ranges.

Five one-way MANOVA's and four follow-up Hotelling's T^2 tests were executed (i.e., one for each range). The results of these MANOVA's are shown in Table 2 and 3. A summary of conclusions based on these results appears in Table 4.

The results in Table 4 indicate that RFA-A and RFA-B are not significantly different except at a range of 42 meters. Also, RFA-A and RFA-C are significantly different at every range considered except 25 meters. Finally RFA-B and RFA-C differ significantly at every range except 25 meters.

A comparison of the multivariate results in Table A-III-30 of the RFA test report appears in Table 5. A summary of some of the more important points of this comparison follows:

a. Overall, the MVA conclusions confirm the univariate findings with the exception of the 42-meter range. The univariate findings show adapter B equivalent to adapter C for each variable, while the MVA findings show adapters B and C to be nonequivalent at a range of 42 meters when all variables are taken together. The difference in two findings is a direct result of the univariate analysis' failure to take into account the correlation among the variables.

b. Note that for each range, there are 12 univariate test conclusions as opposed to three multivariate conclusions. The univariate method does not permit the formulation of a joint conclusion with regards to the equivalence (or nonequivalence) of the three adapters for a particular range as does the MVA method. Further, a given set of univariate conclusions, one for each variable under consideration, will not always agree with the MVA conclusions when the variables are being considered simultaneously.

c. For the reasons indicated above the MVA approach is superior to a series of univariate analyses when the variables are correlated.

The example illustrates that the methodology and programs developed are applicable in determining correlation between variables; performing univariate analyses when no correlation exists; and conducting multivariate analyses when correlation is present.

The following two figures (1 and 2) depict the flow chart of the computer program used to conduct the analyses of the example just illustrated.

The procedures outlined above have been programmed in FORTRAN IV. The response data is the input and the program uses four subroutines. "HOTEST" tests the dispersion matrix for homogeneity and compound symmetry. "FTEST I" and "FTEST 2" perform the MANOVA's and compute the F-values.

"HOTEL" performs the Hotelling's T^2 on each combination of two treatments.

All four subroutines are used for each set of input data when the data can be classified in two ways. However, when the data can be classified in only one way, FTEST 2 is not executed. Also, when a value obtained from FTEST 1 is not significant, the results from HOTEL may be ignored.

A user package has been developed which explains the control cards and the correct procedure for the input data. It also includes a program listing (PROGRAM MULTVAR) which is currently on the CDC 6400 at Fort Leavenworth, Kansas. The original program was designed for the IBM 360 computer using FORTRAN IV. That program can be obtained through DDC by requesting the report whereas a copy of the MULTVAR Program can be obtained by contacting the Management Division of the USAIB and requesting it.

TWO-WAY MANOVA RESULTS
RFA COMPARISONS

SOURCE	S	M	N	XOBS	XTAB	RESULT	CONCLUSION
ROW (device)	2	.5	215	.212	.045	SD	Compare devices by range using 1-way MANOVA
COLUMN(range)	4	-.5	215	.570	.042	SD	
RXC(interaction)	4	1.5	215	.079	.060	SD	

Table 1

Decision Rule: If $X_{OBS} \geq X_{TAB}$ there is a significant difference (SD) at the .05 level of significance; otherwise no significant difference (NSD) exists.

ONE-WAY MANOVA RESULTS BY RANGE
RFA COMPARISONS

RANGE	df1	df2	ADJUSTED df1	ADJUSTED df2	F_{OBS}	F_{TAB}	RESULT	CONCLUSION
25	8	168	1	29	2.13	2.88	NSD	A = B = C
42	8	168	1	29	4.32	2.88	SD	Isolate differences between devices by range using Hotelling's T^2 tests.
75	8	168	1	29	3.60	2.88	SD	
100	8	168	1	29	4.71	2.88	SD	
125	8	168	1	29	5.65	2.88	SD	

Table 2

Decision Rule: If $F_{OBS} \geq F_{TAB}$, there is a significant difference (SD) at the .10 level of significance; otherwise no significant difference (NSD) exists.

HOTELLING's T^2 TEST RESULTS BY RANGE
RFA COMPARISONS

				A vs B			A vs C			B vs C		
RANGE	df1	df2	T^2TAB	T^2OBS	RESULT	CONCLUSION	T^2OBS	RESULT	CONCLUSION	T^2OBS	RESULT	CONCLUSION
42	4	84	8.41	13.16	SD	$A \neq B$	28.47	SD	$A \neq C$	13.05	SD	$B \neq C$
75	4	84	8.41	1.20	NSD	$A = B$	24.59	SD	$A \neq C$	22.19	SD	$B \neq C$
100	4	84	8.41	5.02	NSD	$A = B$	38.52	SD	$A \neq C$	20.17	SD	$B \neq C$
125	4	84	8.41	6.98	NSD	$A = B$	49.91	SD	$A \neq C$	21.82	SD	$B \neq C$

Table 3

Decision Rule: If T^2OBS $\geq$ T^2TAB, there is a significant difference (SD) at the .10 level of significance; otherwise no significant difference (NSD) exis

SUMMARY OF RFA COMPARISONS

BY RANGE COMPARISON				
25	42	75	100	125
A = B = C	A ≠ B ≠ C	A = B A ≠ C B ≠ C	A = B A ≠ C B ≠ C	A = B A ≠ C B ≠ C

Table 4

COMPARISON OF UNIVARIATE ANOVA CONCLUSIONS
TO MVA CONCLUSIONS OF RIMFIRE ADAPTER TEST

RANGE	UNIVARIATE CONCLUSIONS				MVA CONCLUSIONS
	MOE	A vs B	A vs C	B vs C	
	$\bar{O}$	A = B	A = C	B = C	
	$\bar{R}$	A = B	A = C	B = C	
25	$\overline{ES}$	A = B	A = C	B = C	A = B = C
	$\bar{D}$	A = B	A = C	B = C	
	$\bar{O}$	A ≠ B	A = C	B = C	
	$\bar{R}$	A = B	A = C	B = C	
42	$\overline{ES}$	A = B	A ≠ C	B = C	A ≠ B ≠ C
	$\bar{D}$	A ≠ C	A = C	B = C	
	$\bar{O}$	A = B	A = C	B = C	A = B
	$\bar{R}$	A = B	A ≠ C	B ≠ C	A ≠ C
75	$\overline{ES}$	A = B	A ≠ C	B ≠ C	B ≠ C
	$\bar{D}$	A = B	A = C	B = C	
	$\bar{O}$	A = B	A ≠ C	B = C	A = B
	$\bar{R}$	A = B	A ≠ C	B ≠ C	A ≠ C
100	$\overline{ES}$	A = B	A ≠ C	B ≠ C	B ≠ C
	$\bar{D}$	A = B	A ≠ C	B = C	
	$\bar{O}$	A = B	A ≠ C	B ≠ C	A = B
	$\bar{R}$	A = B	A ≠ C	B ≠ C	A ≠ C
125	$\overline{ES}$	A = B	A ≠ C	B ≠ C	B ≠ C
	$\bar{D}$	A = B	A ≠ C	B ≠ C	

Table 5

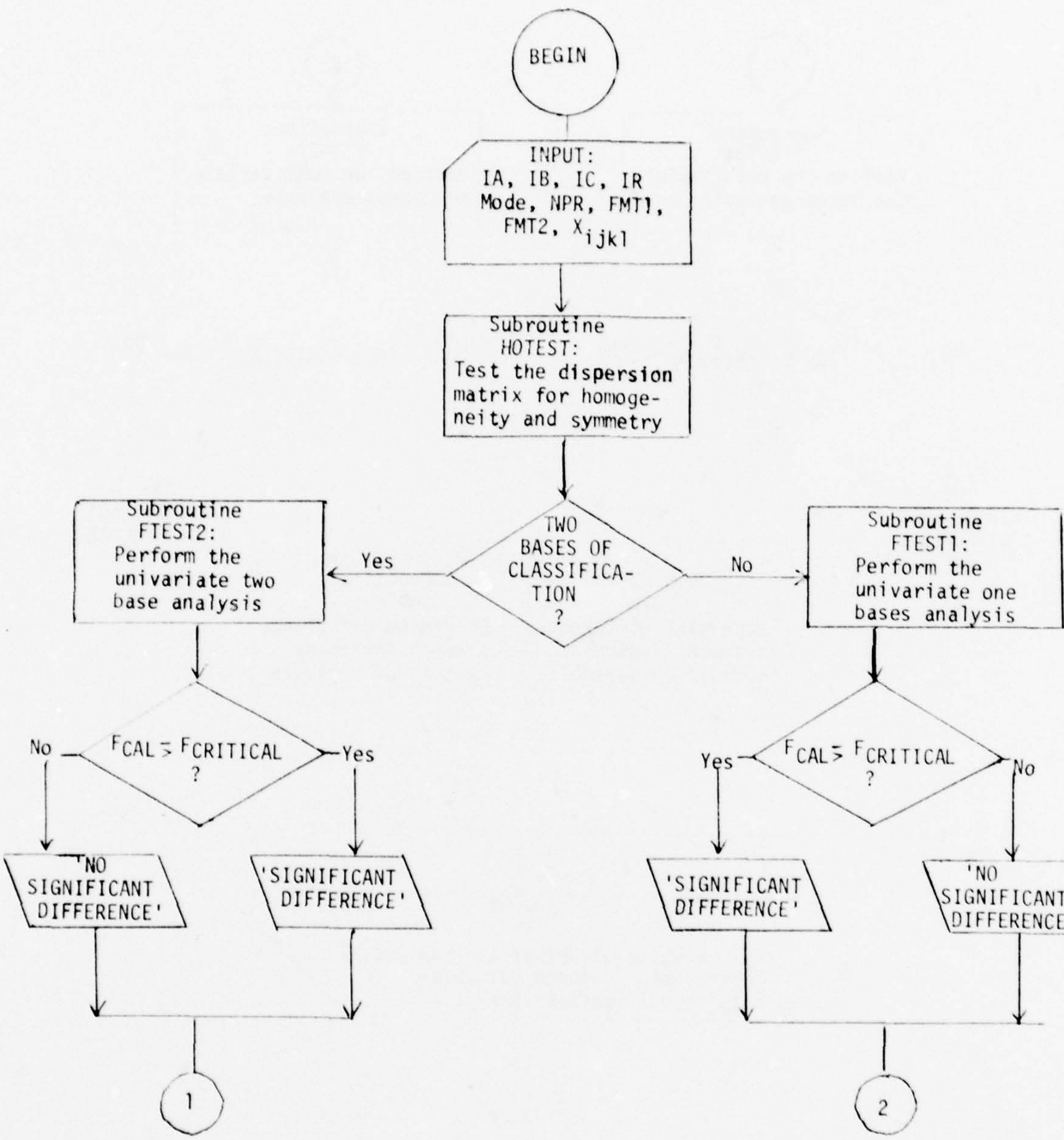

Figure 1. Flow-Chart of MANOVA

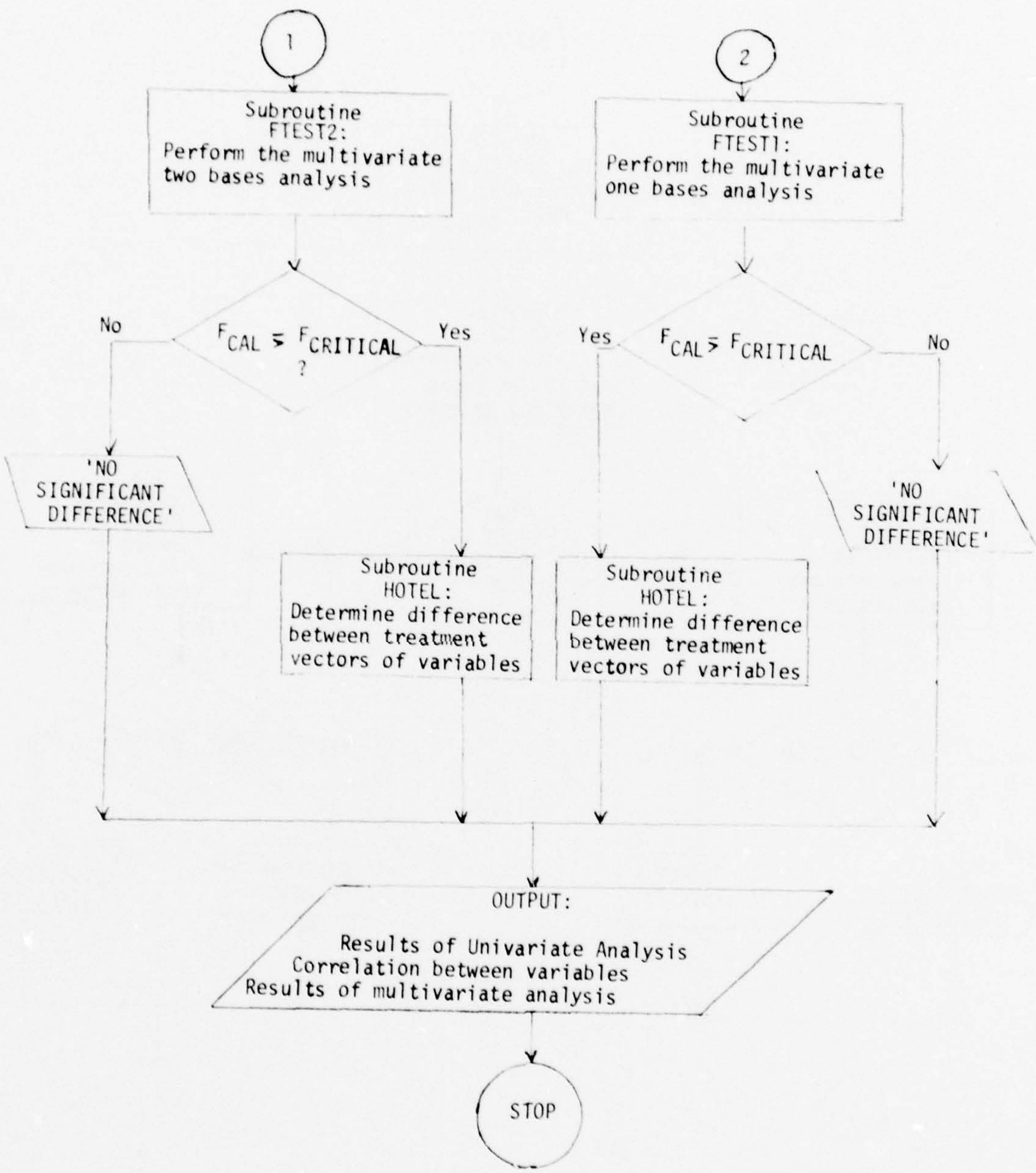

Figure 2. Flow-Chart of MANOVA (Concluded)

TITLE: A Cost Optimal Approach to Selection of Experimental Designs for Operational Testing Under Conditions of Constrained Sample Size

AUTHORS: MAJ S. W. Russ, Jr., U.S. Army Communications, Electronics Engineering and Installation Agency, Dr. Douglas C. Montgomery, Georgia Institute of Technology, and Dr. Harrison M. Wadsworth, Jr., Georgia Institute of Technology

ABSTRACT: The expected additional system cost (EASC) concept is proposed as a criterion for designing operational tests. This cost is shown to consist of four components; fixed costs of testing, sampling costs, expected costs due to type I error, and expected costs due to type II error. The problem of selecting the experimental design to minimize the EASC under conditions of constrained sample size is formulated as a nonlinear optimization problem. An algorithm for solving this optimization problem is proposed. The procedure is demonstrated for a hypothetical operational test based on a 2^3 factorial design with one covariate.

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INTRODUCTION

The department of the Army acquires new major systems through a highly structured and formalized process. This process relies heavily on the results of both development and operational tests. Development tests (DT) are designed to determine whether or not the system meets its technical specifications. Operational tests (OT) are concerned with evaluating the operational value of a system in terms of improved performance capabilities over either current standard or competitor developmental systems. The U.S. Army Operational Test and Evaluation Agency (OTEA) has the responsibility for planning, conducting, and evaluating all OT's of major Army systems. OTEA is continuously required to design and analyze the results of OT's based on small sample sizes, due to limitations on the availability of resources such as time, money, or prototype units.

This paper describes a procedure for selecting the design of an OT based on a criterion of minimum expected additional system cost due to the testing procedure. The work is limited to univariate (single measure of effectiveness) linear statistical models, with continuous, quantitative factors. The general approach is to develop a mathematical model which has as its objective function, expected additional system cost (EASC). The EASC is defined as the sum of four cost elements. These are:

(a) Fixed cost of testing

(b) Sampling cost

(c) Expected cost due to a type I error

(d) Expected cost due to a type II error.

The major class of experimental design considered is the factorial experiment with all factors fixed. The methodology allows the treatment of a covariate in addition to the primary response. Consequently, the primary method of statistical analysis for the resulting test design would be either the analysis of variance or the analysis of covariance. See Montgomery [2] or Winer [5] for a general discussion of factorial designs and their statistical analysis.

DEVELOPMENT OF THE COST MODEL

The general form of the cost model is

$$EASC = C_0 + \sum_{i=1}^{N} C_i + C_\alpha \alpha + C_\beta \beta \qquad (1)$$

where EASC = Expected cost of additional testing

C_0 = Fixed cost of testing

N = Number of observations

C_i = Cost of sampling for observation i

C_α = Penalty cost of a type I error

C_β = Penalty cost of a type II error

α = Probability of a type I error

β = Probability of a type II error

The procedure considers the EASC necessary to make a decision regarding main effects only. That is, the decision was of the type needed to determine the advisability of adopting a proposed device over a standard device or equipment. Consequently the factor "system type" in the experiment appears at only two levels and the hypothesis of interest are

$$\begin{aligned} H_0&: \mu_1 - \mu_2 = 0 \\ H_1&: \mu_1 - \mu_2 = d > 0 \end{aligned} \tag{2}$$

The null hypothesis H_0 states that the proposed device is not significantly better than the standard for comparison (SFC). The alternative hypothesis states that the proposed device is better than the SFC by an amount d, the performance margin required for adoption of the proposed device. The required performance margin, d, must of course be specified in order to compute the probability of making a type II error, that is, the probability of failing to reject the null hypothesis when the proposed device is better.

Figure 1 illustrates the errors and penalty costs in operational tests required for evaluation of the last two terms in the cost model. The other model components would usually be well known for any specific test situation.

For a general factorial experiment involving K factors the cost model (1) becomes

$$\begin{aligned} \text{EASC} = C_0 + \sum_{\varepsilon_1=1}^{L_1} \cdots \sum_{\varepsilon_K=1}^{L_K} n_{\varepsilon_1 \cdots \varepsilon_K} C_{\varepsilon_1 \cdots \varepsilon_K} + \\ C_\alpha \alpha + C_\beta \beta(\alpha, \lambda, \nu_t, \nu_e) \end{aligned} \tag{3}$$

where

L_i = number of levels of the i^{th} factor, X_i,

ε_i = ε^{th} level of the i^{th} factor,

$n_{\varepsilon_1 \ldots \varepsilon_K}$ = number of observations in the $\varepsilon_1 \ldots \varepsilon_K^{th}$ cell,

$C_{\varepsilon_1 \ldots \varepsilon_K}$ = cost of an observation in the $\varepsilon_1 \ldots \varepsilon_K^{th}$ cell,

α = significance level,

λ = noncentrality parameter,

ν_t = degrees of freedom between treatments,

ν_e = degrees of freedom for error.

The noncentrality parameter λ is required to compute the type II error probability β. It is a function of the degree to which Ho is false, the number of factors considered in the design, and the sample size. Russ [4] gives computing formulas for λ for unbalanced factorial designs involving a single covariate.

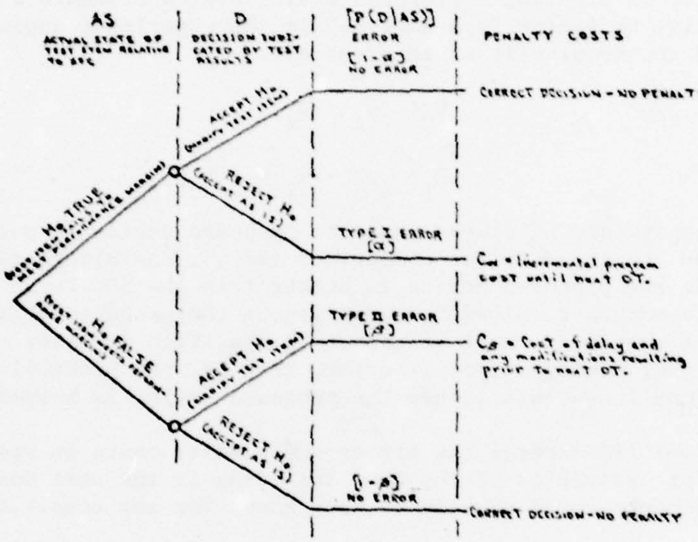

Figure 1. Errors and Penalty Costs in Operational Testing

Parameter Estimates Needed. The following parameter estimates are needed prior to the design of OT-1, the first stage operational test.

1. All cost coefficients
2. Error variance for the response variable in a completely random design
3. Correlation coefficients between the response variable and each covariate as well as all control factors
4. The ratio of the average variation of each factor about its fixed level to its population variance.

The values of these parameters would usually come from developmental tests conducted on the devices or from similar operational tests conducted previously. They may also be obtained from a series of pre-tests. Other sources of data include combat and field training exercise after action reports, TRADOC studies, intelligence studies and simulation experiments. The estimates for subsequent test phases (OT-II), etc.) would be obtained from the first phase (OT-1).

An Example: The 2^K Factorial. To illustrate the specific form of the cost model, a 2^K factorial design with all factors $(X_1, X_2, \ldots X_K)$ fixed and a single covariate (Z) will be presented in detail. The factor "system type" is denoted by X_1. Assuming that the hypotheses (2) are of interest, the problem becomes one of choosing the number of observations $n_{ij}(i=1,2,\ j=1,2,\ldots 2^{K-1})$ and α to minimize

$$\mathrm{EASC} = C_0 + \sum_{i=j}^{2} \sum_{j=1}^{2^{K-1}} C_i n_{ij} + C_\alpha \alpha + C_\beta \beta(\alpha, \lambda, \nu_t, \nu_e) \tag{4a}$$

subject to

$$\sum_{j=1}^{2^{K-1}} n_{ij} \leq S_i \quad (i=1,2) \tag{4b}$$

$$n_{ij} \geq 2 \quad \forall i,j \tag{4c}$$

$$0 < \alpha < 1 \tag{4d}$$

$$n_{ij} \text{ integer} \tag{4e}$$

where

$$\nu_t = 1 \tag{5a}$$

$$\nu_e = \sum_{i=1}^{2} \sum_{j=1}^{2^{k-1}} n_{ij} - (2^K+1) \tag{5b}$$

$$\sigma_0^2 = \sigma_Y^2 \left[1 - \rho_{X_2Y}^2 \left(1 - \frac{\bar{\sigma}_{X_2}^2}{\sigma_{X_2}^2}\right)\right] \cdots \left[1 - \rho_{X_KY}^2 \left(1 - \frac{\bar{\sigma}_{X_K}^2}{\sigma_{X_K}^2}\right)\right] (1 - \rho_{ZY}^2) \tag{5c}$$

$$\lambda = \frac{d^2}{\sigma_0^2} \left(\frac{h_1 h_2}{h_1 + h_2}\right) \tag{5d}$$

$$h_i = \frac{2^{K-1}}{\sum_{j=1}^{2^{K-1}} (1/n_{ij})}, \quad i=1,2 \tag{5e}$$

and

$$\beta = P\{F_{(1+\lambda)^2/(1+2\lambda),\ \nu_e} < [1/(1+\lambda)]F_{\alpha,1,\nu_e}\} \tag{5f}$$

In (4b), S_1 and S_2 represent the constraints imposed on the maximum number of observations that may be taken with the new system (i=1) and the SFC (i=2), respectively. (4c) is imposed to insure that all cells in the design have at least two replicates, permitting an estimate of experimental error for each cell, and also avoiding the difficulties in interpretation sometimes caused by unbalanced factorials with some empty cells. Also, note that (5a) results from the factor "system type" appearing at two levels, and (5b) is the error degrees of freedom with unequal sample sizes adjusted for a single covariate. (5c) expresses the design error variance σ_0^2 in terms of the variance of the response variable σ_Y^2, the correlation coefficient between the factor X_i and the response ρ_{X_iY}, the average variance of X_i around its fixed levels $\bar{\sigma}_{X_i}^2$, and the population variance of X_i denoted by $\sigma_{X_i}^2$. This relationship is based[1] on Feldt [1]. (5d) and (5e) define the noncentrality parameter for an unbalanced 2^K factorial design. Finally, (5f) is based on a central F approximation to the noncentral F distribution.

Model Optimization. An optimization algorithm based on the partial enumeration method of Neuhart and Bradley [3] was used to solve (4). The algorithm uses a sequential analysis of the functional relationships between the constants, parameters, and variables of the model to eliminate infeasible and nonoptimal alternatives. A complete description of the algorithm and a FORTRAN IV program listing is given in [4].

Analysis of the Objective Function. We shall now briefly discuss the behavior of the objective function and the relationships between the decision variables n_1, n_2 (the optimal allocation of observations to system types 1 and 2, respectively) ν and α. Figure 2 shows several cost factors and rates of change of cost factors plotted as functions of $(T_1, T_2|\alpha,N)$, the individual treatment sample sizes when the total sample size and α are fixed. Since T_1 is bounded (due to sample size restrictions), only a portion of figure 2 will actually occur. Also, since T_1 takes only integer values, only discrete points within that segment can occur. Figure 3 illustrates these segments of the EASC curve which are obtained from the simulated data for several different values of N, the total sample size. It should be noted that increasing the value of N shifts the segment of the EASC curve from right to left with respect to Figure 2.

Figure 4 illustrates the effect of increasing the significance level, α. The figure shows that as α increases all of the curves in Figure 2 are compressed to the left. This is because as α increases, for fixed N, the rate of change of β with respect to T_1 increases.

Selecting for each value of N the optimal allocation of observations, $(\bar{n}_1, \bar{n}_2)$, results in the EASC values shown in Figure 5. Note that as the significance level increases, the optimal number of observations initially increases, then decreases. This is the result of the variations in the rate of change of β with respect to N for given values of α and N. Where this rate is high enough to off-set the increase in sampling cost, increasing N will reduce EASC. Once this rate decreases to the point where

$$C_\beta \left(\frac{\Delta\beta}{\Delta N}\right) < \frac{\Delta SC}{\Delta N}$$

then increasing N will increase EASC.

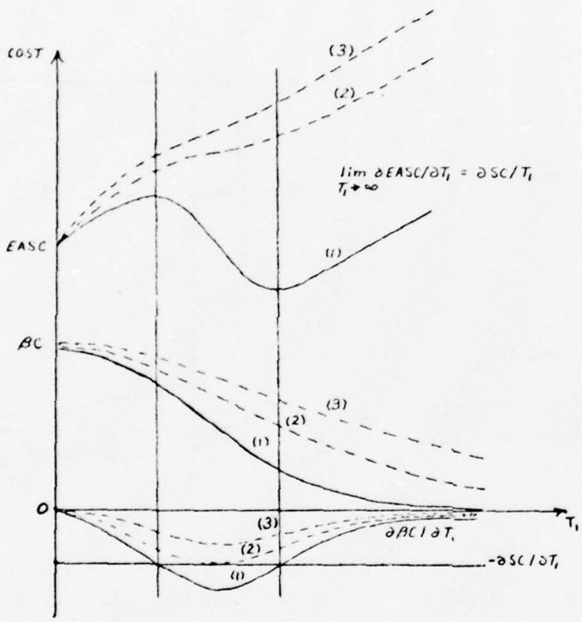

Figure 2. EASC as a Function of $(T_1, T_2|\alpha, N)$ - General Form

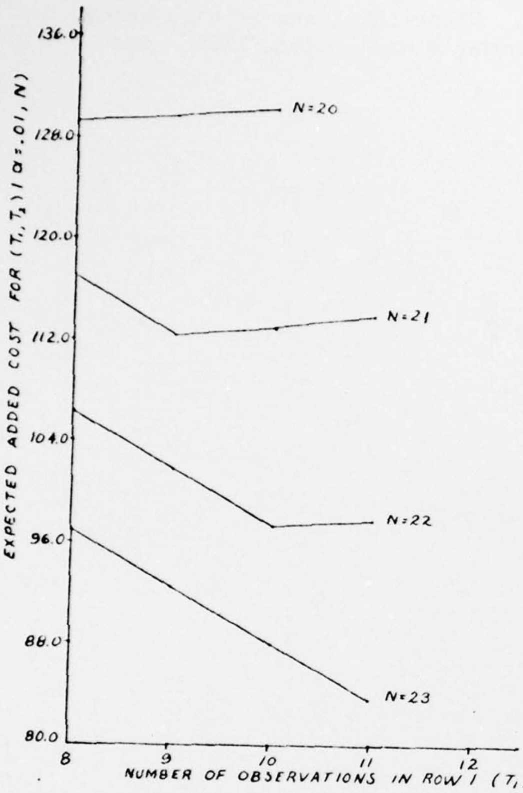

Figure 3. EASC for $(T_1,T_2|\alpha = .01,N)$

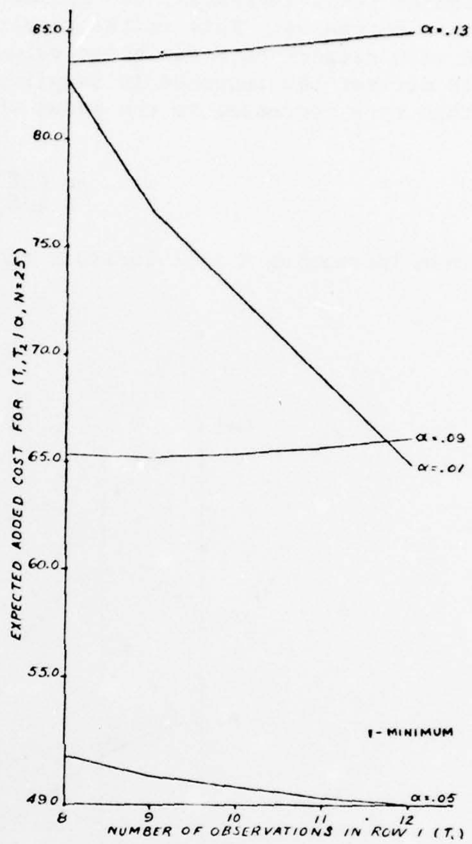

Figure 4. EASC as a Function of $(T_1,T_2|\alpha,N)$ - Computed Examp

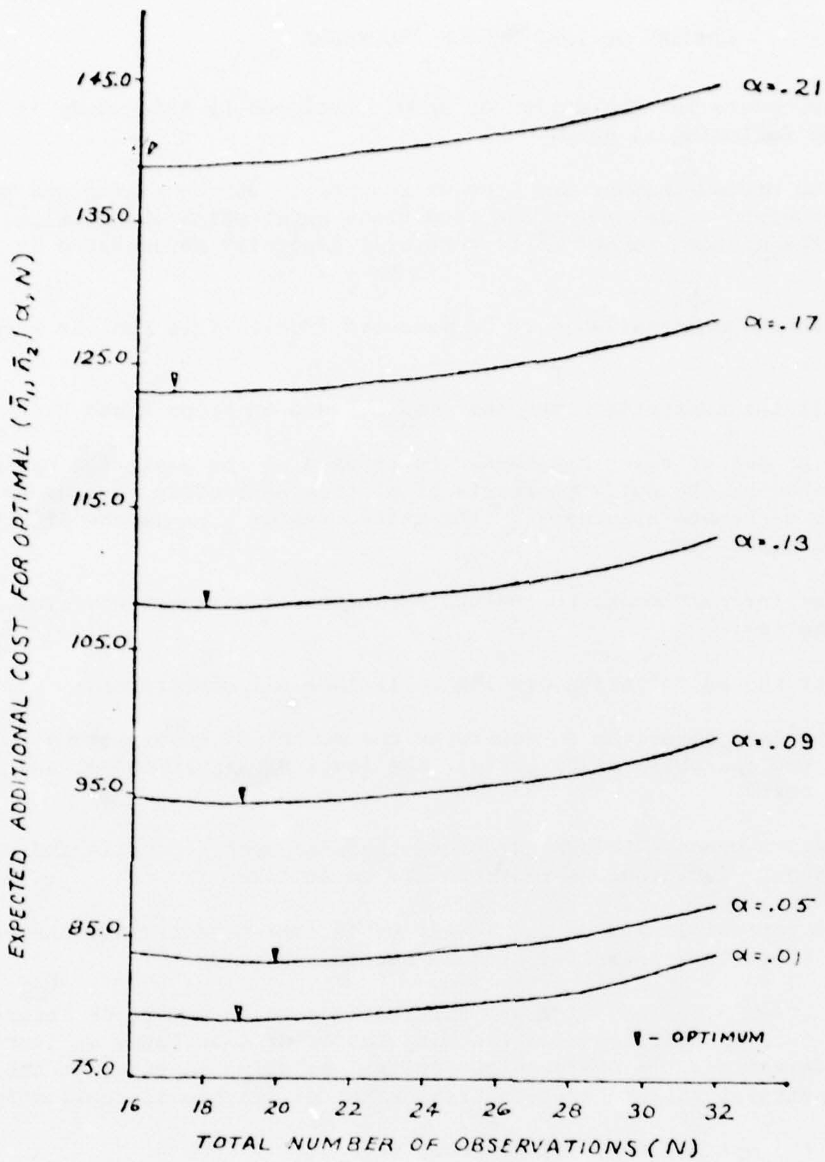

Figure 5. EASC as a Function of N for Optimal $(T_1, T_2 | \alpha, N)$

SUMMARY OF TEST DESIGN PROCEDURE

The basic procedure for the design of an OT developed by this study is summarized by the following 14 steps.

1. Determine minimum number and type of factors to be considered and how they are to be combined to determine the conditions under which observations will be taken. The minimum number of factors will generally be dictated by the test issues.

2. Determine response variable to be measured (MOE). This must be a continuous variable.

3. Formulate the appropriate response model based on steps 1 and 2.

4. Select the set of exact hypotheses to be used as the basis for optimization. Normally, this would be the null hypothesis of no treatment effect versus an exact form of the alternate hypothesis: the tested system exceeds the SFC by the required performance margin.

5. Determine the cost model to include estimates of all cost coefficients and primary parameters.

6. Formulate the optimization problem to include all constraints.

7. Apply the EASC algorithm to determine the number of observations to be taken in each row and their distribution, the level of significance, and the power of the test.

8. Use a random process to assign observations to specific cells and to determine the sequence in which observations are to be taken.

9. Vary the control limits on the levels of factors to determine the optimum control required if control is anticipated to become a problem.

10. Repeat steps 5,6, and 7 for any alternatives which may be of interest to the experimenter such as addition of a blocking factor or covariate; an increase in the number of observations, if the previous optimal solution occurred at the upper limit of this constraint for one or both treatments; or fractional replication.

11. Select the optimal feasible alternative.

12. Begin experimentation.

13. Correct estimates of input parameters as test data becomes available.

14. Repeat Step 7 and other steps as necessary to determine the effect, if any, of the corrected parameter estimates on the optimal solution.

AN EXAMPLE

The algorithm was demonstrated by a hypothetical example in which operational tests were to be designed to evaluate the overall military worth of a new ground-to-

air tactical missile system, TAAMS, which is under development as a replacement for the HAWK missile system. The specific illustration concerns tests for the guidance system.

The critical issue for evaluation is the accuracy of the guidance system. Ambient temperature, altitude of target, and speed of the target are the most likely factors to have a significant effect on the accuracy. The maximum numbers of TAAMS and HAWK missiles that may be fired in each phase of the OT to evaluate the guidance system are 12 and 20 respectively. The measure of effectiveness (MOE) is stated as the mean miss distance from the target.

A 2^3 completely randomized factorial design was selected with ambient temperature, Z, treated as the covariate. The two independent variables were altitude of the target, X_2, and speed of the target, X_3. These two variables are treated as control variables while ambient temperature was considered a covariate since it could not be controlled. Factor X_1 is the missile type.

The test designer then uses the proposed procedure to determine the number of firings to be used for each missile type and their distribution among the 2^3 cells of the design. Estimates of cost coefficients and variability estimates required for use of the procedure are first obtained. These are shown in Table 1.

Figure 6 shows the results of the use of the EASC program with the input values listed in Table 1. The optimal values shown in Figure 7 were found to be, $\alpha = 0.29$, $N = 16$, $T_1 = 8$, $T_2 = 8$ and $\beta = 0.2207$. This resulted in an EASC of $8.907 M.

Table 1. Initial Input Data for OT-I

Cost Coefficients (million dollars)	Primary Parameters	
$C_0 = 1.000$	$\sigma^2_Y = 4.000$	$\bar{\sigma}^2_{X_2} = 2.000$
$C_\alpha = 10.000$	$d = .200$	$\bar{\sigma}^2_{X_3} = 2.000$
$C_\beta = 10.000$	$\rho^2_{X_2Y} = .500$	
$c_1 = .250$	$\rho^2_{X_3Y} = .500$	$\sigma^2_{X_2} = 20.000$
$c_2 = .100$	$\rho^2_{ZY} = .500$	$\sigma^2_{X_3} = 10.000$

During a planning meeting a new control unit costing $7,000 was proposed for the target drones. This control unit would reduce altitude variations by 50%. The new value of the control variance for altitude, $\sigma^2_{X_2}$, was then inputed to the EASC program. All other parameters were left the same. This gave a new optimal solution of $8.987 M, a reduction of $10,000. This was used to justify the purchase of the new control unit and the first test phase was conducted.

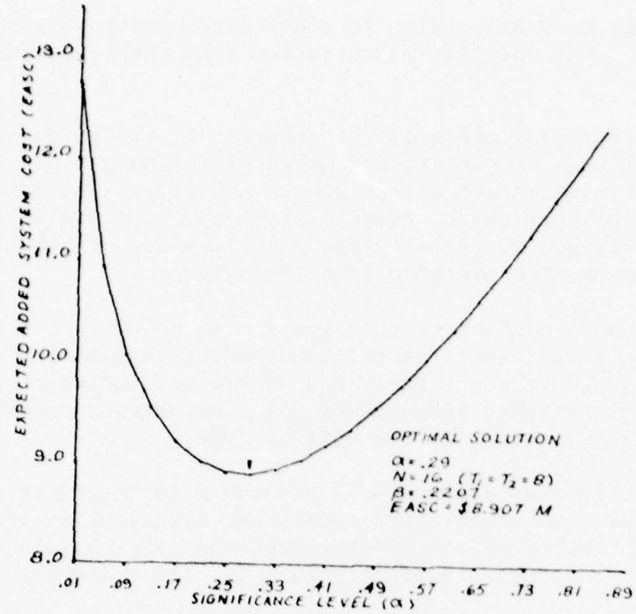

Figure 6. Optimal (EASC/α for Initial OT I Design

The results of the first phase are used to revise the parameter estimates for subsequent phases. The input data for OT II are shown in Table 2 and Figure 7 illustrates the results of this run of the EASC program. It is to be noted that the error costs, C_α and C_β, are changed for the OT II tests. Following the evaluation shown in Figure 7, the performance margin, d, was reduced from 0.200 to 0.150. This necessitated a new program run and resulted in a new set of values. The new values were:

$$\alpha = 0.21$$

$$\beta = 0.2583$$

$$n = 18$$

$$T_1 = 8$$

$$T_2 = 10$$

$$\text{EASC} = \$12.074 \text{ M}$$

For OT III, new estimates of the input data were determined. These included significant increases in C_α and C_β since an error would now become critical. Results of OT III will be used to decide whether to put the TAAMS missile into production. The new data shown in Table 3 and the output is graphed in Figure 8.

Table 2. Initial Input Data for OT II

Cost Coefficients (million dollars)	Primary Parameters	
$C_0 = 1.000$	$\sigma_Y^2 = 2.500$	$\bar{\sigma}_{X_2}^2 = .800$
$C_\alpha = 20.000$	$d = .200$	$\bar{\sigma}_{X_3}^2 = 1.400$
$C_\beta = 15.000$	$\rho_{X_2 Y}^2 = .700$	$\sigma_{X_2}^2 = 20.000$
$c_1 = .250$	$\rho_{X_3 Y}^2 = .600$	$\sigma_{X_3}^2 = 10.000$
$c_2 = .100$	$\rho_{ZY}^2 = .650$	

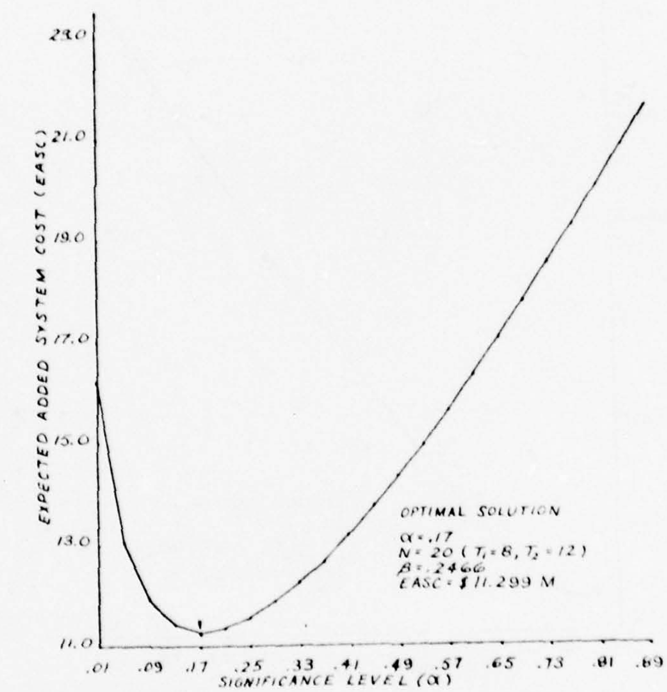

Figure 7. Optimal (EASC/a) for Initial OT II Data

Table 3. Initial Input Data for OT III

Cost Coefficients (million dollars)	Primary Parameters	
C_0 = 1.000	σ^2 = 2.500	$\bar{\sigma}^2_{X_2}$ = .800
C_α = 500.000	d = .150	$\bar{\sigma}^2_{X_3}$ = 1.400
C_β = 150.000	$\rho^2_{X_2Y}$ = .600	$\sigma^2_{X_2}$ = 20.000
c_1 = .350	$\rho^2_{X_3Y}$ = .600	$\sigma^2_{X_3}$ = 10.000
c_2 = .100	ρ^2_{ZY} = .550	

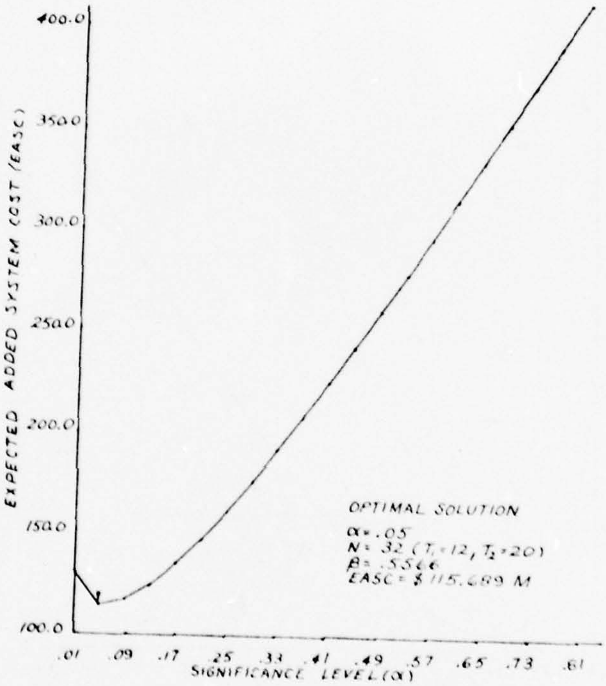

Figure 8. Optimal (EASC/α) for Initial OT III Data

Prior to testing, a new speed control device is introduced on the target drones which reduces the variance in speed, $\bar{\sigma}^2_{X_3}$, by 28.5%. This new value is then used for the program and a new optimal solution is obtained. This is:

$$\alpha = 0.05$$

$$\beta = 0.5527$$

$$N = 32$$

$$T_1 = 12$$

$$T_2 = 20$$

$$EASC = \$115.107\ M$$

This reduced the expected cost by $582,000. The cost of the 32 new drones is $320,000 and therefore the new drones were justified.

The results above indicate using the maximum number of firings for both missile systems. Because of this result the program was run again to determine the effect on EASC of increasing the allowable number of HAWK missiles to 21. The results were observed to be:

$$\alpha = 0.05$$

$$\beta = 0.5502$$

$$N = 33$$

$$T_1 = 12$$

$$T_2 = 21$$

$$EASC = \$114.831\ M$$

This reduction of $276,000 in EASC could be obtained by an expenditure of $100,000 for the additonal missile and thus the additional HAWK could be justified.

CONCLUSION

This paper has developed a criterion of expected additional system cost as a measure of the relative efficiency of alternative experimental design procedures for operational tests conducted with sample size constraints. A hypothetical example was used to demonstrate the methodology. The decision alternatives that may be evaluated using this methodology can be extended beyond those shown in the example. By varying parameters, many other situations, such as the addition or deletion of a covariate and estimates of the effects of parameter misspecification, can be evaluated.

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10 May 1977

TITLE: Battlefield Visualization Graphics Analysis Techniques

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AUTOVON 552-5258/5140

ABSTRACT: The potential use of interactive computer display graphics applied to terrain analysis, tactical maneuver planning, test analysis, and model set up and analysis is described in this paper. Specific examples of contour, vegetation, cultural, road/rail nets, bridge and obstacles, operations overlays, fields-of-fire, etc. produced in planes map overlay format are discussed. Projection of similar overlays from ground and aerial 3-D perspectives are also illustrated. Interactive man-machine interfaces and applications are addressed in context of study applications exercises.

ESTIMATED TIME REQUIRED FOR PRESENTATION: 30-40 minutes.

CATEGORY APPROPRIATE FOR THIS PAPER: Special session topic: New OR/SA Techniques.

Subject: Battlefield Visualization Graphics Analysis Techniques

Authors: Dr. L. G. Pfortmiller, Mr. R. A. Davison

Agency: Combined Arms Combat Developments Activity, Combat Operations Analysis Directorate, Fort Leavenworth, Kansas

A. INTRODUCTION.

1. The rapid growth of technology in ADP and interactive computer graphics represents a vast potential for development of OR/SA techniques applied to combat model simulations and games. At the Combined Arms Center at Fort Leavenworth, Kansas we are taking some initial steps to explore this environment. During this presentation I will illustrate potential uses of interactive computer display graphics applied to terrain analysis, scénario development, tactical maneuver planning, test analyses, and combat model set-up and post analysis.

2. To do so I have chosen three recent studies either in process or recently completed at CACDA. These are the Division Restructure Study, The Family of Scatterable Mines (FASCAM) COEA Study, and the Analysis of the Antitank Missile Test (ATMT) conducted at Fort Hunter Liggett, California by CDEC. The initial example is the Division Restructure Study. The application is the use of computer graphics in the tactical planning and scenario development phase of setting up the CARMONETTE model to be used in the analysis effort.

B. TERRAIN ANALYSIS.

1. The terrain grid shown (slide 2) is a 10 km x 8 km piece of terrain north and just west of FULDA. The display slides were taken with a 35mm camera directly off of the tektronix 4014 CRT display screens. (Ektachrome-X ASA64 film with 1 second exposure was used.) The terrain grid lines are shown at 100 meter intervals. The Z-elevation is scaled by a factor of 3. The location of the villages of NIEDERROSO (lower left) and NIEDERROLLA (upper) are shown by the asterisks. The view is from a point approximately 20 km back at a height of 3,000 ft, with a 15X magnification. The terrain elevation data is obtained from the defense mapping agency (DMA) in UTM grid aligned format. At CACDA we currently have large portions of central V and VII corps areas, parts of the North German plains, South Korea, Mid East (CATTS terrain), and selected US test areas: Fort Hunter Liggett, Fort Hood, and Fort Irwin (not on-line yet). These terrain data bases are stored on-line (disc storage) on the TRADOC Data Processing Field Office (DPFO) CDC 6400/6500 computer system for rapid access by the graphics software.

2. Using a digitizer board and working from a map, we input overlays of the rivers (FULDA) and road nets for the terrain area (slide 3). The AUTOBAHN is triple lines, double lines are major paved roads, dashed double lines are secondary roads. The various overlays developed can be built up in different fashions. This slide (slide 4) shows a combination of the river, road, and rail nets together with an operations overlay. The Blue defense is developed on the left while the threat axis of approach is shown coming from the east towards the left.

3. This view (slide 5) is a long range contour perspective of the same terrain area only looking from the east towards the west, i.e., from the direction of the threat. The contours are drawn at 20m intervals. Vertical elevation exaggeration is the same as before, with the viewer standing 20 km back at a height of 3,000 ft. The river/road overlays can also be displayed on this perspective again (slide 6). The river and road nets with the operations overlay is shown in perspective without the terrain grid (slide 7).

4. The standard 2D contour map is also included in the computer software (slide 8). This is the same 20 meter contour map now viewed in 2 dimensional 1 to 50,000 UTM grid map format. The river and road net overlay can also be applied as seen in slide 9.

5. Using a vegetation/cultural feature code supplied by DMA allows a representation of these features to be applied on the map area (slide 10). The dark asterisk areas show forestation features. The '=' signs are urban or built-up areas (villages, etc.).

C. DEFENSE PLANNING AND SCENARIO DEVELOPMENT. Using this type of map and preceding perspective capabilities, the tactician and scenario developer can produce a tactical plan for defense of the area. This defensive plan is shown in schematic form on this overlay (slide 11). The next step in the DRS scenario development for the CARMONETTE model is to plan each defensive area in detail at the individual weapon level. We will show how this is done using a typical defensive team area.

1. First we blow up the terrain area 1:25,000 scale (slide 12) in front of the team area. The weapon locations in the team are shown as letters. D's are Dragons, A's are tanks, and T's are TOW systems. The area represented is approximately a 4 x 5 km area.

2. For each weapon position we then can generate a LOS map. The first shown (slide 13) is for a Dragon position. The scan is set to 500 meters past the maximum effective range of the Dragon (1,000 + 500 meters). The drawn in lines indicate intervisibility with tank/APC targets located in those regions. The Dragon is given a 90^{o} field of view (FOV) here, with the LOS checked at 1^{o} increments within that FOV.

a. All of the Dragon LOS areas are shown next. Notice that the LOS of the rearward Dragon positions are limited to 500 meters or less except for about a 10^{o} field straight ahead to the east (slide 14).

b. A similar set of coverage fans is shown for the three tank positions (maximum range of 1500 + 500 meters). Again note the mask regions and field of fire restrictions (slide 15).

c. The 8 TOW positions coverages out to 3,500 meters are shown in this slide (slide 16). Again, the reduced fields are evident with the maximum TOW range usable only to the far left flank coverage.

d. To illustrate the coverage blockage, the forestation and cultural features are overlayed on the TOW coverage fans. The villages, forestation on the hilltops, and land masks are easily identified (slide 17).

3. Similar procedures are used for weapon placement on the remaining teams to plan the entire defense. These positions are then input into the CARMONETTE model together with the balance of the tactical plan. The threat plan is developed using the same techniques. This is as far as we have gone in the Division Restructure Study planning to date.

D. ANALYSIS OF MODEL RESULTS. We now switch to a previous study, the FASCAM COEA, conducted by the Combat Operations Analysis Directorate at CACDA, to illustrate the analysis aspects of graphics, but first we will go through a similar scenario planning phase.

1. First, a long range view of the terrain (slide 18), this time 5 km x 10 km deep. The terrain is just northeast of FULDA. The model used was DYNTACS-X. The view is from the defense, again with the same elevation exaggeration used in the previous slides. Now a view from the defender's right flank (slide 19). Another from the left flank (slide 20). And rotating the terrain 180°, a view from the threat side (slide 21). Again, these terrain graphics are used to look for anticipated threat approaches, major hill masks, flanking fire position, etc., in a very detailed terrain analysis.

2. Now we return to the 2D contour map (slide 22) given for a 5 x 10 km area using a UTM grid system.

a. The 4 defensive teams were positioned as shown in slide 23. The active defense is being portrayed in the scenario and later I shall show planned withdrawal routes and subsequent team locations.

b. Next, slide 24 shows the coverage of the area for a TOW position in Team One. Now we go down to the TOW position on the ground and generate a perspective view of the terrain in front of this position (slide 25). The field of view of 60° with the elevation enhanced by 3. Notice we have removed the hidden lines in the view. The perspective drawing contains about 4,800 vectors (some are not visible). The previous long range terrain perspectives did not have the hidden lines removed. Removing these hidden lines increases the required computer time.

c. Another TOW position from a second team area is shown on this view (slide 26). Note the flanking long range fields of fire as they become quite important in the analysis of the model results. This (slide 27) is the corresponding TOW ground perspective view. In this position the forward slope is towards the left.

3. In the next view (slide 28) we have added planned minefield locations, shown on the right as 200 meter boxes with numbers. The minefields are planned in conjunction with the weapon coverage to insure covering fire or obstacles to deny the threat cover in potential defilade positions. Corresponding artillery fire aim points were generated by similar considerations. We also show the planned withdrawal

routes and subsequent occupied positions. Finally, TOW coverage from one of the secondary positions is shown in conduct of the active defense. Notice again the flanking coverage obtained.

4. We now switch to a long perspective to illustrate the threat approach (slide 29). This is from the threat direction and shows selected Blue defender positions.

a. The next series of slides (slides 30, 31, 32) show successive positions of threat force approach trails as anticipated in the attack planning. Boxes are drawn at 30 second intervals along the trails.

5. Similar views can be generated from any perspective. To illustrate we show two ground views. The first (slide 33) is the same team one TOW position as shown before. The approaching threat trails are shown when visible to the defender. The same display for the TOW from position 2 is shown in slide 34. The previous scenario preparation data and planning information was used to set up various run sets for the DYNTACS-X model.

6. The next series of slides show examples of how the graphics display were used in the analysis of model results. The terrain grid in slide 35 is the same as previously shown. These results depict the time event history of individual model replications.

a. This gives the cumulative history of events to a battle time of 600 seconds. The pairing lines represent kills. Boxes at the end indicate the firing Blue weapon. An X with a pairing line is a Red weapon killing a Blue defender weapon. The minefield boxes are FASCAM delivered fields. Encounters are indicated by a # sign; kills by an X.

b. In slide 36 the battle time has increased to 1200 seconds. Regular stars indicate a Blue element killed by Red artillery. Jagged stars indicate a Red element kill by Blue artillery. Across the top is a 200 second update of a killer/victim scoreboard including LER, mine encounters, and mine kills. By 1,200 seconds team one has withdrawn to its secondary position. The defenders at the lower left have just begun to open fire on threat elements on their left flank.

c. The situation at end of the simulation run at 1,600 seconds is shown by this final slide (slide 37) in the series. A summary is printed at the lower part of the screen. In the battle portrayed, it is interesting to note the preponderance of fire from flanking positions which correspond to the increased fields of fire seen in the LOS fans. Additionally, few Blue kills of Red elements occur from the initial positions.

7. The use of graphics and the time dimension portrayal (which we can't really show using slides) has proved invaluable in model checkout and analysis. Both responsiveness and confidence in results were greatly increased even though we were using rudimentary application software and were developing as we went along.

8. The next two slides illustrate another display technique suited for intervisibility studies. Here (slide 38) we show simulated threat vehicle approach trails towards a series of defenders indicated by numbers on the right. These trails were generated by the DYNTACS-X mobility model for the HELLFIRE COEA Study conducted by CACDA. Using the LOS/Mobility driver code of DYNTACS, we can then display only the visible paths for selected defender elements. This overlay (slide 39) shows the intervisibility from defender position element no. 1. When these displays are generated on the CRT screen, the approach is time sequenced. Thus, both the static and dynamic geometry of intervisibility is readily available for analysis. Statistics involving segment length correlation effects and multiple intervisibility sequences can be visually examined in the original framework, before data aggregation techniques are used.

E. FIELD EXPERIMENT ANALYSIS. The next series of slides very quickly illustrates an application to field experiment test analyses. This (slide 40) is a long range view of a 5 x 10 km area at Fort Hunter Liggett California. The "Site A and B (TETAM)" area is to the lower left for those of you familiar with the area and the TETAM tests. The example test we used is the ATMT test conducted by CDEC at Fort Hunter Liggett from October through December 1975 to examine TOW gunner tracking capability against evasive target vehicles. Using the digitized terrain as a background, we added the test data taken which provides a recreation of the trial geometry and time sequence.

1. This is a ground perspective view from a TOW gunner position along a ridge line at Site A (slide 41). In this next slide (slide 42) we have now overlayed an approaching target vehicle's path on the terrain. Successive boxes are drawn at 10 second intervals if the target is visible as determined by the terrain LOS model.

2. Slide 43 is another view to illustrate the trial as viewed from a simulated helicopter position to the left flank and forward. The combination of the test data, position location data and the terrain base allows for a significant increase in understanding the conduct of a field experiment, a rapid data reduction process, and an increased perception of the tactical parameters influencing the results.

3. We are currently expanding and exploring field experiment analysis using graphics techniques in support of the Division Restructure battalion test at Fort Hood, Texas.

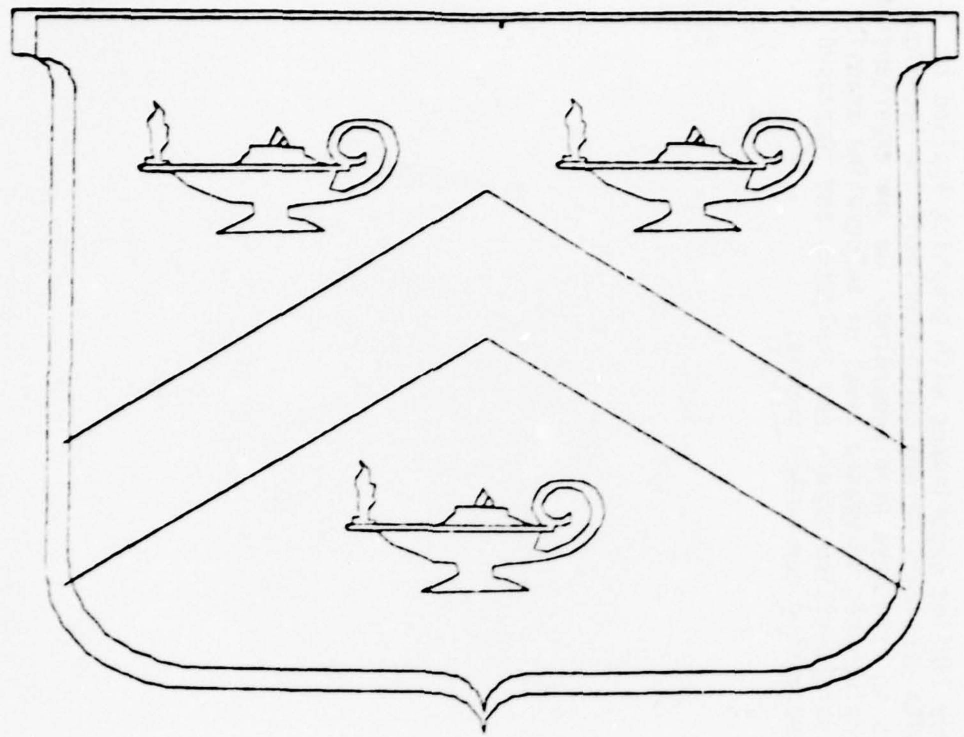
F. SUMMARY.

a. The next three slides (slides 44, 45, 46) illustrate a portion of our on-line data base. They are 20 km x 20 km long range perspectives looking east towards the border in the central Europe V corps region. We are constantly increasing our on-line terrain data base and expect to expand it significantly in scope as well as detail in connection with on-going DMA digitizing programs.

2. The previous slides shown are representative of the preliminary work we are doing in graphics. Our equipment consists of two Tektronix 4051 graphics display calculator systems and a 4014 19" display terminal. We will be increasing our graphics capability significantly with the addition of two Tektronixs 4081 intelligent graphic terminal systems and four 19" Ramtek color display systems driven off of a Varian V77 mini-computer system. This all will be linked to the DPFO CDC 6400/6500 at Fort Leavenworth.

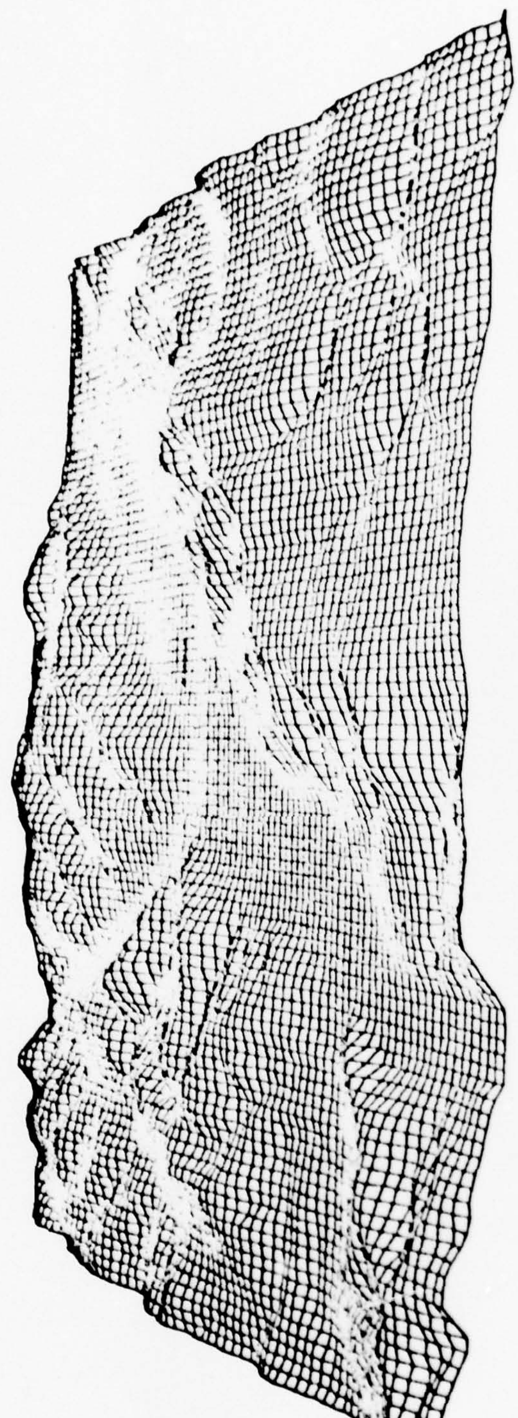
3. Based on our limited experiences with graphics applied to traditional Army ORSA problems, we are quite confident the technique is sound. Our results to date have been rewarding. We are experiencing an increased responsiveness, increased level of information transfer between study advisors/decision makers and analysts, and increased confidence in understanding our combat models.

COMBINED ARMS



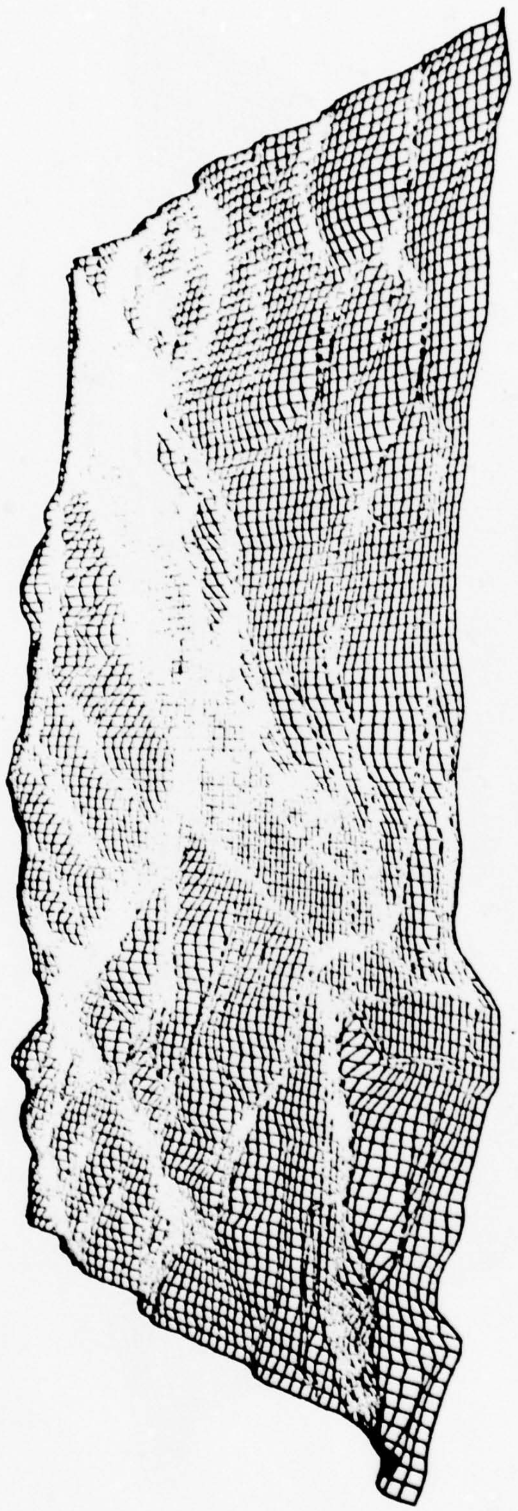
COMBAT DEVELOPMENTS ACTIVITY

SLIDE 1

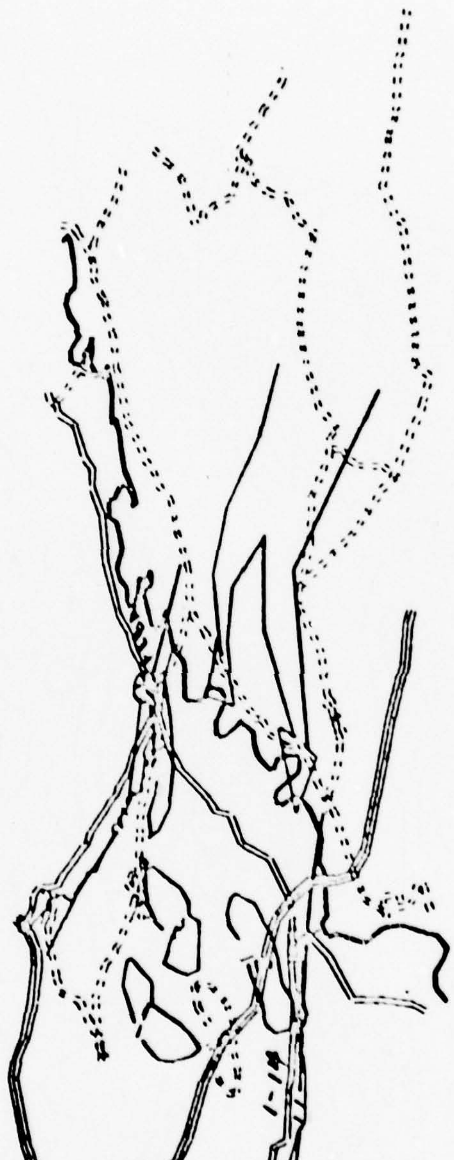


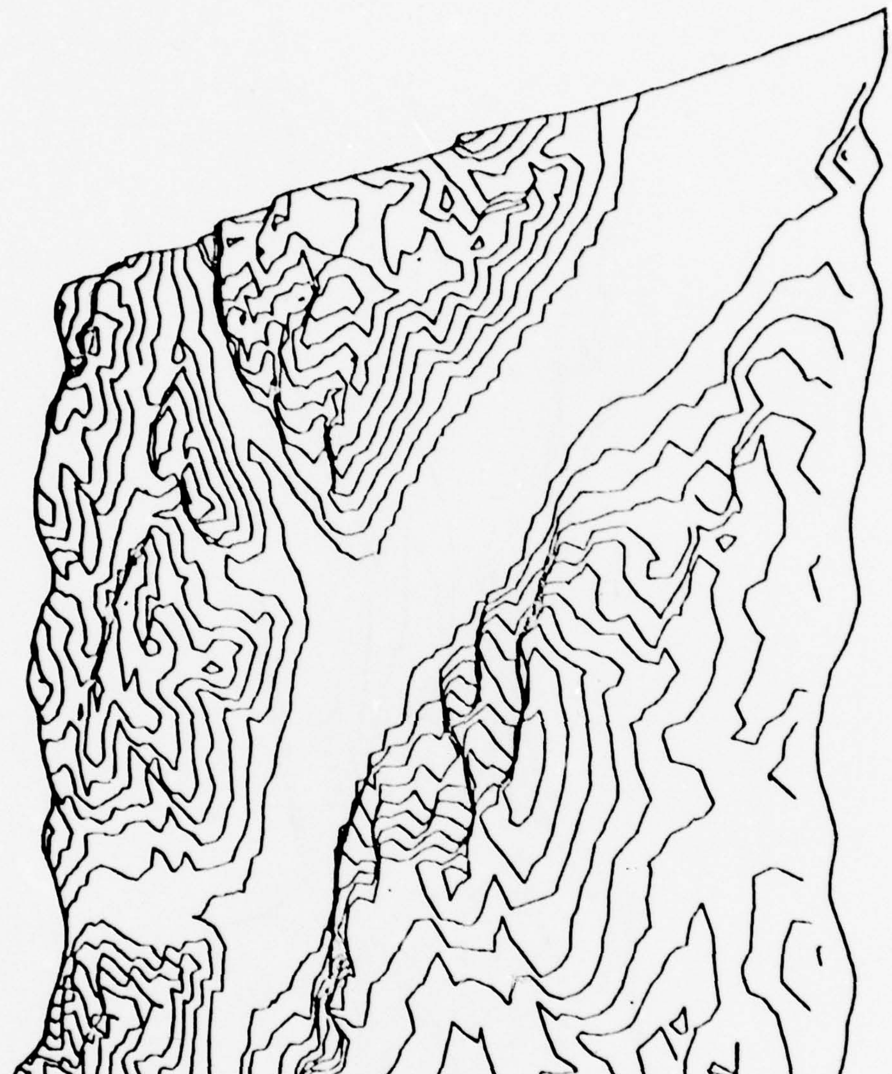
SLIDE 2

SLIDE 3

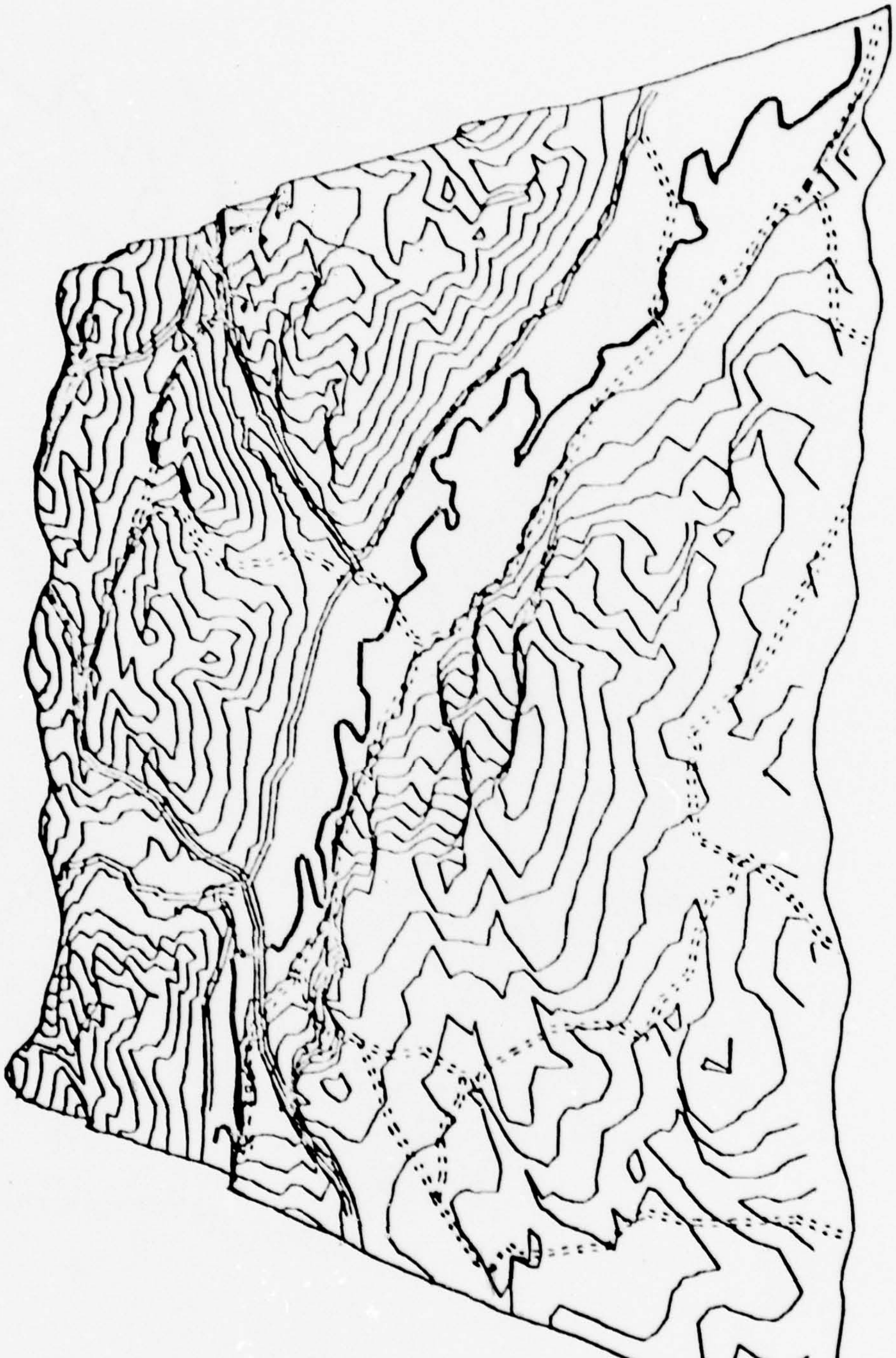


SLIDE 4



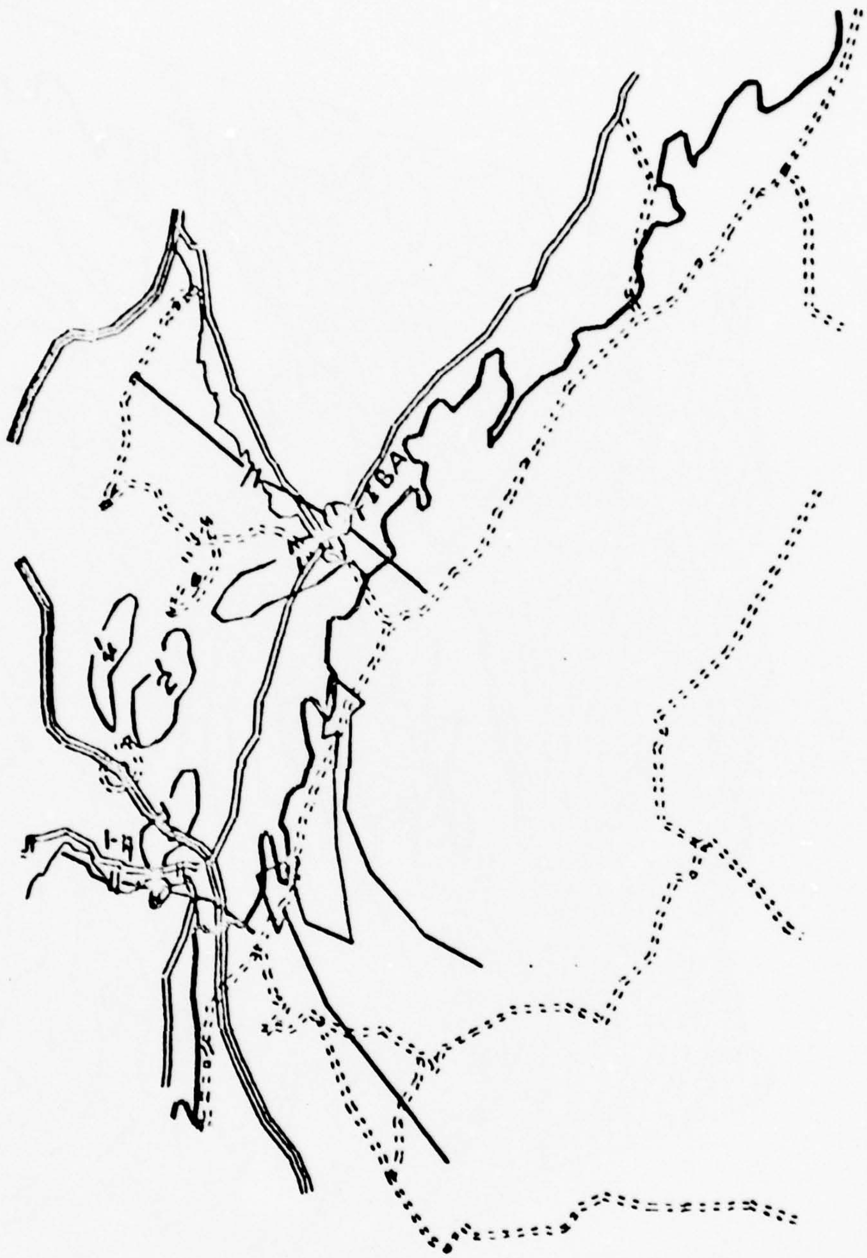


SLIDE 5



SLIDE 6

SLIDE 7

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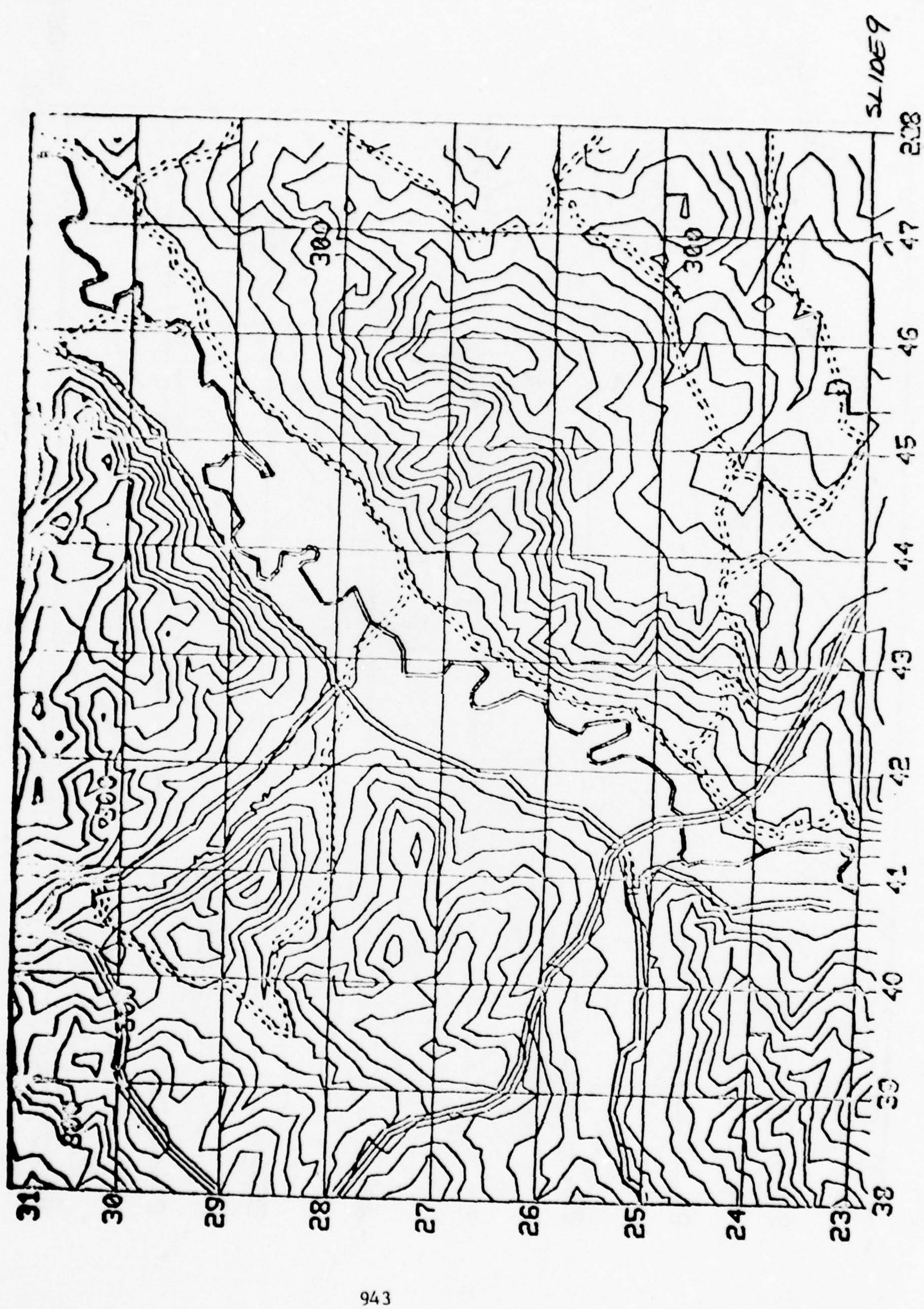
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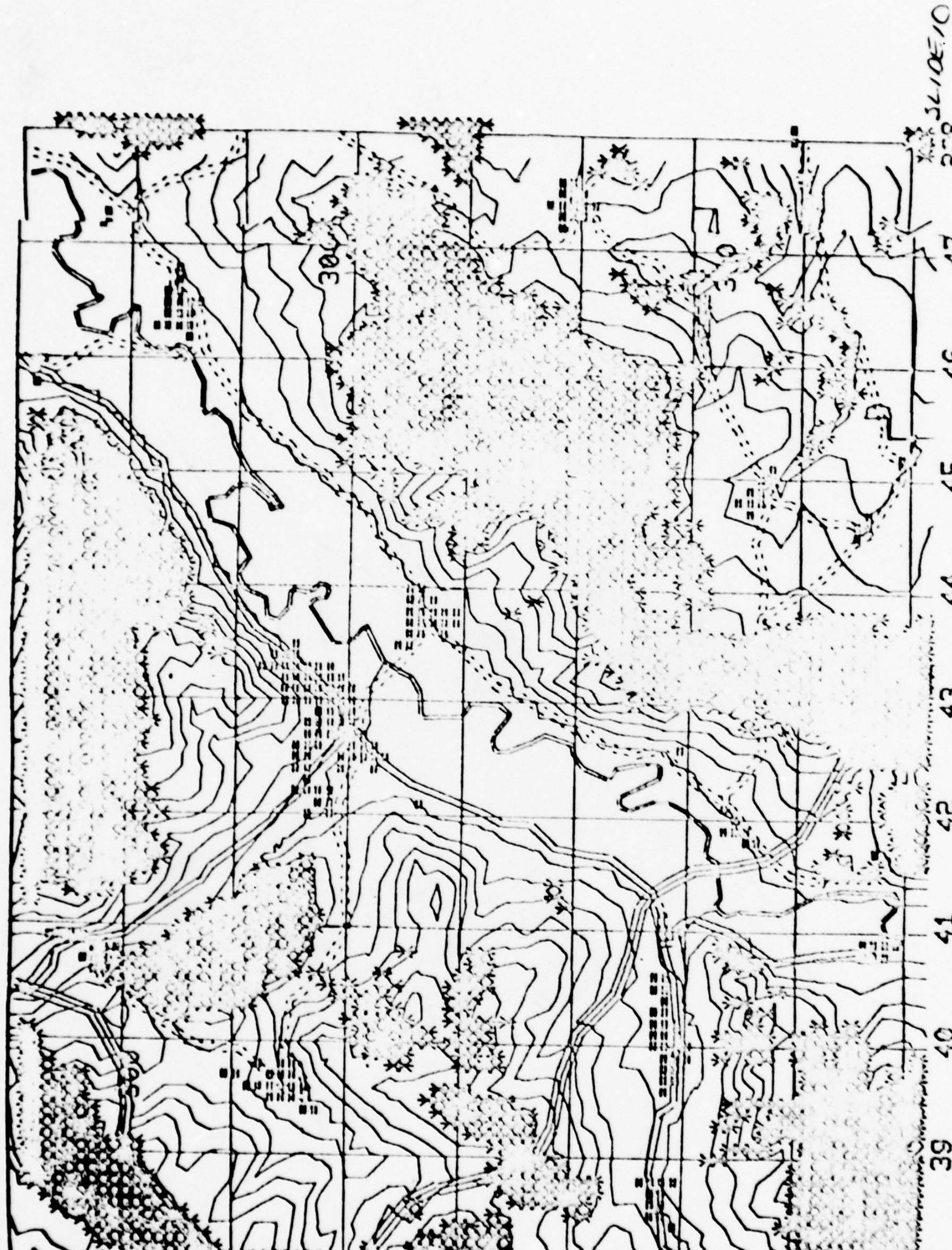
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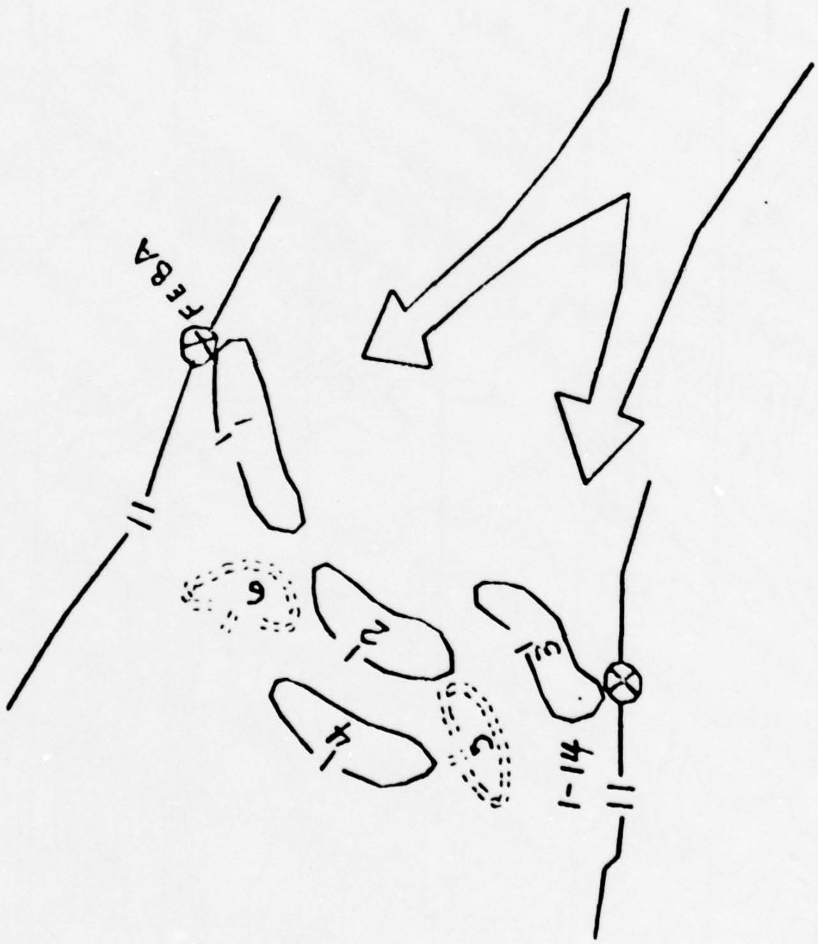
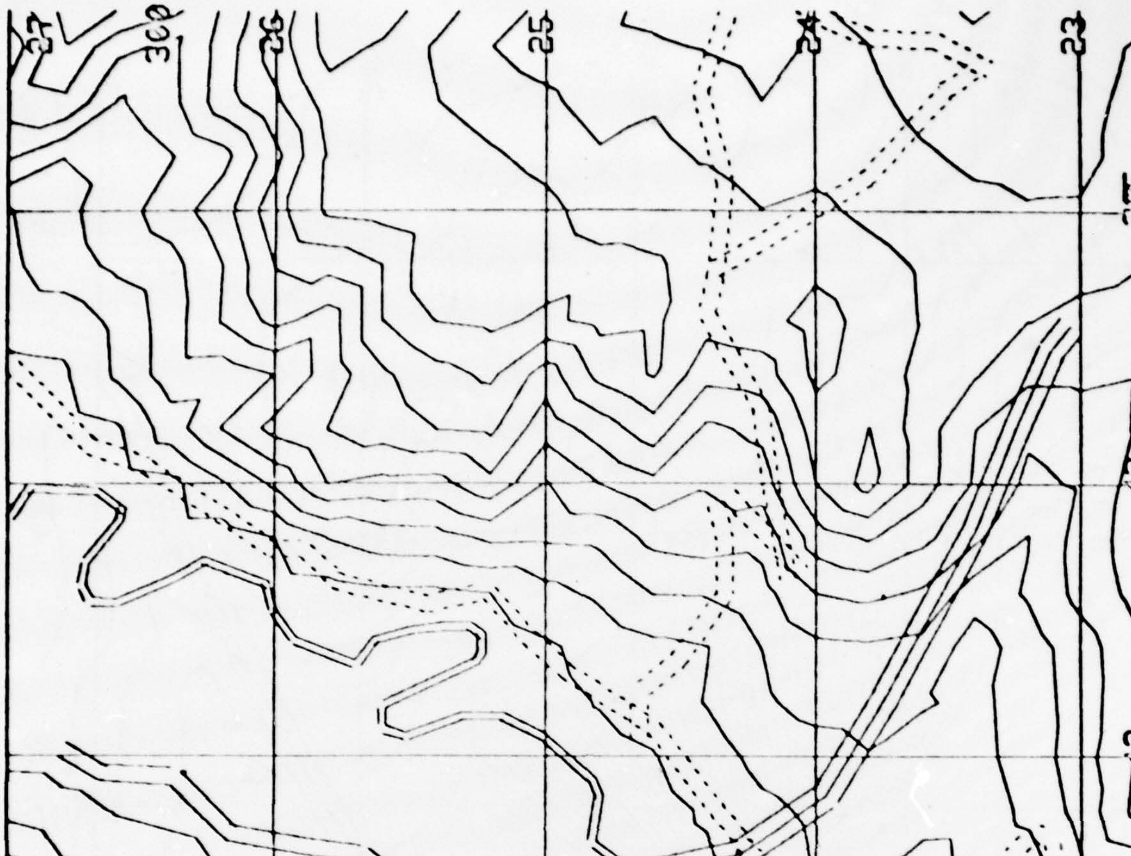
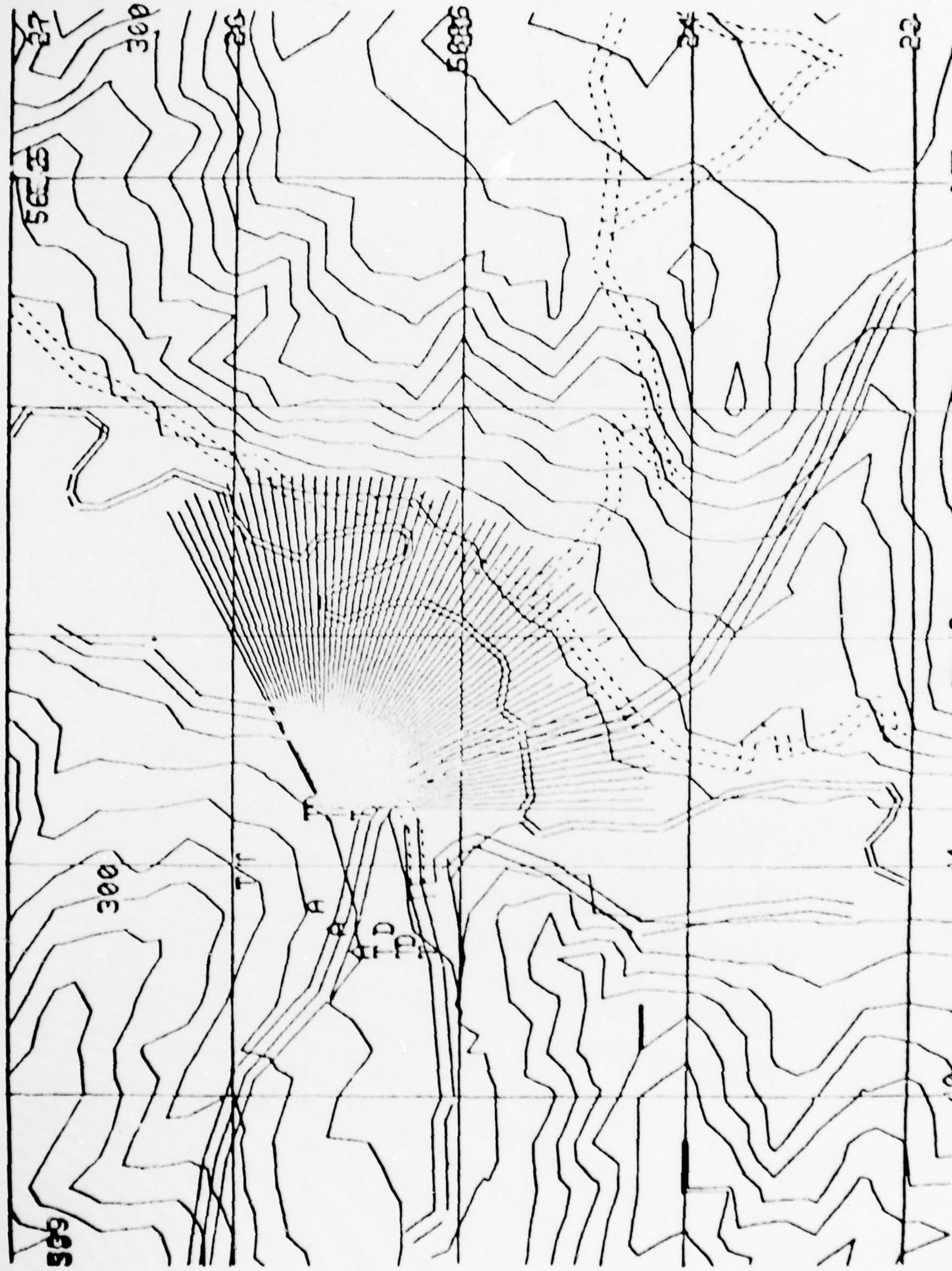
SLIDE 11

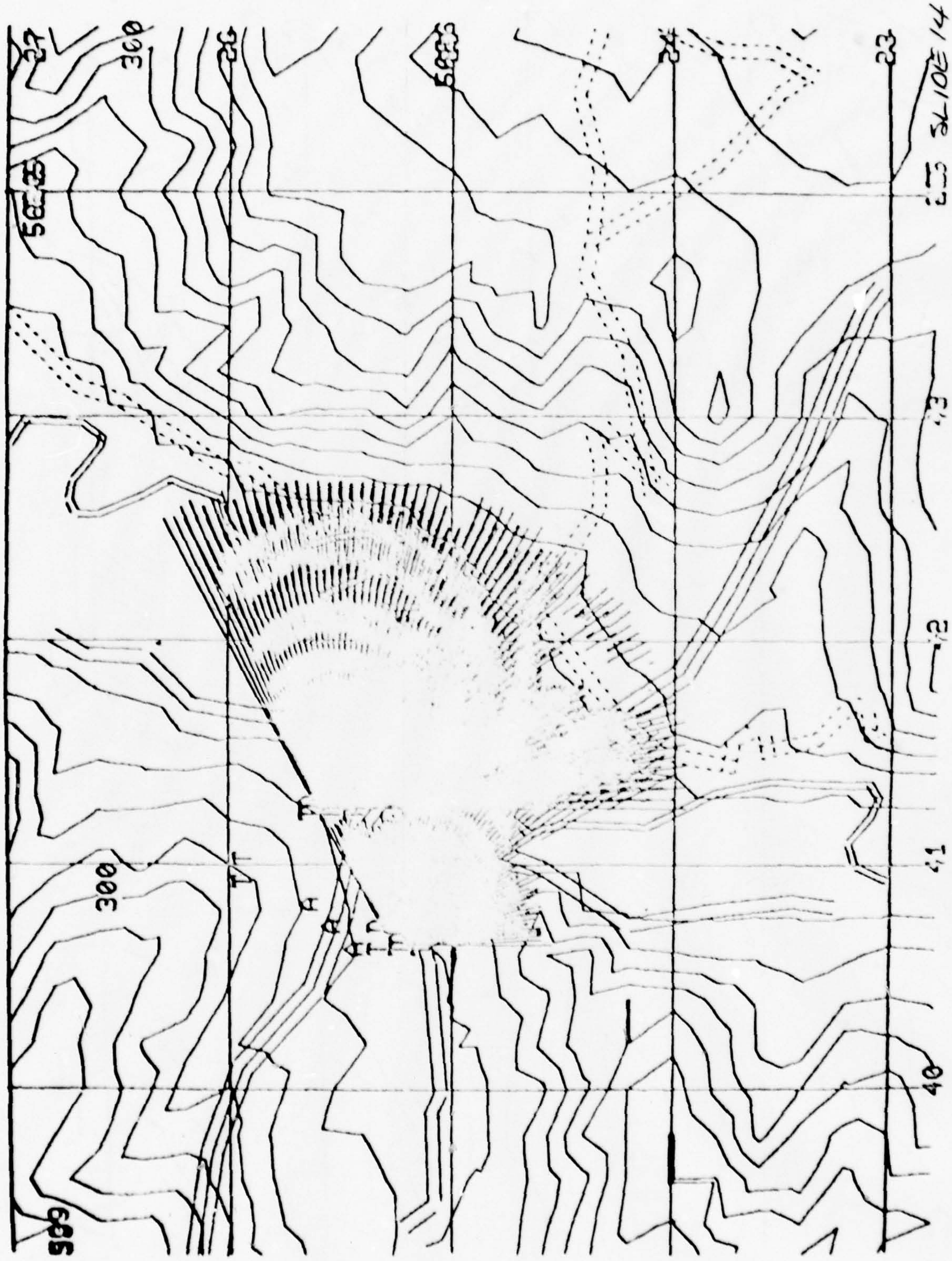




SLIDE 12



SLIDE 13


SLIDE 14
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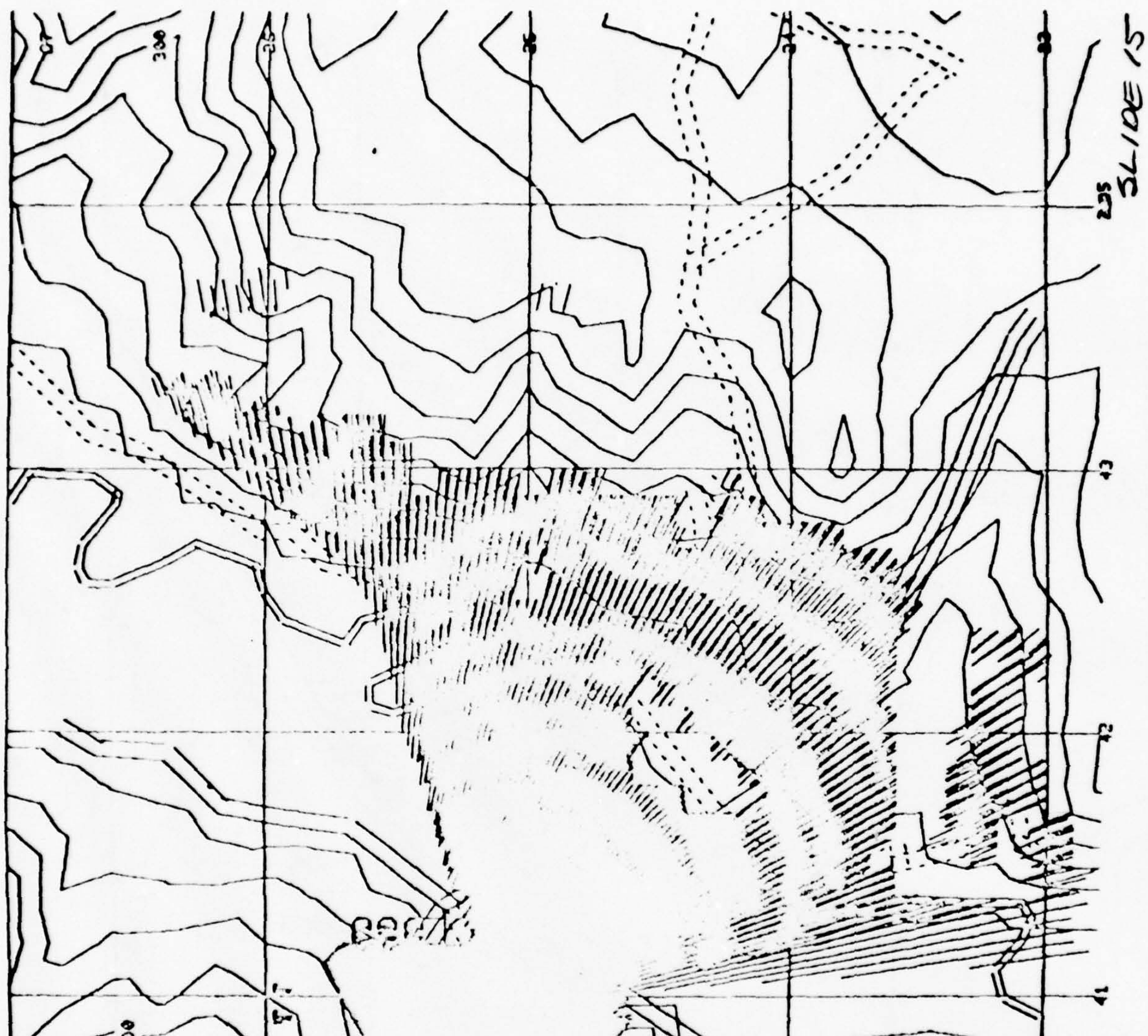
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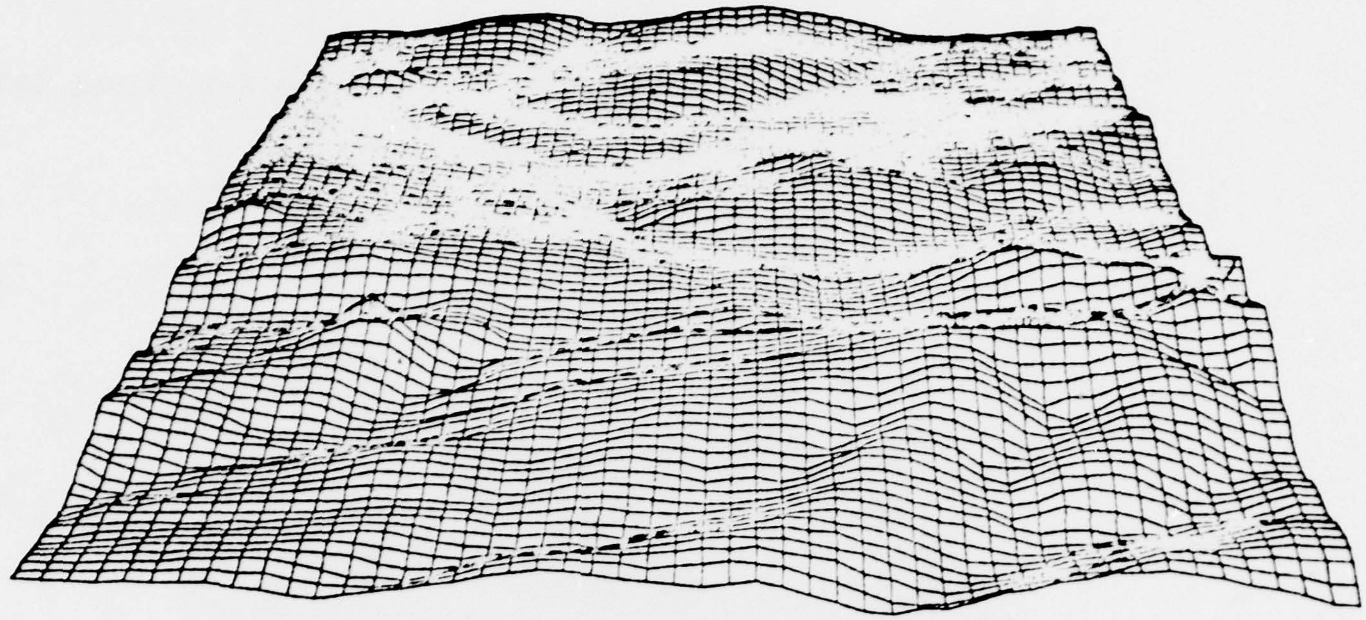
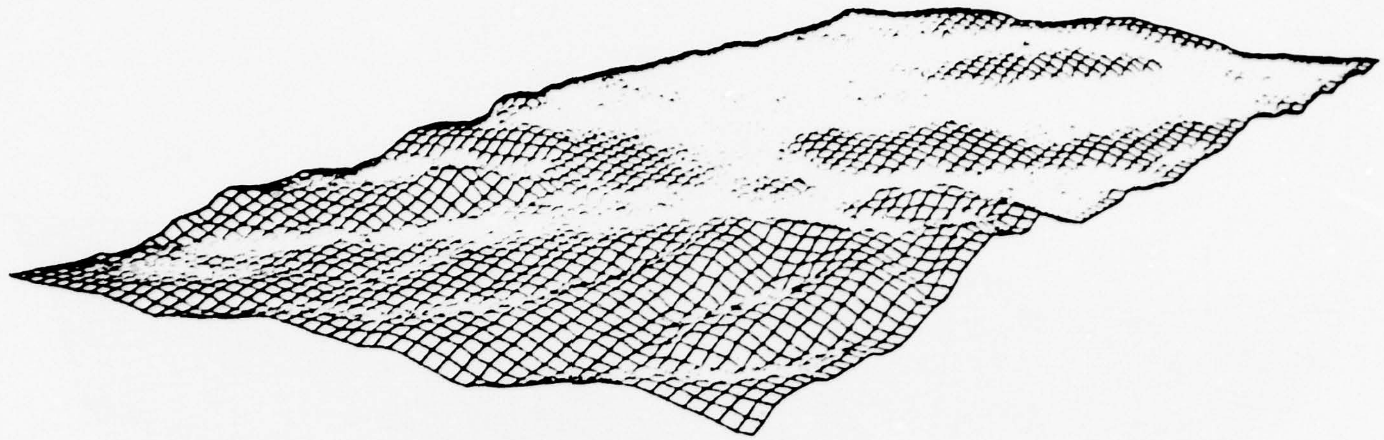


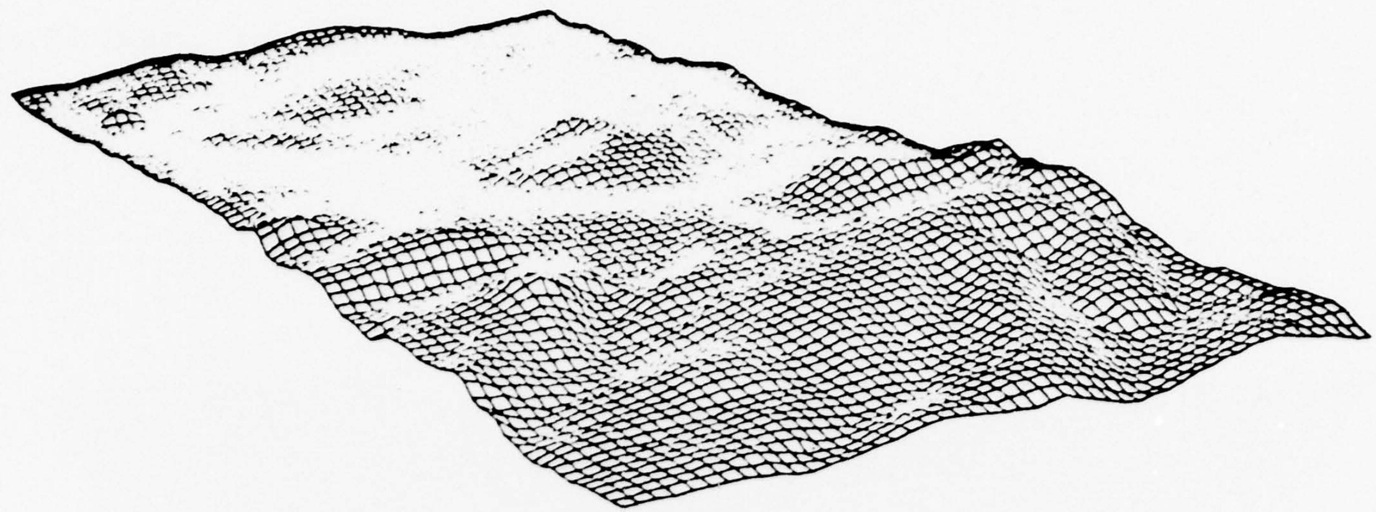
PLOT TRAILS 1-Y 0-N

SLIDE 18



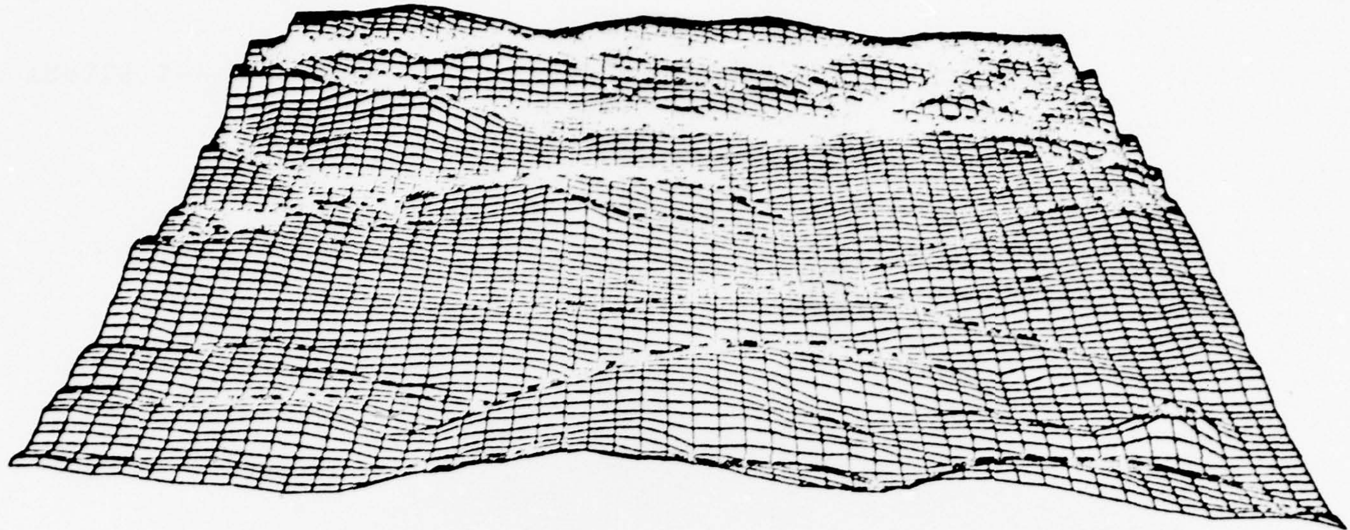
PLOT TRAILS 1-Y 0-N

SLIDE 19



PLOT TRAILS 1-Y 0-N

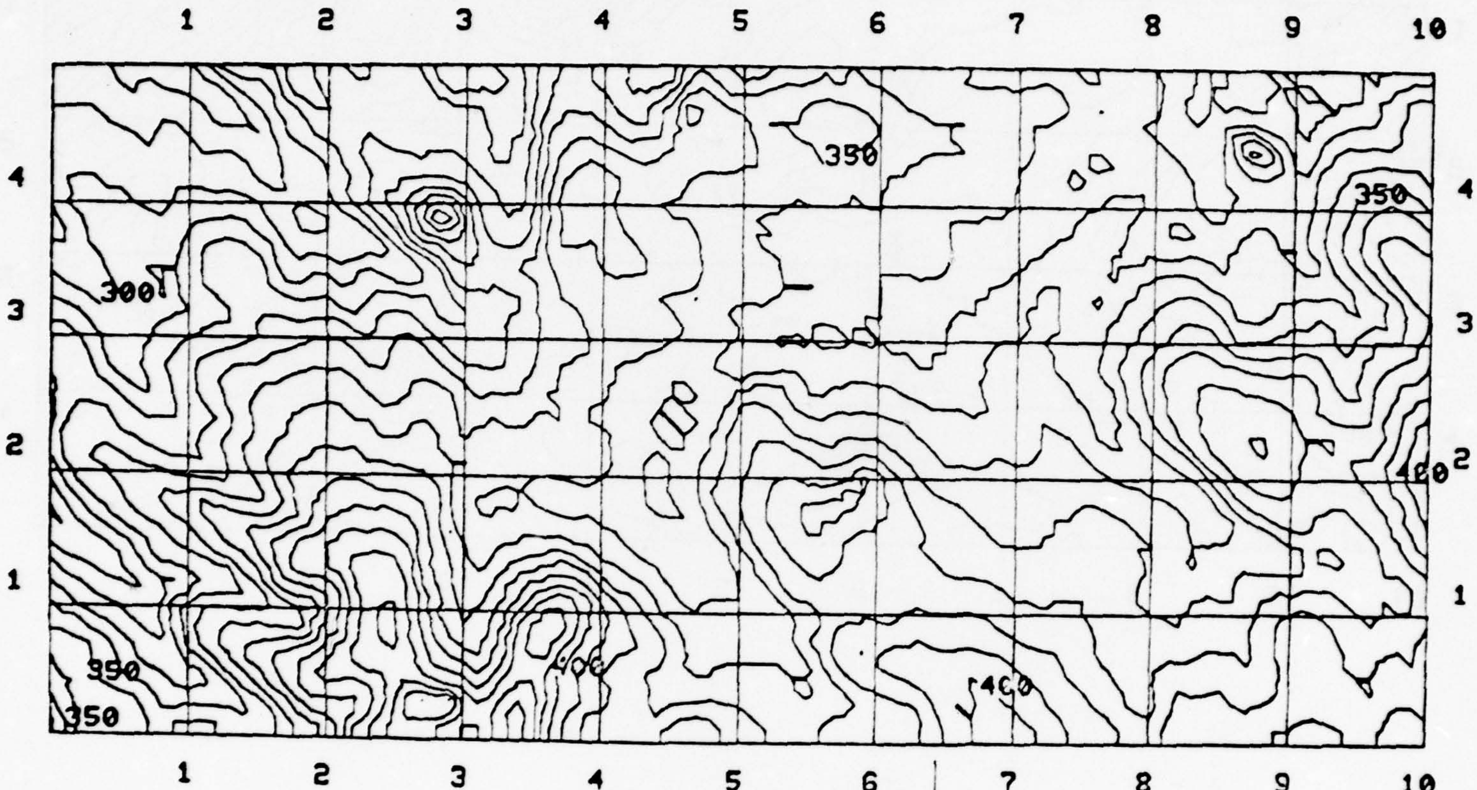
SLIDE 20



PLOT TRAILS 1-Y 0-N

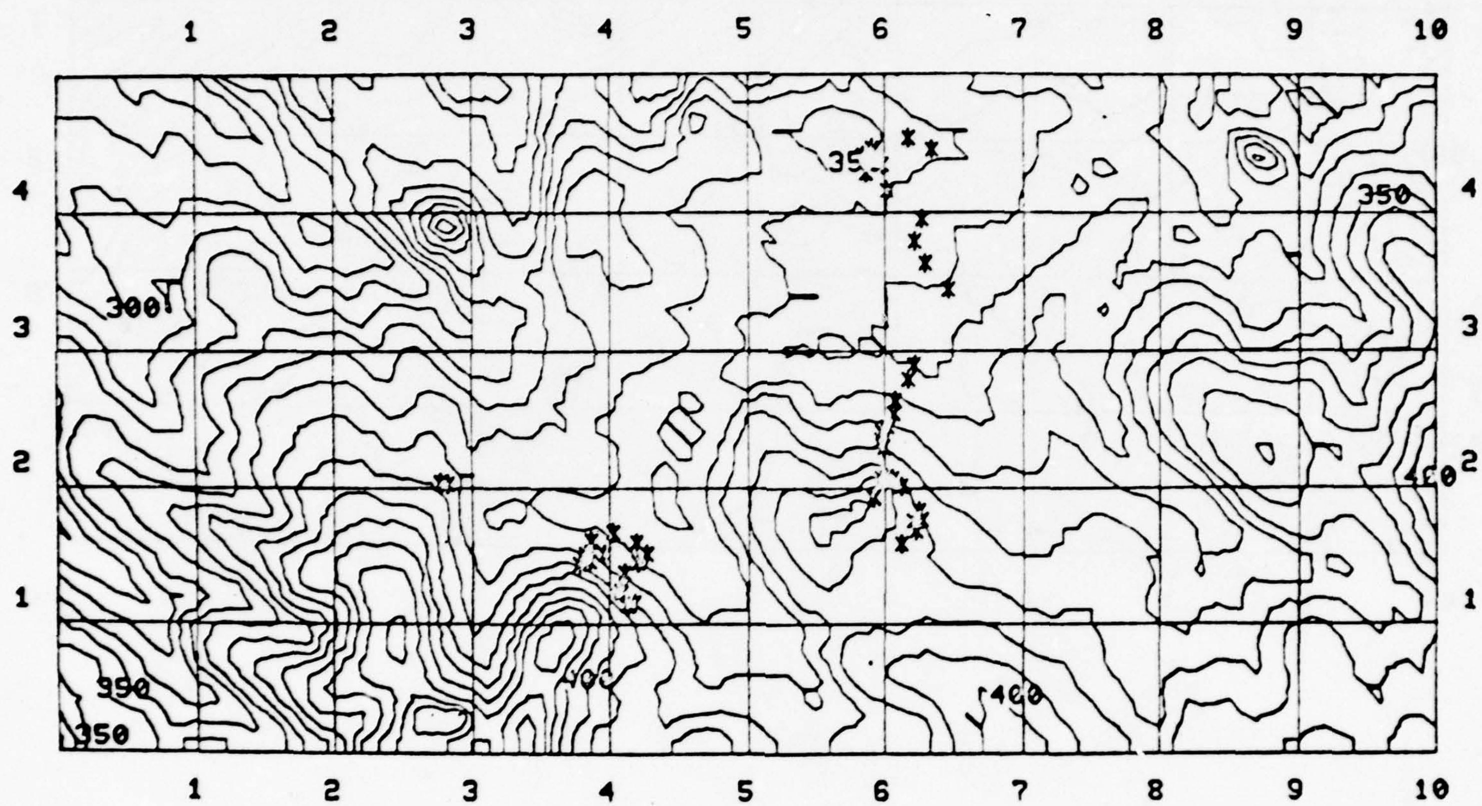
SLIDE 21

XOM-52400 YOM-92665



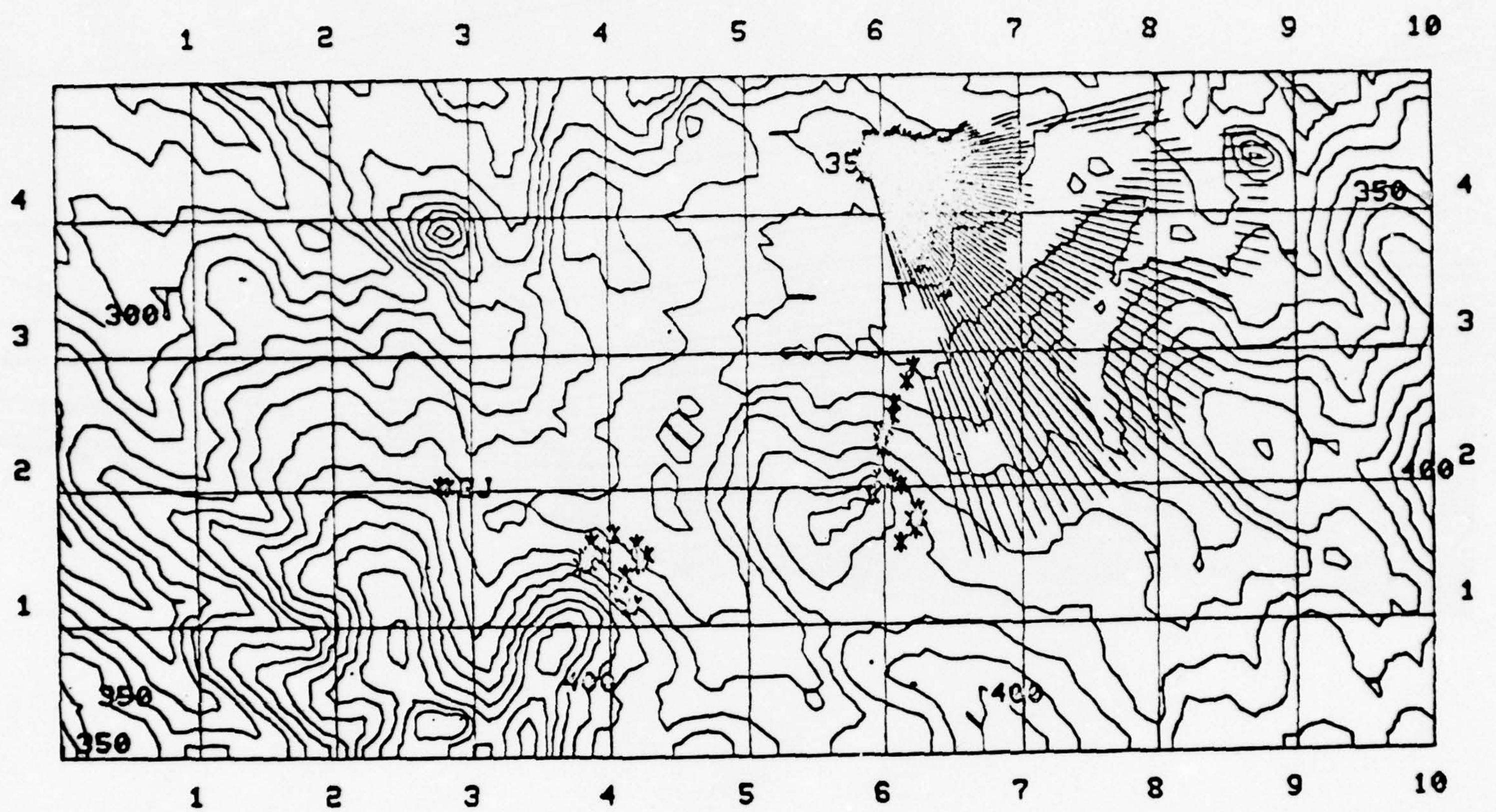
SLIDE 22

XOM=52400 YOM=92665

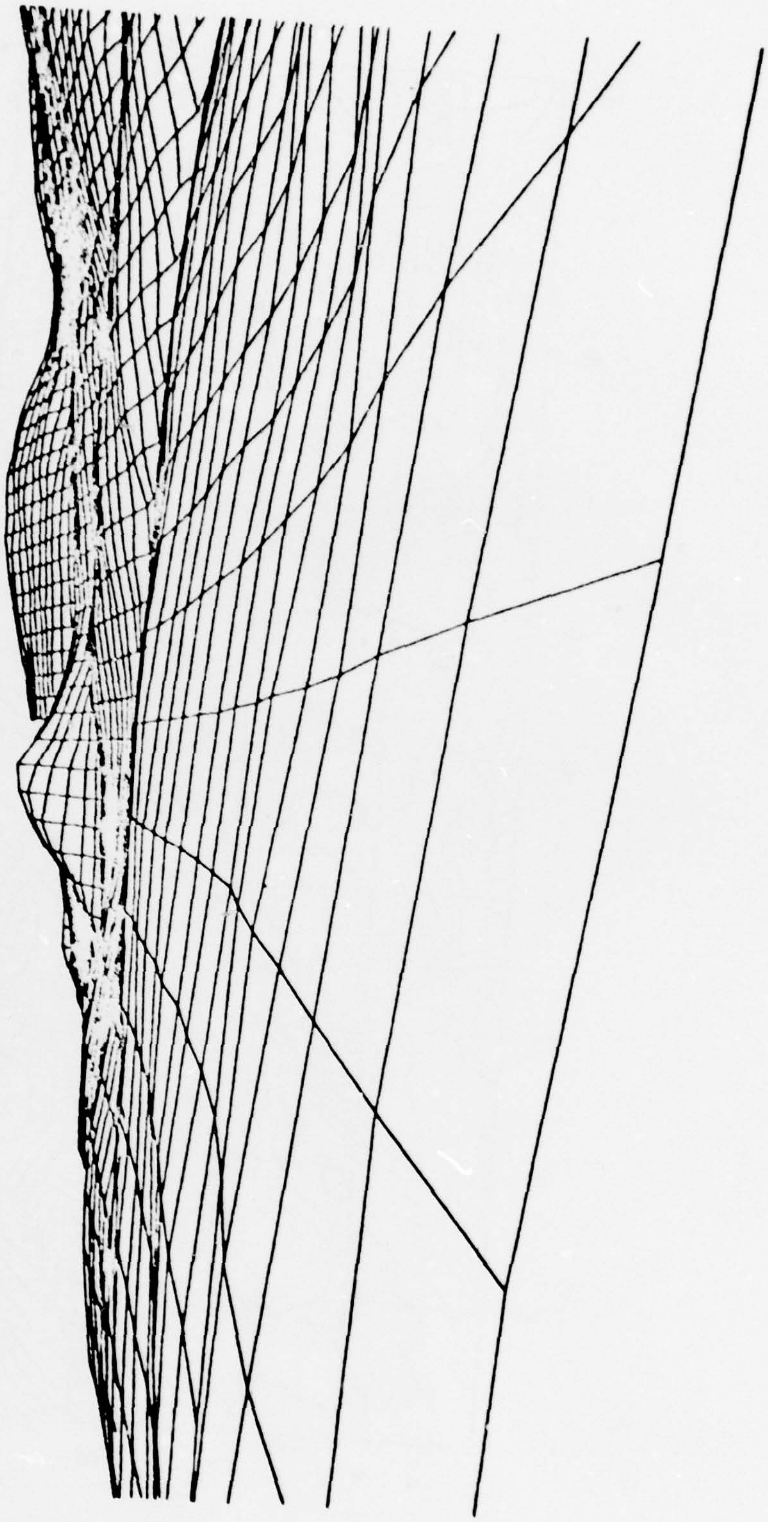


SLIDE 23

XOM-52400 YOM-92665



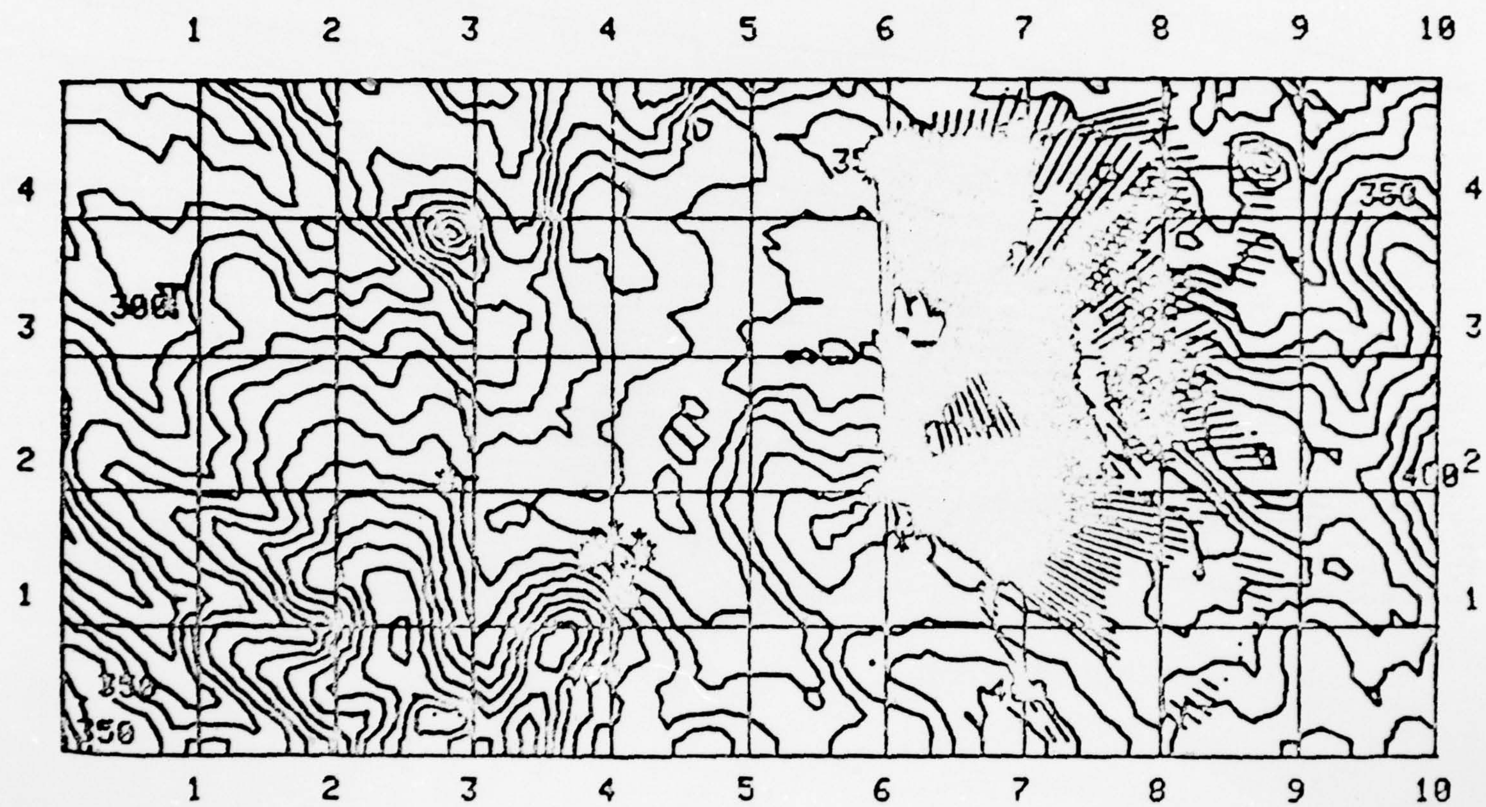
SLIDE 24



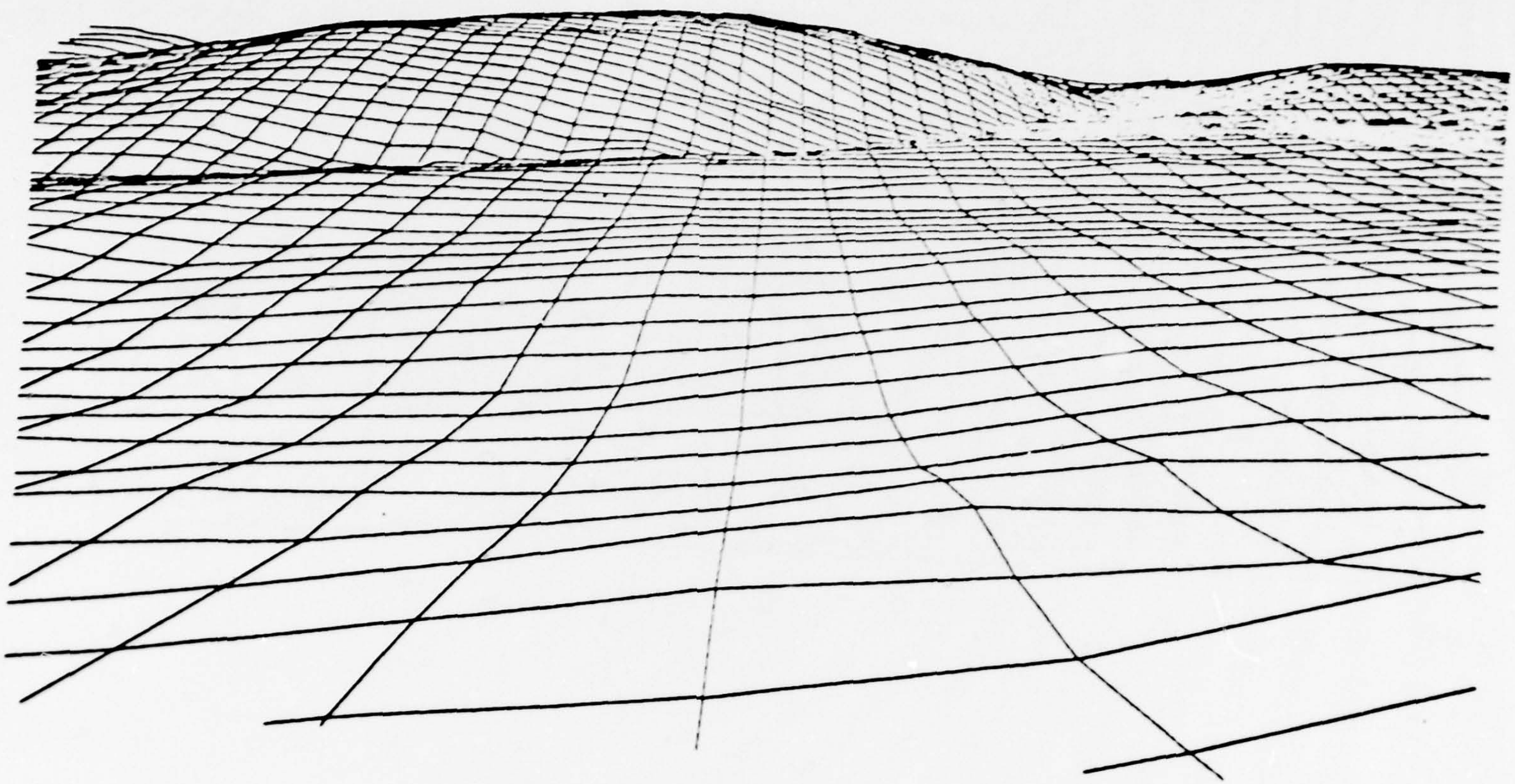
SLIDE 25

3J

XOM=52400 YOM=92665

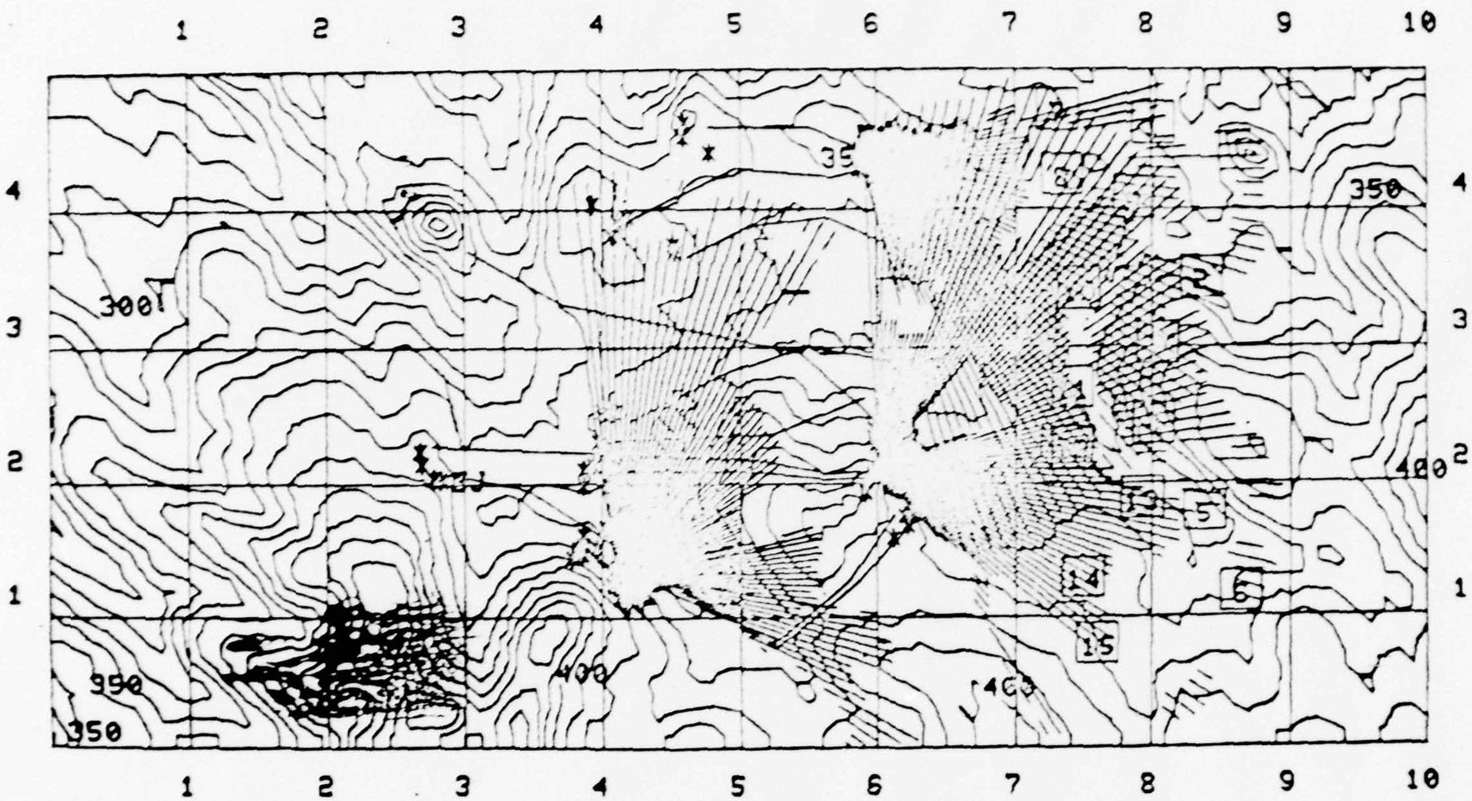


SLIDE 26

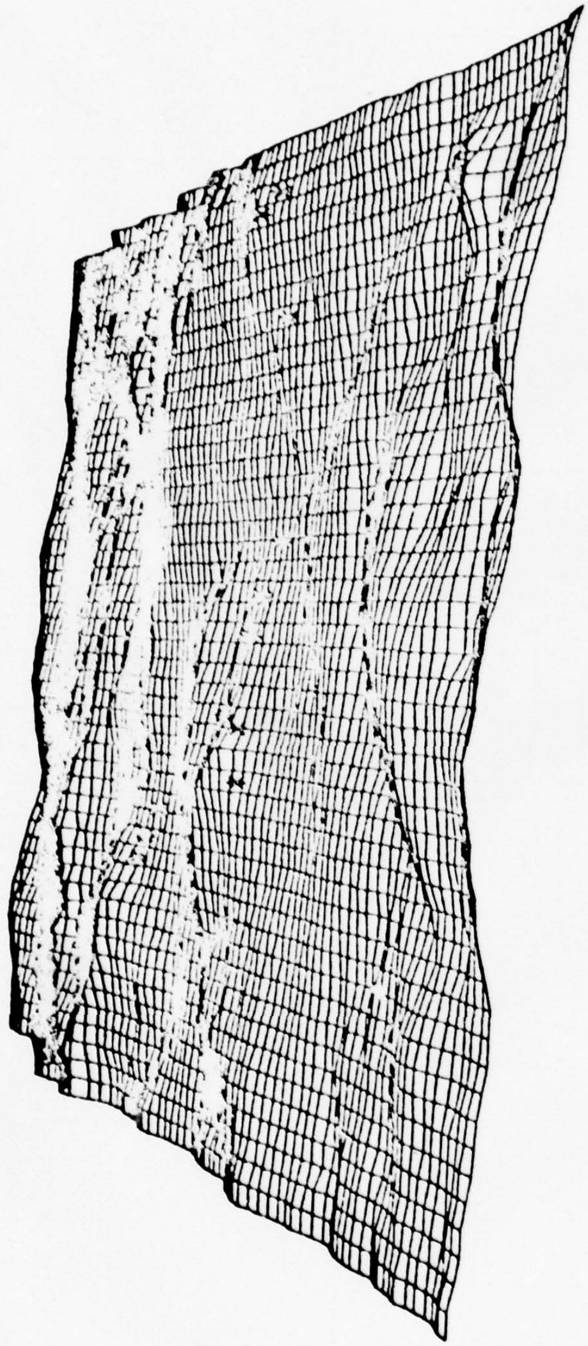


SLIDE 27

XOM=52400 YOM=92665

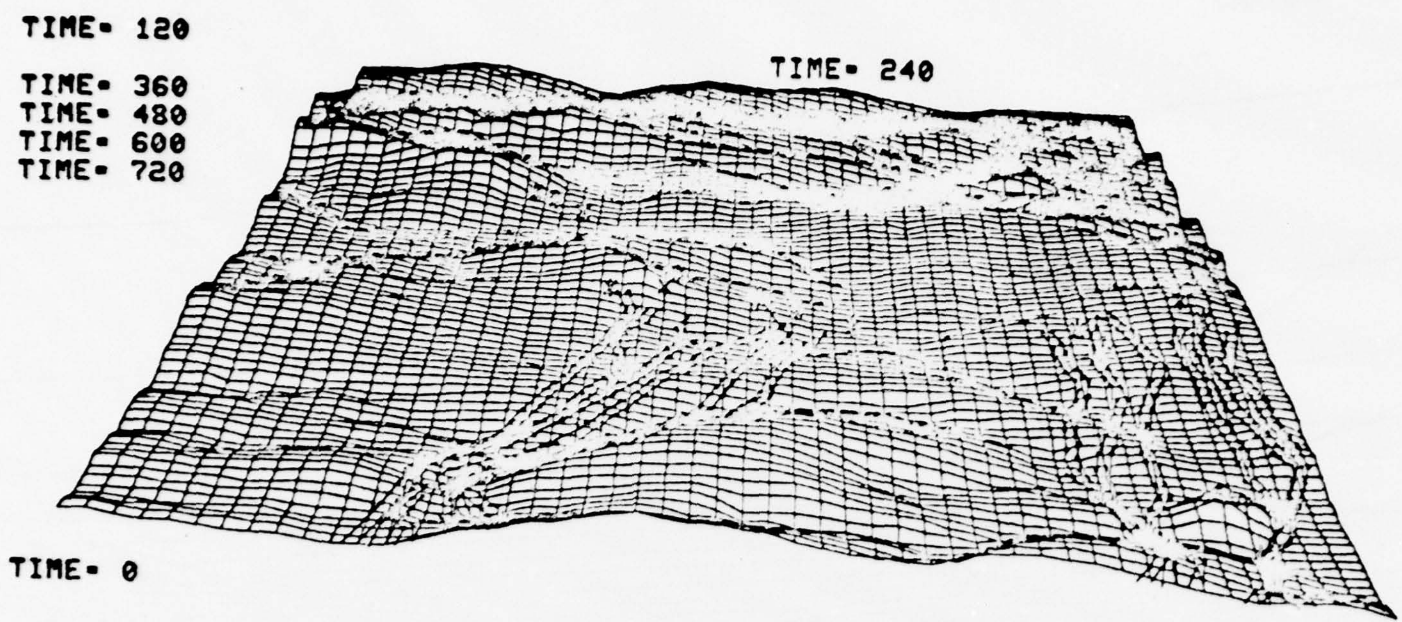


SLIDE 28



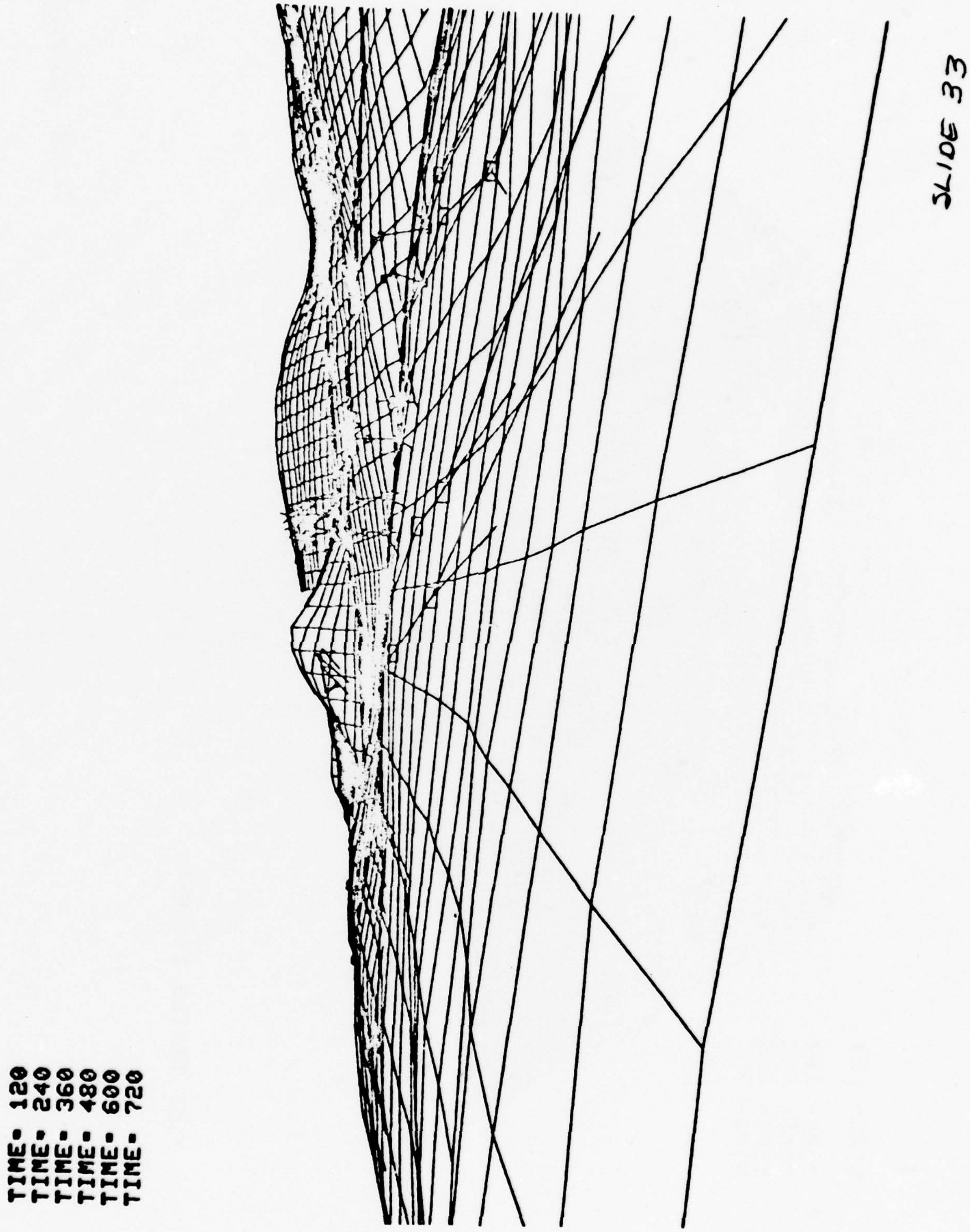
PLOT TRAILS 1-Y 0-N

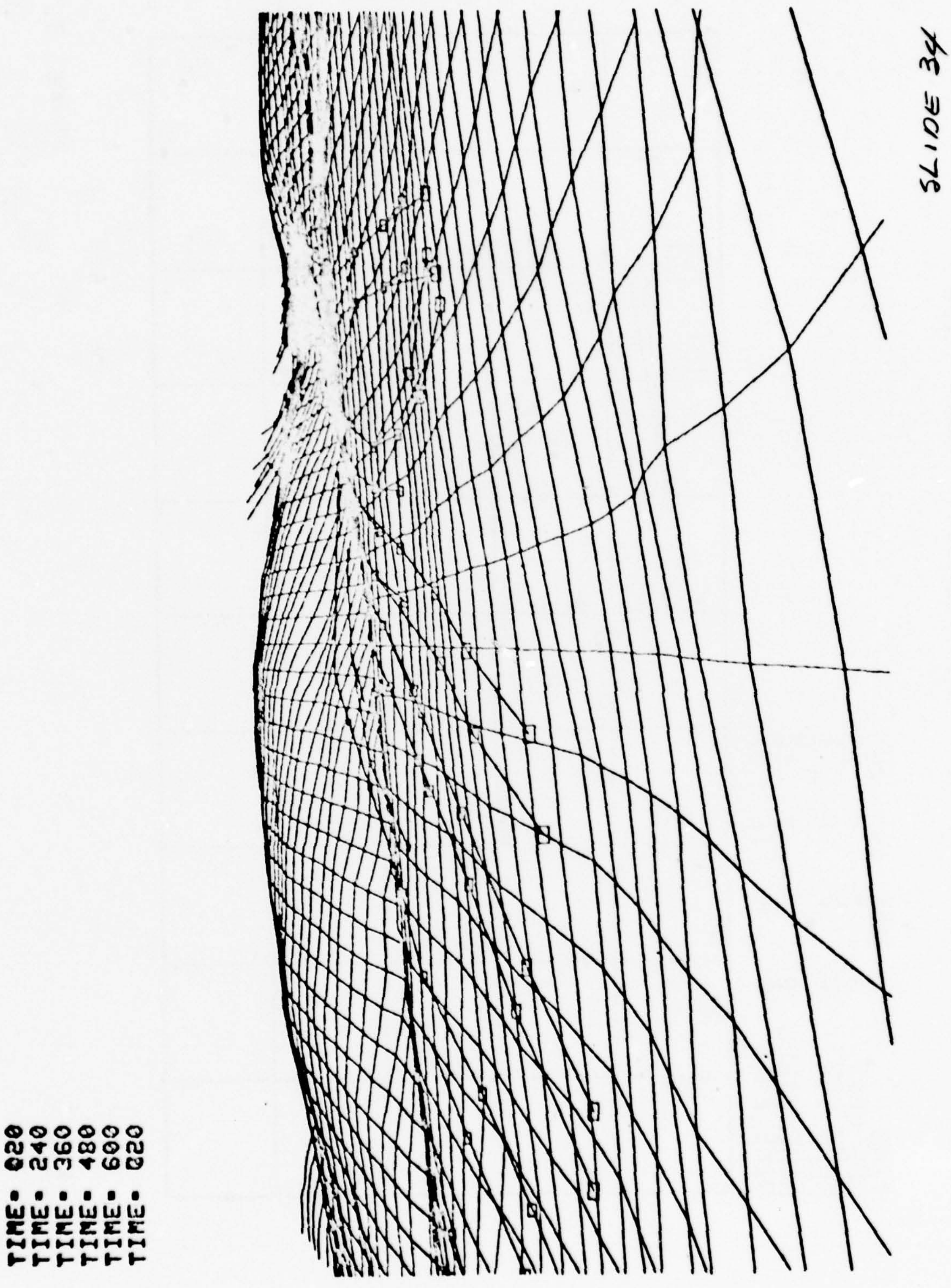
SLIDE 29



PLOT TRAILS 1-Y 0-N

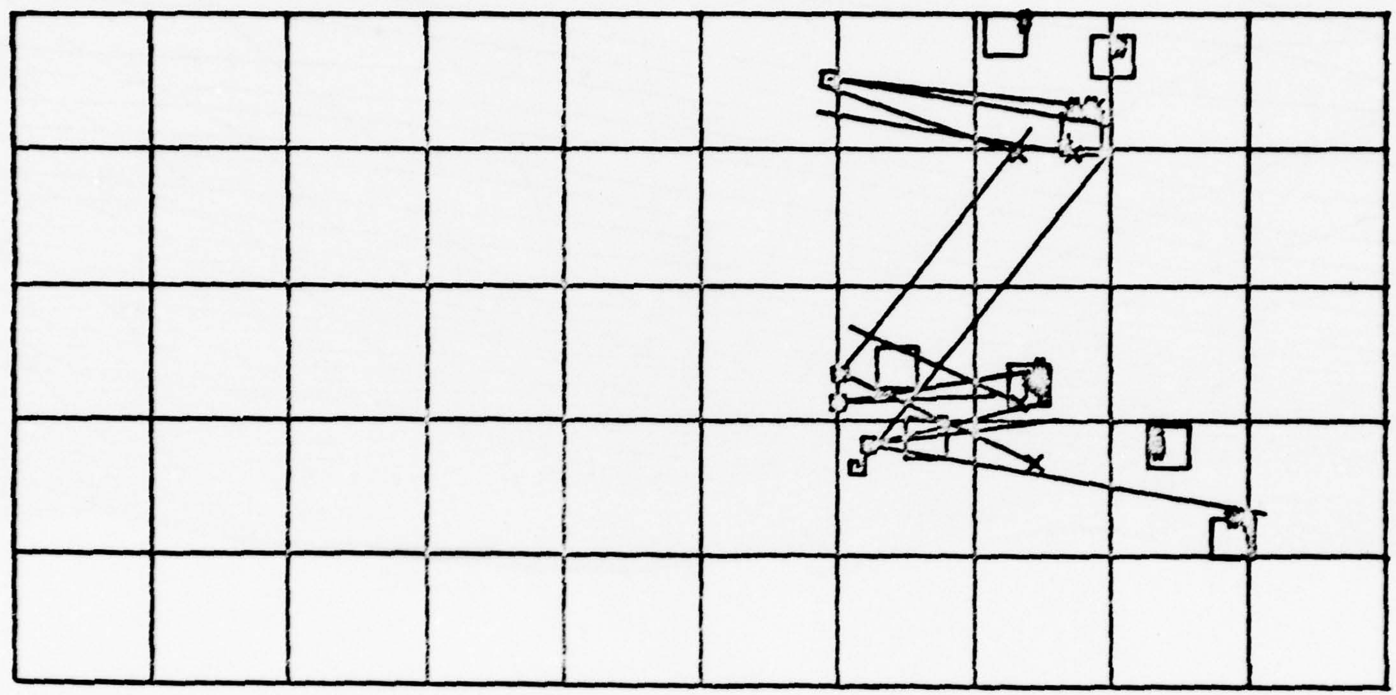
SLIDE 30, 31, 32


TIME= 120
TIME= 240
TIME= 360
TIME= 480
TIME= 600
TIME= 720
SLIDE 33


TIME= 120
TIME= 240
TIME= 360
TIME= 480
TIME= 600
TIME= 720
SLIDE 34

MF CM DENSITY DET ID#

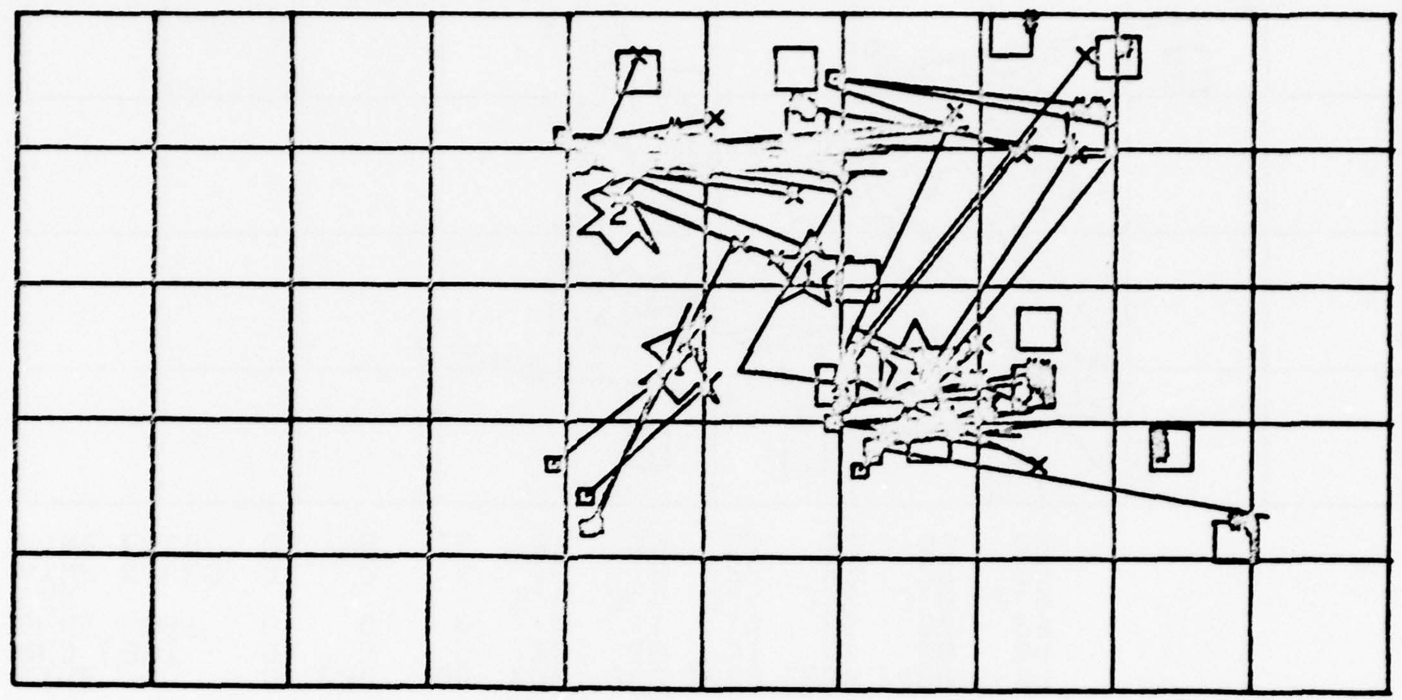
TIME	0	200	400	600
RED LOST	0	3	9	25
BLUE LOST	0	0	0	5
LER				5.0
MINE KILLS	0	2	6	16
MINE ENCS	0	6	16	37



SLIDE 35

MF CM DENSITY DET ID#

TIME	0	200	400	600	800	1000	1200
RED LOST	0	3	9	25	38	51	57
BLUE LOST	0	0	0	5	11	15	26
LER				5.0	3.5	3.4	2.2
MINE KILLS	0	2	6	16	18	22	23
MINE ENCS	0	6	16	37	39	51	

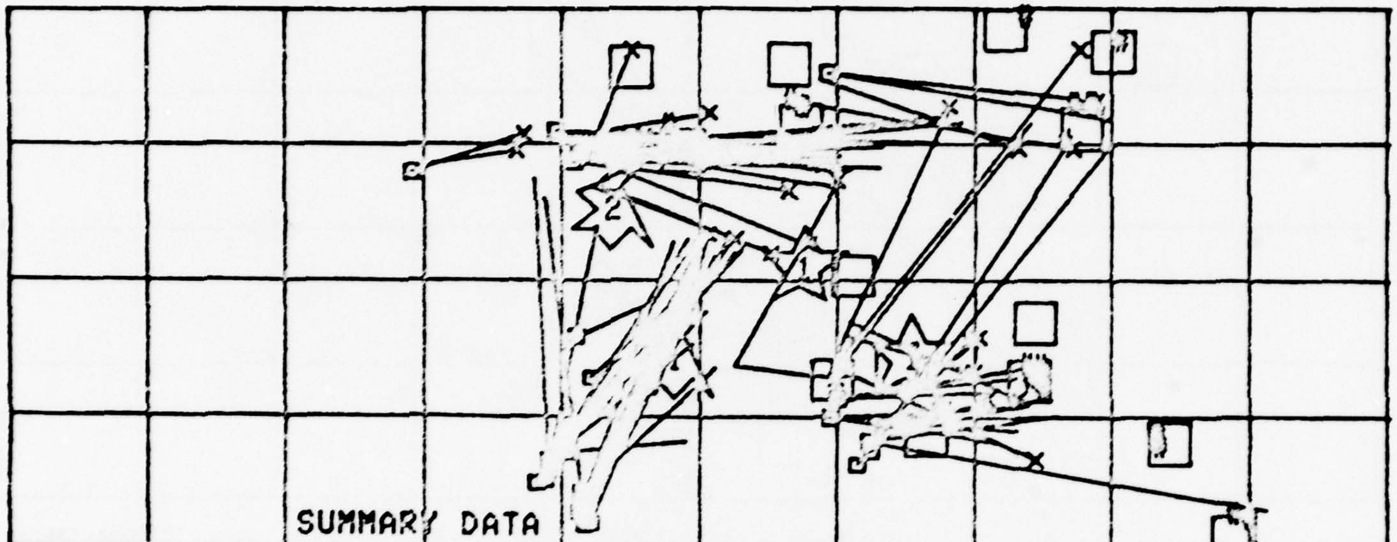


SLIDE 36

FILE NO. 1
KILL TYPE CHOSEN 1
FEATURES CHOSEN:
32 MF CM DENSITY DET ID#

TIME 1600

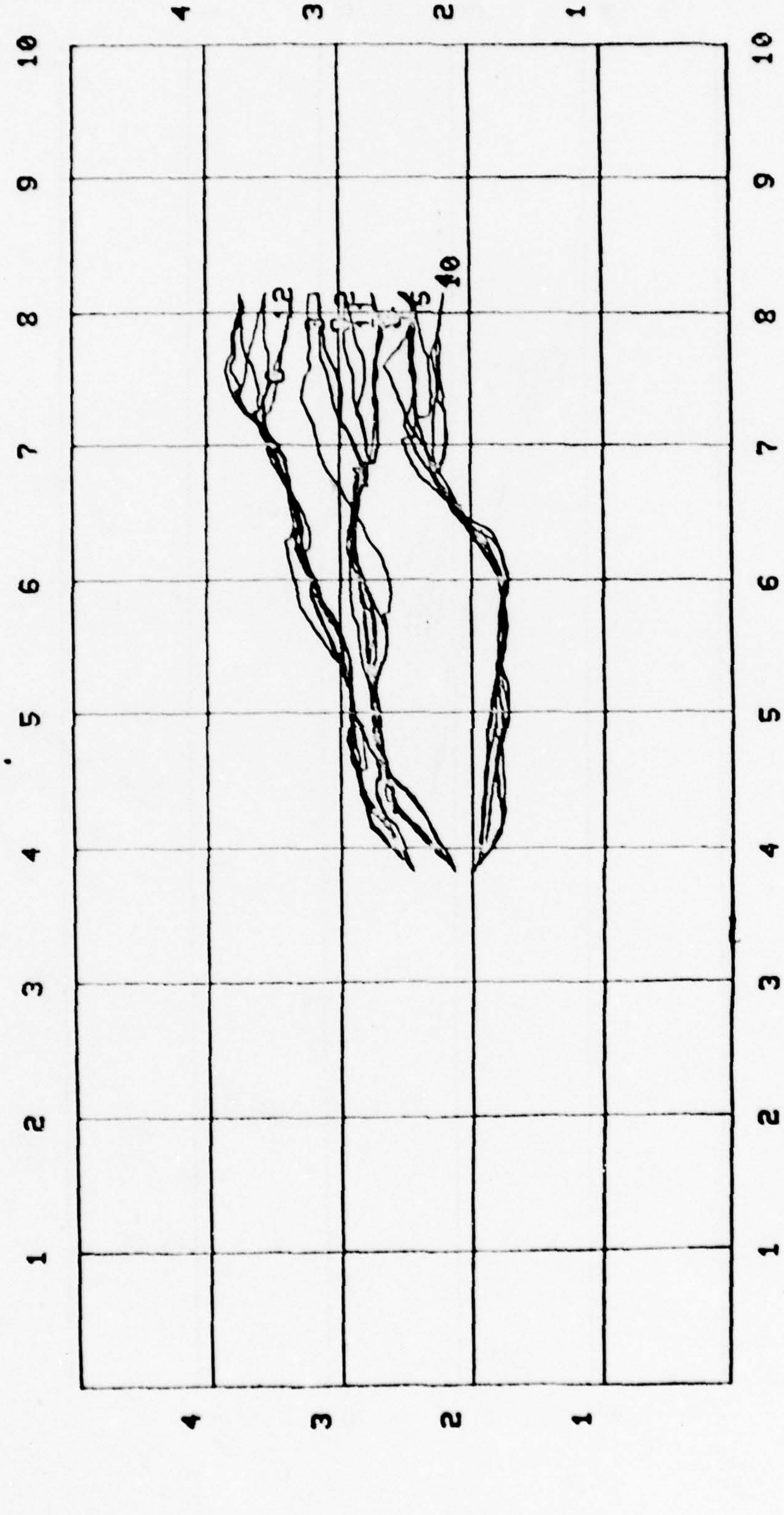
TIME	0	200	400	600	800	1000	1200	1400	1600
RED LOST	0	3	9	25	38	51	57	80	84
BLUE LOST	0	0	0	5	11	15	26	27	29
LER				5.0	3.5	3.4	2.2	3.0	2.9
MINE KILLS	0	2	6	16	18	22	23	23	23
MINE ENCS	0	6	16	37	39	51	52	53	53



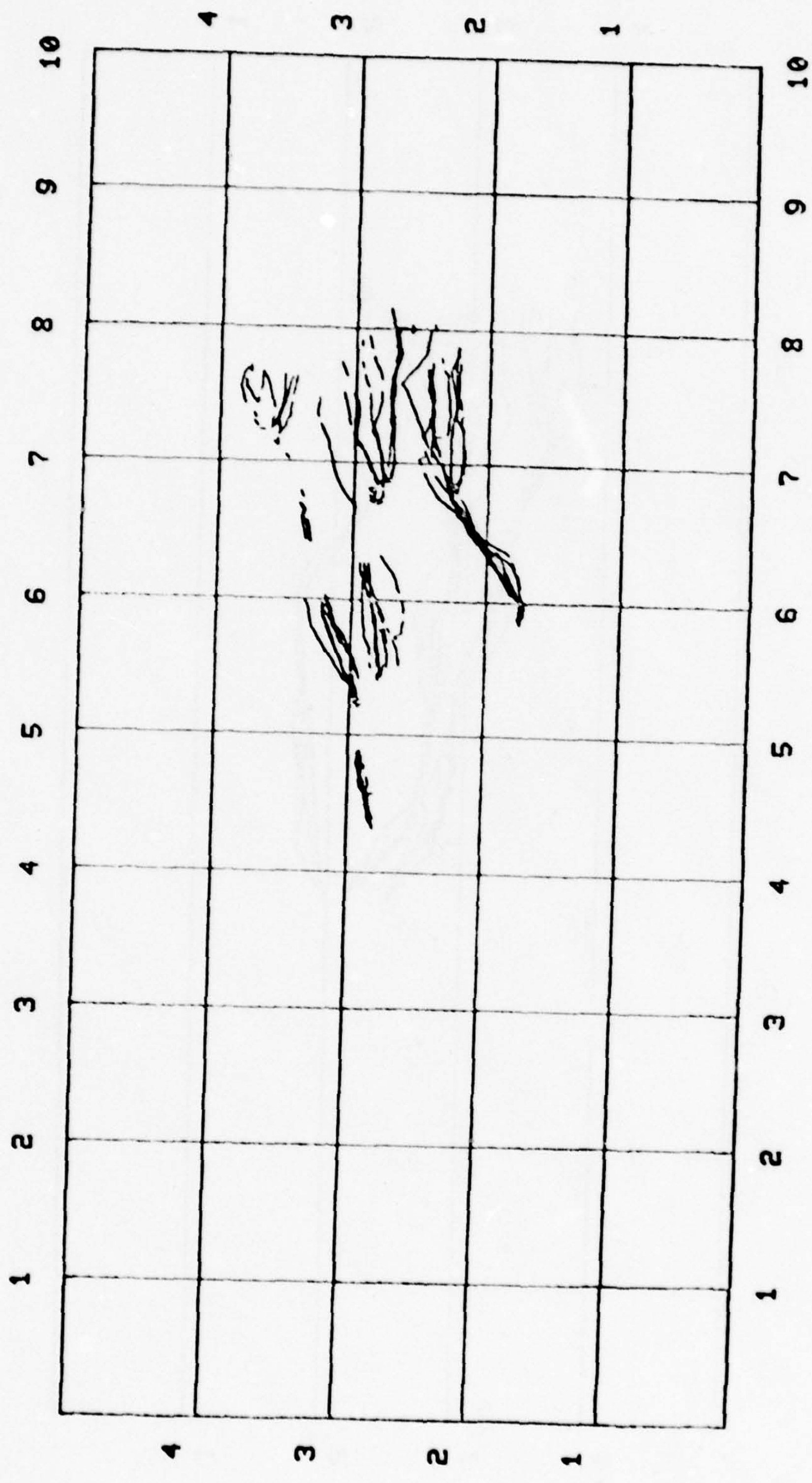
SUMMARY DATA

RED LOSSES	BLUE LOSSES	LER	MF ACT	MINE ENC	MINE KILLS	TIME	KT
84	29	2.90	15	53	23	1600	1

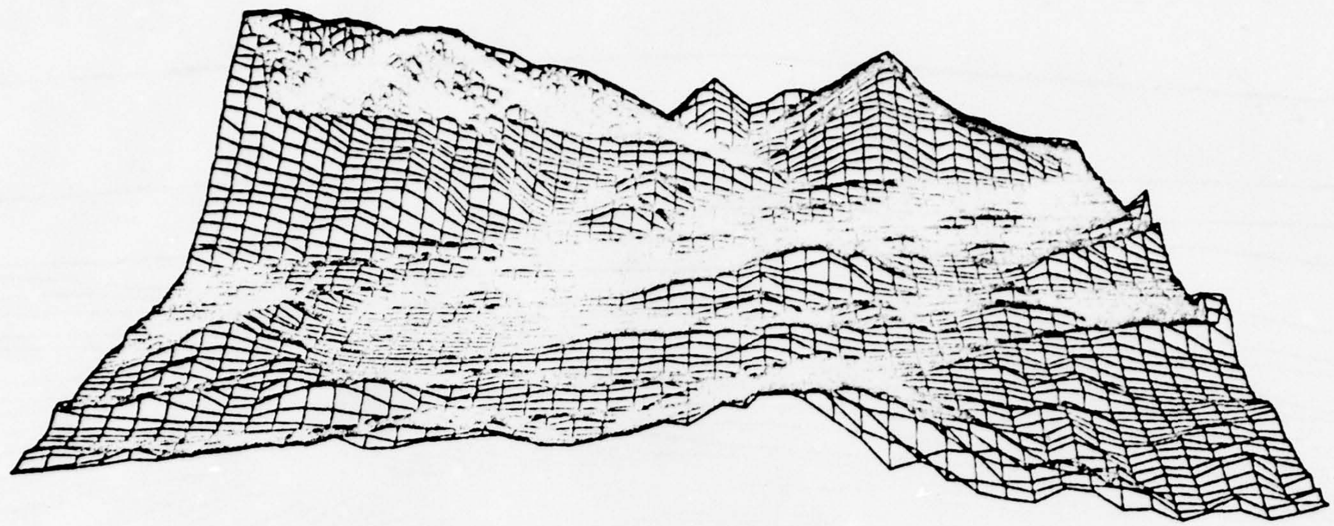
SLIDE 37



JL/OE 38

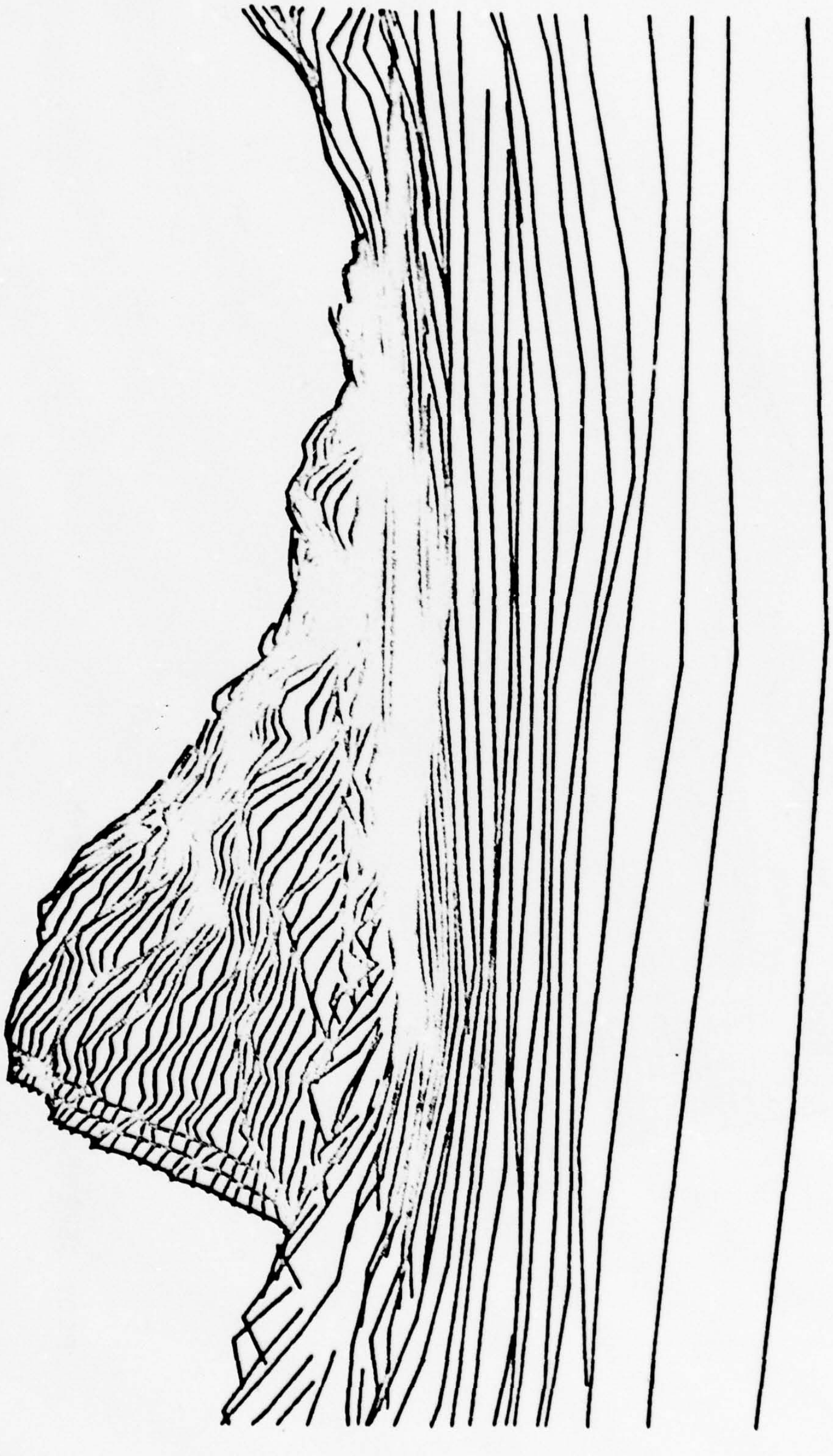


SLIDE 39

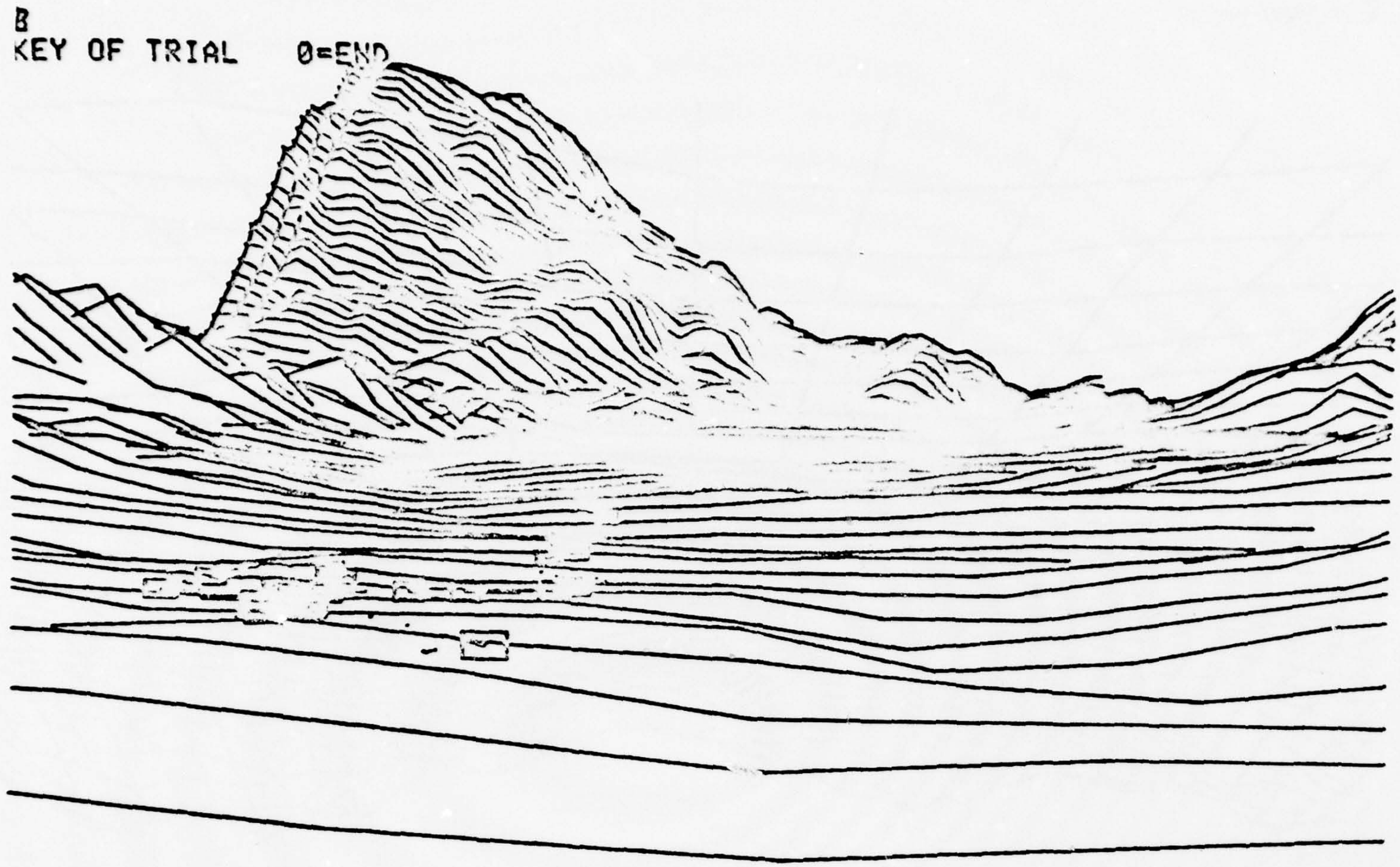


PLOT DEFENDER POSITIONS 1-Y 0-N

SLIDE 40

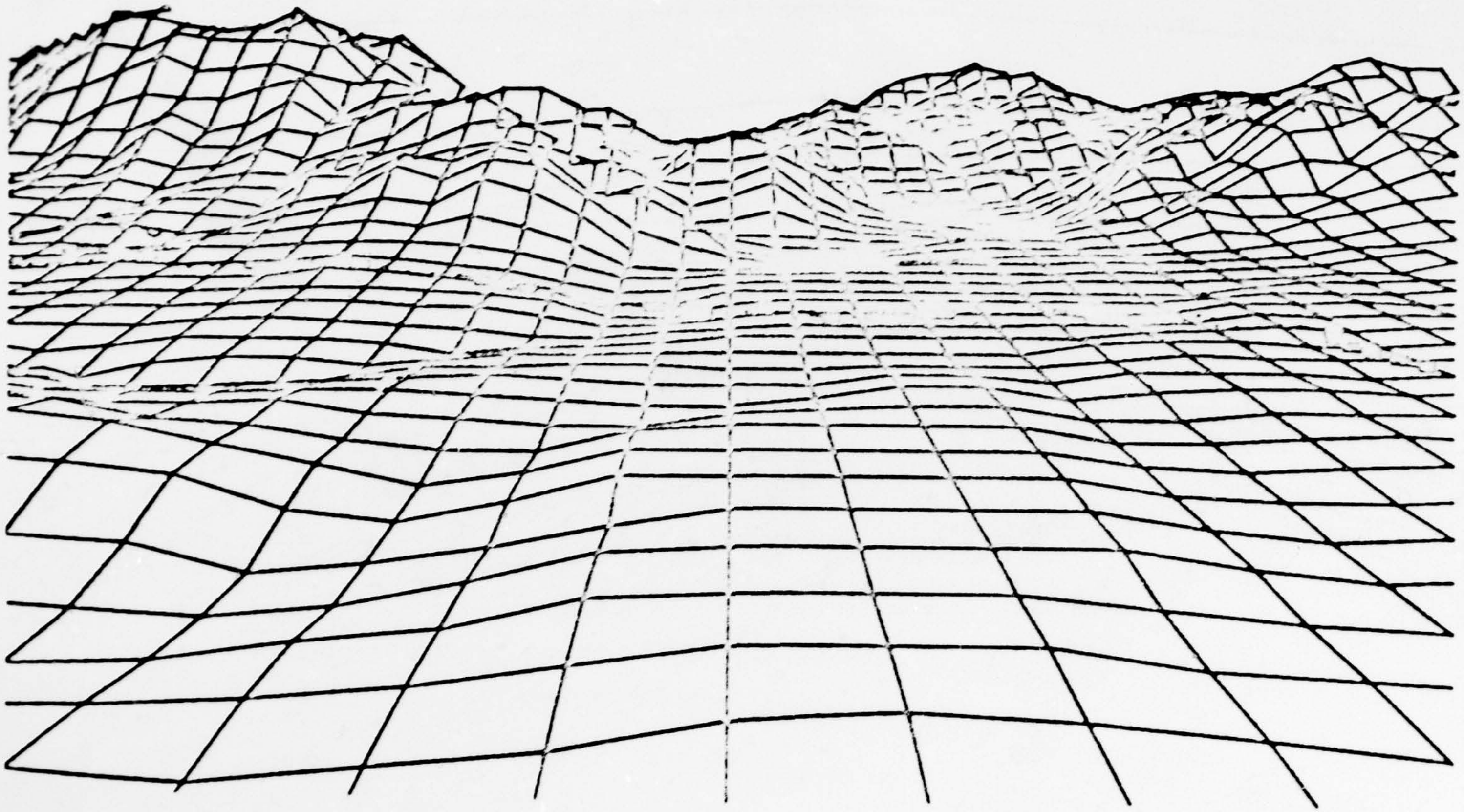


SLIDE 41



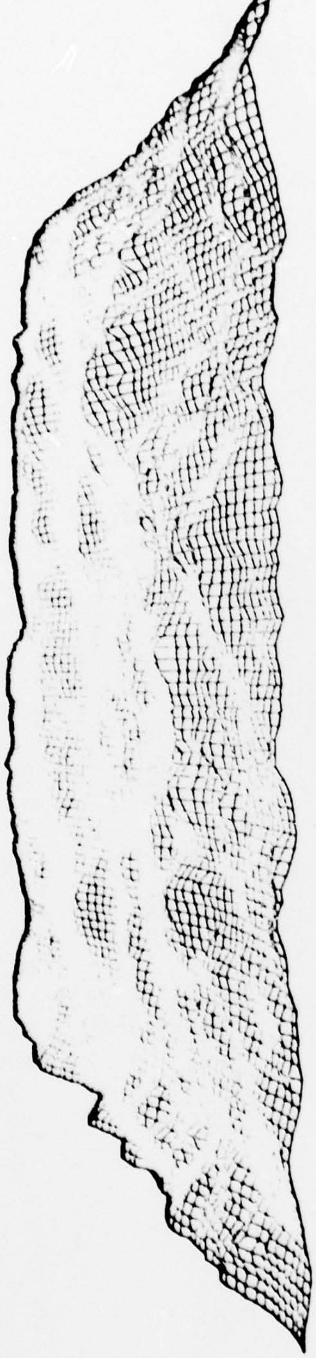
SLIDE 42

12
KEY OF TRIAL   0=END



SLIDE 43

SLIDE 43



SLIDE 44

SLIDE 45

TITLE: A Mathematical Programming Model for an Aircraft Modification Program

AUTHOR: Dr. Frank Fox

ABSTRACT: A large fleet of aircraft is to be modified by the installation on each aircraft in the fleet of a certain number of engineering change proposal (ECP) kits. The aircraft are deployed in smaller subfleets, called field units, at various locations around the world, and the kit installations are to take place at a single contractor facility. Therefore, each of the aircraft must be taken out of operation in the field and sent in to the contractor facility for modification.

Each field unit has an authorized strength of aircraft with a specified operational readiness to maintain, permitting only a certain number of aircraft to be away from each field unit at a given time. Furthermore, ECP kits become available over a period of time. At the beginning of the program only certain types of kits are available, and the last type of kit doesn't become available until some time later. Therefore, an aircraft sent in for modification early in the program will not get all of the ECP kits and must be sent back at least a second time.

In order to maintain operational readiness, the aircraft should be sent in as few times as possible and brought back as quickly as possible. On the other hand, aircraft can be allowed to wait at the contractor facility until additional ECP kits become available and can be installed. Hence, there are reasons for bringing the aircraft back from the contractor facility as quickly as possible and also reasons for leaving them there for extra periods of time.

An optimum solution to the problem of how many aircraft should be sent in and for how long they should remain can be developed using mathematical programming. In this paper an integer programming model consisting of constraints and an objective function is developed, making it possible to maximize the number of kits installed on the fleet of aircraft in a given period of time. The solution obtained from the model permits a kit installation schedule to be developed. The schedule will give for each field unit the number of aircraft to be sent in each month and the month in which they will be returned to the field unit. The model has application to scheduling problems of the type described above and will be useful in the solution of similar problems.

A MATHEMATICAL PROGRAMMING MODEL FOR AN AIRCRAFT MODIFICATION PROGRAM

DR. FRANK FOX

US ARMY AVIATION RESEARCH
AND DEVELOPMENT COMMAND

1. INTRODUCTION

A large fleet of aircraft is to be modified by the installation on each aircraft in the fleet of a certain number of engineering change proposal (ECP) kits. The aircraft are deployed in smaller sub-fleets, called field units, at various locations around the world; and the kit installations are to take place at a single contractor facility. Therefore, each of the aircraft must be taken out of operation in the field and sent in to the contractor facility for modification.

Each field unit has an authorized strength of aircraft with a specified operational readiness to maintain, permitting only a certain number of aircraft to be away from each field unit at a given time. Furthermore, ECP kits become available over a period of time. At the beginning of the program only a certain type of kits are available, and the last type of kit doesn't become available until some time later. Therefore, an aircraft sent in for modification early in the program will not get all of the ECP kits and must be sent back at least a second time.

In order to maintain operational readiness, the aircraft should be sent in as few times as possible and brought back as quickly as possible. On the other hand, if an aircraft is allowed to wait at the contractor facility until additional ECP kits become available and can be installed, it can be returned to the field unit with more modifications, making it better able to perform its mission and perhaps also eliminating a future trip to the contractor facility for those modifications for which it waited. Hence, there are reasons for bringing the aircraft back from the contractor facility as quickly as possible and also reasons for leaving them there for extra periods of time.

An optimum solution to the problem of how many aircraft should be sent in and for how long they should remain can be developed using mathematical programming. In this paper an integer programming model consisting of constraints and an objective function is developed, making it possible to maxmize the number of kits installed on the fleet of aircraft in a given period of time. The solution obtained from the model permits a kit installation schedule to be developed. The schedule will give for each field unit the number of aircraft to be sent in each month and the month in which they will be returned to the field unit. The model has application to scheduling problems of the type described avove and will be useful in the solution of similar problems.

2. ASSUMPTIONS

When constructing a model it is desired to model the actual physical situation as closely as possible. However, it is often impossible to

model all aspects of a situation in complete detail. Certain assumptions were made here in order to include as much complexity as possible while still keeping the model manageable.

The kit installation program will run for a certain number of time periods. A time period can be any fixed length of time such as a month, quarter, year, etc. For convenience of expression let the time periods be months and let the length of the program be T months.

First, it is assumed that at the beginning of the program all of the aircraft are in need of all the kits. Also it is assumed that the kits are numbered according to availability; the first type of kit available is kit number 1, the second type of kit available is kit number 2, etc. Further, it is assumed that for each type of kit there is a month in which that kit becomes available in whatever quantities it is needed from then on.

In practice the monthly demand for kits is probably uniform enough to make this a reasonable assumption.

Hence, the following table of kit availability can be constructed.

Table 1. TABLE OF KIT AVAILABILITY

MONTH	1	2	3	4	. . .	T
KIT	K_1	K_2	K_3	K_4	. . .	K_T

The table has the following interpretation. All aircraft shipped from the contractor facility in the 1st month will have recieved kits 1 through K_1, those shipped in the 2nd month will have received kits 1 through K_2, those shipped in the 3rd month will have received kits 1 through K_3, etc. If all the kits become available before the T^{th} month, there will be some month c such that from the c^{th} month on all aircraft shipped from the contractor facility will have all kits.

Implicit in this assumption is the assumption that while an aircraft is at the contractor facility, it receives all of the kits which are available before it is shipped back. Furthermore, it is assumed that the installation time for all groups of kits is the same, no matter how many kits there are in a group.

In order to decrease the number of possibilities represented by the variables it is assumed that an aircraft will be sent in no more than twice. Also no aircraft at a field unit will be sent in the second time until every aircraft at the field unit has been sent in once. Furthermore, no aircaft will be sent in the second time until it can have all of its remaining needed kits installed without any waiting time (in other words until the earliest month in which it would be shipped from the contractor facility is the c^{th} month or later).

Let X_{ijk} = the number of aircraft sent in for the first time from the i^{th} field unit in the j^{th} month which are shipped back from the contractor facility in the k^{th} month, and let

Y_{ijk} = the number of aircraft sent in for the second time from

the i^{th} field unit in the j^{th} month which are shipped back from the contractor facility in the k^{th} month.

The subscripts j and k are used to indicate the month j in which the aircraft are sent in for modification and the month k in which the aircraft are shipped back from the contractor facility. A given subscript j may have many associated values of k which in effect allows the aircraft to stay at the contractor facility for various lengths of time, premitting it to wait at the contractor facility until additional kits become available and can be installed.

3. PERMISSIBLE VARIABLES

In the variable X_{ijk}, the subscript j respresnts the month in which the aircraft are shipped to the contractor facility, and the subscript k represents the month in which the aircraft are shipped from the contractor facility. Therefore, the value of j must always be less than the value of k, limiting the possibilities for subscripts to certain values.

Let S_i be the transportation time for an aircraft to be shipped from the i^{th} field unit to the contractor facility, and assume that the transportation time for shipment back to the field unit is also S_i. Let p be the installation time for a group of kits. Recall that p is independent of the number of kits in a group.

If an aircraft is shipped in during the j^{th} month, it will arrive at the contractor facility in the $j + S_i{}^{th}$ month and take p months to be processed. Therefore, the first month in which it can be shipped from the contractor facility is the $j + S_i + p^{th}$month. However, since an aircraft is allowed to wait at the contractor facility for the availability of additional kits, it may not be shipped back in the $j + S_i + p^{th}$ month. That month or any later month is a possible shipping month. Hence, the variable X_{ijk} cannot have a subscript k with $k < j + S_i + p$, thereby limiting the variables which can occur to those listed in the following table.

Table 2. TABLE OF PERMISSIBLE SUBSCRIPTS FOR X_{ijk}

$X_{i,1,1+S_i+p}$	$X_{i,1,2+S_i+p}$	$X_{i,1,3+S_i+p}$	...	$X_{i,1,T}$
	$X_{i,2,2+S_i+p}$	$X_{i,2,3+S_i+p}$	...	$X_{i,2,T}$
		$X_{i,3,3+S_i+p}$	...	$X_{i,3,T}$
				$X_{i,T-S_i-p,T}$

Recall that there is some month c such that all aircraft shipped from the contractor facility in the c^{th} month or a later month will have had all the kits installed. Further, it was assumed that aircraft are not sent in for a second time until all kits are available. Specifically, the aircraft which are sent in for the second time will not be sent in before the month which permits c to be the first month in which they could be shipped from the contractor facility. Since the shipping time

from the i^{th} field unit is S_i and the processing time is p, in the variable Y_{ijk} it is necessary that $j \geq c - S_i - p$. Furthermore, since an aircraft sent in for the second time receives all the remaining needed kits, there is no need for it to stay at the contractor facility longer than the p months necessary to install the kits; in other words, there is no waiting time for additional kits. Therefore, the subscripts on Y_{ijk} are limited to those listed in the following table.

Table 3. TABLE OF PERMISSIBLE SUBSCRIPTS FOR Y_{ijk}

$Y_{i,c-S_i-p,c}$	$Y_{i,c-S_i-p+1,c+1}$	...	$Y_{i,T-S_i-p,T}$

An alternative to having a list of permitted subscripts would be to allow all subscripts to be used and to impose the constraint that the inadmissible variables have the value zero. However, this method was not used in order to keep the number of variables and constraints used to a minimum.

4. AIRCRAFT AVAILABILITY CONSTRAINT

At each field unit there are a certain number of aircraft. In essence, the aircraft availability constraint prevents more aircraft from being sent in from a given field unit than there are aircraft at that field unit. However in applying this constraint, it is not satisfactory to simply count the number of a aircraft sent in, because some aircraft are sent in twice. Therefore, it is necessary to have two separate constraints, one for those aircraft being sent in for the first time and one for those being sent in for the second time.

Let N_i = the number of aircraft stationed at the i^{th} field unit at the start of the modification program. For each field unit, the total number of aircraft sent in for the first time during the T months of the program must be less than or equal to N_i. As an inequality the constraint is written

$$\sum_j \sum_k X_{ijk} \leq N_i \; , \qquad (1a)$$

for each field unit i. The left-hand side of (1a) represents the total number of aircraft sent in for the first time during the T months of the program. In order to avoid cumbersome expressions in the limits of the summation notation, the limits are sometimes ommitted. If no limits are given a summation, it is to be understood that the sum is over all permissible values of the subscript. For example, the left hand side of equation (1a) represents the sum over all permissible subscript j and k for X_{ijk}, in other words over all values listed in Table 2.

If all the kits become available for installation before the T^{th} month, there will be some month c such that all aircraft shipped for the contractor facility in the c^{th} month or a later month will have all the kits installed. In other words, any group X_{ijk} of a aircraft with $k \geq c$ will have all the kits installed the first time they are sent in and will not go in a second time. Therefore, not all the aircraft are possibilities for being sent in a second time, and the number of aircraft avail-

able to be sent in a second time must be decreased by the number which have all the kits installed the first time. The number of aircraft from the i^{th} field unit which have all the kits installed the first time is

$$\sum_j \sum_k X_{ijk} ,$$

Hence the constraint is written as

$$\sum_j \sum_k Y_{ijk} \leq N_i - \sum_j \sum_k X_{ijk} , \qquad (1b)$$

for each field unit i.

5. OPERATIONAL READINESS CONSTRAINT

In each field unit some of the aircraft can be away from the field unit for ECP kit installation. However, in order to maintain the required operational readiness of the fleet, there is a limit on the number of aircraft which can be away at any given time.

Let M_i = the number of aircraft which can be away from the i^{th} field unit for ECP kit installation when there are Ni aircraft assigned to the field unit. The operational readiness constraint is that the number of aircraft away from the i^{th} field unit in each month must be less than or equal to M_i.

An expression is needed for the number of aircraft which are away from the i^{th} field unit during the r^{th} month. Those which will be away during the r^{th} month are those which are sent in during the r^{th} month or earlier and which have not returned by the r^{th} month. In order to arrive back at the field unit in the r^{th}month, an aircraft must be shipped from the contractor facility in the $r - S_i$ th month. Hence, for the variable X_{ijk} if $k > r - S_i$, the aircraft will not have returned to the field unit by the r^{th} month.

The number of aircraft away from the i^{th} field unit during the r^{th} month will first be established for the variables X_{ijk} by month for the months in which they are sent in from the field unit. The number away from the i^{th} field unit in the r^{th} month which were sent in during the 1^{st} month is

$$X_{i,1,q_1} + X_{i,1,q_1+1} + X_{i,1,q_1+2} + \cdots + X_{i,1,T} = \sum_{k=q_1}^{T} X_{i1k}$$

where q_1 is the larger of the two numbers $r - S_i + 1$ and $1 + S_i + p$. It is necessary that $q_1 \geq r - S_i + 1$ so that the aircraft will not have returned to the field unit by the r^{th} month, and it is necessary that $q_1 \geq 1 + S_i + p$ because $S_i + p$ is the minimum time for aircraft to be sent in and have kits installed.

The number away from the i^{th} field unit in the r^{th} month which were sent in during the 2^{nd} month is

$$X_{i,2,q_2} + X_{i,2,q_2+1} + X_{i,2,q_2+2} + \cdots + X_{i,2,T} = \sum_{k=q_2}^{T} X_{i2k}$$

where q_2 is the larger of $r - S_i + 1$ and $2 + S_i + p$.

Similar expressions can be written for the 3rd, 4th, ..., r-1st months.

The number away from the i^{th} field unit during the r^{th} month which were sent in during the r^{th} month is

$$X_{i,r,q_r} + X_{i,r,q_r+1} + X_{i,r,q_r+2} + \ldots + X_{i,r,T} = \sum_{k=q_r}^{T} X_{irk}$$

where $q_r = r+S_i+p$.

For the variables Y_{ijk}, the number of aircraft sent in from the i^{th} field unit before the r^{th} month which have not returned by the r^{th} month is

$$Y_{i,q,q+S_i+p} + Y_{i,q+1,q+S_i+p+1} + \ldots + Y_{i,r,r+S_i+p} = \sum_{j=q}^{r} Y_{i,j,j+S_i+p}$$

where q is the larger of $r-2S_i-p+1$ and $c-S_i-p$. It is necessary that $q \geq r - 2S_i - p + 1$ so that the aircraft will not have returned to the field unit by the r^{th} month, and it is necessary that $q \geq c-S_i-p$, since $c-S_i-p$ is the smallest permissible value of j for Y_{ijk}.

Finally the constraint can be written as

$$\sum_{k=q_1}^{T} X_{i1k} + \sum_{k=q_2}^{T} X_{i2k} + \ldots + \sum_{k=q_r}^{T} X_{irk} + \sum_{j=q}^{r} Y_{i,j,j+S_i+p} \leq M_i \qquad (2)$$

for each field unit i and for each month r where

$q_j = \max \{ r-S_i+1 \ , \ j+S_i+p \}$ and $q = \max \{ r-2S_i-p+1, \ c-S_i-p \}$.

6. <u>CONTRACTOR FACILITY CAPACITY CONSTRAINT</u>

The contractor facility capacity constraint places a limitation on the number of aircraft which can be at the contractor facility in any given month. Let F_r be the maximum number of aircaft which can be at the contractor facility during the r^{th} month.

Now an expression is needed for the number of aircraft at the contractor facility during the r^{th} month. An aircraft from the i^{th} field unit will be at the contractor facility in the r^{th} month if it has been shipped in by the $r-S_i{}^{th}$ month so that it will have time to arrive, and if it is shipped back after the r^{th} month. In other words, if the variable X_{ijk} has $j \leq r-S_i$ and $k > r$.

For aircraft sent in for the first time the number of aircraft at the contractor facility in the r^{th} month is

$$\sum_i X_{i,1,r+1} + X_{i,1,r+2} + \ldots + X_{i,1,T} +$$
$$X_{i,2,r+1} + X_{i,2,r+2} + \ldots + X_{i,2,T} +$$
$$\vdots$$
$$X_{i,r-S_i,r+1} + X_{i,r-S_i,r+2} + \ldots + X_{i,r-S_i,T} = \sum_i \sum_{j=1}^{r-S_i} \sum_{k=r+1}^{T} X_{ijk}.$$

The sum on j includes all possible months in which the aircraft can be shipped in and have arrived by the r^{th} month, namely months 1, 2, 3, ... , $r-S_i$. The sum on k includes all possible shipping months after the r^{th}, namely months r+1, r+2, ..., T. The sum on i includes all the field units. Further, the subscripts are limited to permissible values; for example $X_{i,r-S_i,r+1}$ would not occur unless p=1, since the smallest permissible value of k in X_{ijk} is $j+S_i+p$.

For the aircraft sent in for the second time the number which will be at the contractor facility in the r^{th} month is

$$\sum_i \left(Y_{i,q,q+S_i+p} + Y_{i,q+1,q+S_i+p+1} \cdots + Y_{i,r-S_i,r+p} \right) = \sum_i \sum_{j=q}^{r-S_i} Y_{i,j,j+S_i+p}$$

where $q = \max \{ r-S_i-p+1,\ c-S_i-p \}$. It is necessary that $q \geq c-S_i-p$, because $c-S_i-p$ is the smallest permissible value of j in Y_{ijk}; and it is necessary that $q \geq r-S_i-p+1$ in order that the aircraft will not have been shipped back by the r^{th} month.

Hence, the constraint is written as

$$\sum_i \left[\sum_{k=r+1}^{T} \sum_{j=1}^{r-S_i} X_{ijk} + \sum_{j=q}^{r-S_i} Y_{i,j,j+S_i+p} \right] \leq F_r \tag{3}$$

for each month r where

$q = \max \{ r-S_i-p+1,\ c-S_i-p \}$.

Recall that M_i is the number of aircraft that can be away from the i^{th} field unit in the r^{th} month. Then $\sum M_i$ is the total number which can be away from all the field units in the r^{th} month and hence the maximum number which could be at the contractor facility. If $\sum M_i \leq F_r$ for each month r, then the contractor facility capacity constraint is automatically satisfied, and in that case, it can be omitted in the formulation of the problem.

7. APPLICATION CONSTRAINT

There may be some reason why aircraft from certain field units are selected first by the model for kit installation. For example, the transportation time or cost from an overseas location might be so much greater than those for other units that in the optimization process they would not be selected. However, those overseas aircraft might be the ones whose modification was most desired. Therefore, there is an application constraint which requires that at least a certain number of aircraft from each field unit will have some kits installed.

For example the constraint might require that one-fifth of all the aircraft at each unit have some kits installed; or it might require that all the aircraft at one particular unit have some kits installed and that one-third of the aircraft at the other units have some kits installed.

Care must be exercised, however, not to impose such a demanding constraint that a feasible solution is impossible.

For each field unit i, let A be the number of aircraft which must have at least some kits installed. Then the constraint becomes

$$\sum_{j,k} X_{ijk} \geq A_i \tag{4}$$

for each field unit i, and the summations on j and k are taken over their permissible values.

8. OBJECTIVE FUNCTION

An objective function must be selected which will cause the optimization of some feature of the modification program. Two choices which come immediately to mind are minimizing the cost of the program and minimizing the out of service time of the aircraft. However, if the cost is minimized, the optimum solution is to install no kits. Then the cost is zero and has been minimized. If the application constraint is applied to specify a minimum number of aircraft which must be modified, minimizing the cost will limit the modified aircraft to the minimum specified number. This is contrary to the desired result which is to modify as many aircraft as possible while keeping the cost to a minimum. Minimizing the out of service time produces a similar result, send in no aircraft or send in the minimum number of aircraft.

Therefore, it was decided to use the following objective function:

$$Z = \sum_{i,j,k} \left(A_{ijk}X_{ijk} + B_{ijk}Y_{ijk} \right)$$

where the coefficients A_{ijk} and B_{ijk} are weight factors which don't necessarily sum to one. The relative sizes of the weights can be chosen to indicate the relative importance of having kits installed on the corresponding groups of aircraft. All that remains is to make a suitable choice for the weights. Two possible choices are discussed.

One possible choice for the weights is $A_{ijk} = B_{ijk} = 1$ for all i,j, and k. Then the objective function represents the total number of aircraft modified where aircraft sent in twice are counted twice. Hence, it is possible to maximize the number of aircraft modified.

If $Y_{ijk} = 0$ for all i,j, and k, the objective function becomes $\sum_{i,j,k} A_{ijk}X_{ijk}$. For each variable X_{ijk} the weight A_{ijk} can be chosen to be the number of kits installed on the corresponding group of aircraft. Since the number of kits installed is determined entirely by the subscript k, the coefficients A_{ijk} are obtained from the table of kit availability (Table 1). Hence, the objective function represents the number of kits installed, making it possible to maximize the number of kits installed.

If some of the Y_{ijk} are *not zero*, an objective function representing the number of kits installed can not always be obtained. Aircraft sent in for the second time will have all the remaining needed kits installed. However in general, before a solution to the model is obtained, there is no way of knowing how many kits the aircraft will have installed the first time they are sent in and therefore no way to know how many kits will be installed the second time. In order to obtain an objective function in this case, let A_{ijk} be the number of kits installed on the group X_{ijk} and choose B_{ijk} to be an estimation of the number of kits installed on the group Y_{ijk}. For example, if each aircraft is to have K kits installed during the modification program, B_{ijk} might be taken to be $\frac{K}{2}$ for all i, j, k. Then the objective function will be an approximation to the total number of kits which are installed.

With the objective function just described, a solution to the model can be obtained. From the solution, an improved estimate for the B_{ijk} can be made. These new values for B_{ijk} can be used to obtain a second solution to the model. This iteration process can be continued if desired. However, there is no certainty that improvements will occur after the second solution. Furthermore, the choice of the B_{ijk} may not be critical. Therefore, it may be satisfactory to stop with the second or even the first approximation.

If the number of aircraft processed is maximized, that puts a premium on a large number of aircraft with perhaps only a few kits installed on each. Whereas maximizing the number of kits installed necessitates installing more kits on fewer aircraft. Maximizing the number of aircraft processed might result in an aircraft being sent in for only one kit. While maximizing the number of kits might result in the aircraft waiting at the contractor facility until more kits are available. Therefore, it is considered better to maximize the number of kits installed.

9. SOLUTION

The model is an integer programming model. In all but the most trivial case, the solution would be too involved to obtain manually, and therefore requires a computer procedure. Once a solution is obtained, it consists of values of X_{ijk} and Y_{ijk} for all permissible subscripts i, j, k. Recall that i designates the field unit, j the month in which the aircraft are sent, k the month in which they are sent and X and Y the number of aircraft for the first and second times, respectively. Therefore, a solution to the model is a list by field unit of how many aircraft are to be sent in each month for the first and second times. The list also includes the month in which they are sent back.

10. VARIATIONS

There are at least three important changes which can be made in the model as it has been described so far.

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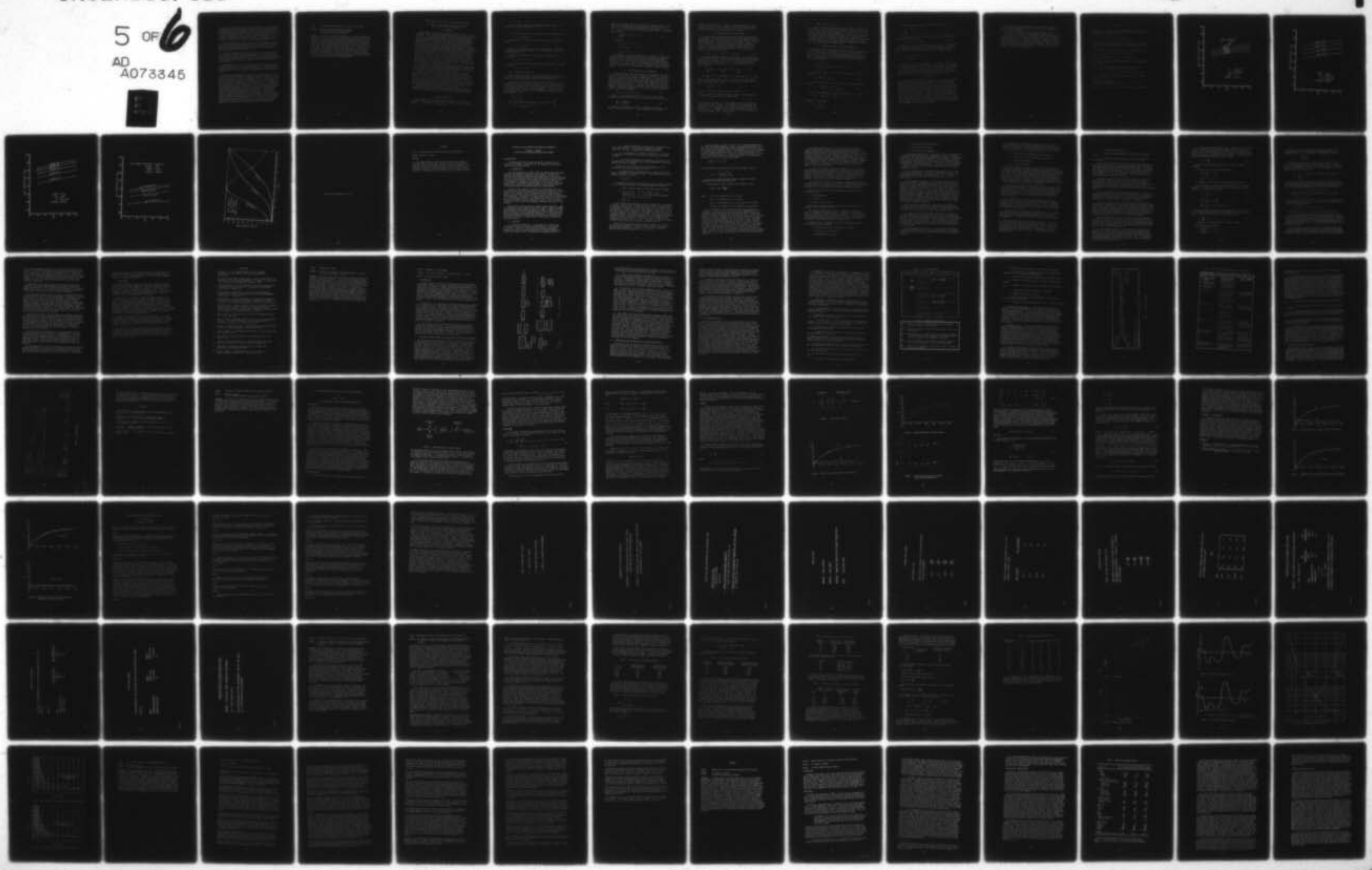

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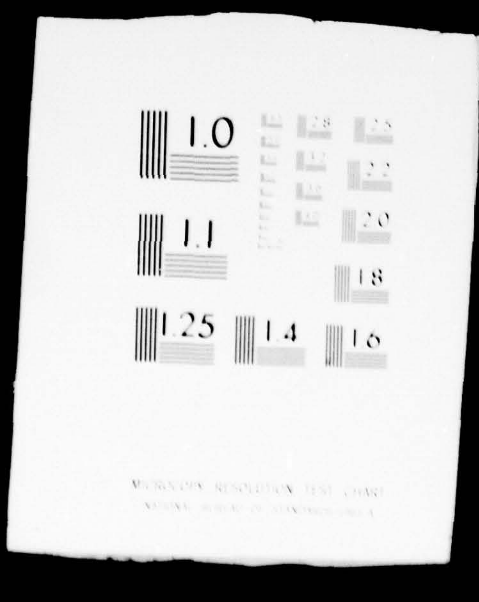

First it is possible to include a cost constraint. It is necessary to determine the cost of installing each kit and the transportation cost of shipping an aircraft from each field unit to the contractor facility. Then an expression can be obtained for the cost expended for an interval of time, for example one year. The cost constraint can then be written, limiting the expenditures in each year to a specified amount. The amount of course could vary from one year to the next. If desired, it is also possible to specify that at least a minimum amount be spent each year.

A second variation which can be made in the constraints is to allow the number of aircraft at a field unit and hence the number of aircraft which can be away from that field unit at any given time to vary. This would permit the introduction of new aircraft into the fleet or the removal of aircraft from the fleet.

The final variation is the most important. When the original model was developed, it seemed that the constraints would be so restrictive that all of the aircraft could not be modified in the specified length of the modification program. Therefore, an objective function was chosen which would maximize the number of aircraft modified or the number of kits installed.

Another approach is to use the application constraint to require that all the aircraft receive all of the modifications. In order to have a feasible solution, some of the other constraints will undoubtedly have to be be relaxed, and the total length of the program may have to be increased.

With all of the aircraft receiving all of the modifications, the old objective function is unusable. Instead the objective function can be chosen to be the total cost of the program. Morever, since it is reasonable to assume that the costs of installing the kits is independent of how the aircraft are brought in for modification, the only variable costs in the modification program are the transportation costs. Furthermore, the only variation in the transportation cost is in the number of trips to the contractor facility. Therefore, one possible objective function is the total transportation cost of the program, and minimizing the transportation cost minimizes the number of trips to the contractor facility. Another possible objective function is the total out of service time while the aircraft are being modified. The out of service time for a group of aircraft X_{ijk} can be determined from the subscripts j and k and from the transportation time S_i. In fact the out of service time for the group X_{ijk} would be $k+S_i-j$. An expression can then be obtained for the total out of service time for all aircraft, and the program can be carried out to minimize the total of service time.

TITLE: Target Oriented Gun Analyses for Feasibility Studies

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ABSTRACT: The objective of these analyses was to examine and model the interactive relationships between gun and projectile design limitations and the constraints imposed by armored target defeat by a KE penetrator. Examination of the interactive aspects of the problem resulted in the need for various relationships describing terminal effects, flight trajectories, flight projectile design, and those of the interior ballistics of the gun. The resulting model, called TOGA, covers the entire course of the ballistic trajectory from the time of firing to target defeat. The computer model provides the user with information concerning gun, projectile, and penetration. The user may specify certain constraints on, for example, chamber volume, muzzle velocity, or penetrator diameter, and obtain gun, projectile, and penetrator parameter values which result in the perforation of a plate of maximum thickness.

TARGET ORIENTED GUN ANALYSIS FOR FEASIBILITY STUDIES

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I. INTRODUCTION

Requirements for the defeat of armored targets by kinetic energy penetrators impose certain constraints on gun design characteristics and parameters. In an effort to determine and quantify these constraints imposed by target defeat requirements a target oriented gun analysis program was undertaken and the resulting relationships programmed for computer analysis. The analysis took into account terminal ballistic relationships, exterior and interior ballistics, thus covering the entire course of the ballistic trajectory from the time of firing to target defeat. The resulting computer program, called TOGA, is written so that the user may specify the thickness of armor plate to be defeated, the program then calculates the trajectory, subject to flight design constraints and assumptions, from the target back to the gun muzzle. The resulting calculated muzzle velocity and flight projectile characteristics are then used as initial conditions for the interior ballistic calculations. TOGA provides the user with values of muzzle velocity, penetrator weight, flight projectile and in-bore weight, propellant weight, chamber volume, expansion ratio and travel as a function of the input parameters: maximum breech pressure, penetrator length-to-diameter ratio, bore diameter, loading density, range.

In addition, the user need not specify the thickness of armor to be defeated but may elect to determine the maximum thickness of armor which can be defeated by a KE penetrator subject to constraints on the shot travel, chamber volume, charge and projectile weight.

TOGA has incorporated within it certain empirical relationships which have been found to be useful, over the years, in various gun analyses and studies. For example, TOGA uses an empirical relationship which relates the projectile weight-to-charge weight ratio to muzzle velocity[1]. The use of such relationships has the effect of restricting the set of candidate solutions to a set of state-of-the-art solutions. Furthermore, such empirical relations restrict the optimization procedures to those methods which do not require derivatives. The method used in TOGA is the so-called Complex Method due to M. J. Box[2].

The gun designer may also wish to have some estimates of breech pressure, shot velocity and travel as a function of time of shot travel. In TOGA these are obtained by a slight variant of the method of Leduc[3].

II. PROBLEM FORMULATION

Let ρ denote the vector whose components describe the fixed properties of the penetrator, e.g., density ρ_p, hardness, material, angle of obliquity, θ, of penetrator to target, etc.,

$$\rho = (\rho_p, \ldots, \theta) \tag{1}$$

and let ϵ denote the propellant and propellant gas values, i.e., density, force, specific heat ratio, etc.,

$$\epsilon = (\rho_e, F, \ldots, \gamma) \tag{2}$$

and τ the fixed properties of the target, such as, density, range,

$$\tau = (\rho_t, R, \text{material}, \ldots) \tag{3}$$

Furthermore, let β denote certain design parameters of interest, namely,

$$\beta = (\Delta, L/D, b, P_x) \tag{4}$$

where Δ is the loading density of the propellant, L/D, the length-to-diameter ratio for the penetrator, b, the bore diameter and P_x the maximum breech pressure.

The depth of penetration of a given target is given by the relationship

$$P = P(V, D; \beta; \tau; \rho; \epsilon) \tag{5}$$

and, for the calculations described here, the form due to Grabarek[4] is assumed, viz,

$$P = f(\rho; \tau; \beta)\, V^c\, D^d \tag{6}$$

with c and d greater than zero.

In the following we seek to define the gun which will deliver a penetrator to the given target, at range R, with values of V and D, such that a target plate of maximum thickness will be perforated.

A penetrator of diameter D striking a target with velocity V will penetrate the target to a depth P given by (5) or (6). The delivery of the penetrator to the target depends upon the flight characteristics of the in-flight projectile and its initial velocity. The initial velocity is the unknown muzzle velocity which is bounded by V_{max} and, in turn, the penetrator and in-flight projectile diameters are bounded by the bore diameter b. Thus,

$$0 \le D \le b$$

$$0 \le V \le V_{max} \tag{7}$$

Hence, when the penetration depth is given by (6) we have

$$0 \le P \le P^* (V_{max}, b; \beta; \tau; \rho; \epsilon) \tag{8}$$

where P* is an upper bound to the thickness of target which can be perforated by a penetrator-gun-projectile complex defined by fixed β, τ, ρ and ε. We seek to determine the maximum P over some region $0 < D \leq b$; $0 \leq V \leq V_{max}$ and where additional constraints are imposed. Our problem can be written as

$$\underset{\{V,D\}}{\text{Maximize}}\ P(V, D; \beta; \varepsilon; \rho; \tau) \tag{8}$$

subject to

$$\begin{aligned} F_1\ (V, D; \beta; \varepsilon; \rho; \tau) &\leq 0 \\ F_2\ (V, D; \beta; \varepsilon; \rho; \tau) &\leq 0 \\ &\vdots \\ F_n\ (V, D; \beta; \varepsilon; \rho; \tau) &\leq 0. \end{aligned} \tag{9}$$

The equations (9) represent functional relationships for the determination of muzzle velocity, flight projectile diameter and weight and the interior ballistic equations necessary to obtain charge weight, projectile weight, chamber volume, expansion ratio and maximum travel. The vectors β, ε, ρ and τ remain fixed throughout any given calculation. In addition, equations (9) also include all constraints placed upon the range of values for any of the calculated parameters. Thus, in general, $P < P^*$.

III. THE EXTERIOR BALLISTIC CALCULATION

In describing the target penetration by a KE penetrator the terminal ballistician uses such descriptors as penetrator diameter D, striking velocity V, and penetrator length-to-diameter ratio. In order to relate these variables to the gun, that is, to determine the launch velocity, or muzzle velocity, we need to define the flight vehicle and its velocity loss from the gun to the target. Thus, we make assumptions about the design of the flight vehicle[5]. For example, the angle of the nose cone, number of fins and how they are fitted and scaled according to penetrator diameter. With these assumptions we can estimate the drag coefficient C_D by a relationship of the form

$$C_D = a_1 V^2 + a_2 V + a_3 + a_4 \cdot \alpha \tag{10}$$

where α is the length-to-diameter ratio of the flight vehicle. Furthermore we assume that the velocity-range relation can be taken as

$$\frac{dV}{dR} = \frac{-\rho_o C_D S_F V}{2 W_{SP}} \tag{11}$$

where ρ_o is the density of air, S_F - a shape factor and W_{SP} is the weight of the flight projectile. Thus integrating (11) with (10),

backwards from the target, at range R, to the gun, where R = 0, we obtain the muzzle velocity. In addition, we know the weight, diameter and average density of the flight projectile. These values provide initial values for the interior ballistic calculations.

IV. INTERIOR BALLISTICS CALCULATIONS

For the interior ballistics calculation we seek to determine the charge weight, C, projectile weight, W_p, travel, XM, chamber volume, CV, and expansion ratio, EXR, subject to a known value of the muzzle velocity and constant β and ϵ. Even with given β, ϵ and muzzle velocity V there exists a multitude of possible solutions for the above parameters. Therefore we make two additional assumptions, namely, we utilize an empirical relationship, developed by Grollman, which relates muzzle velocity to charge-to-projectile-weight ratio. Thus, given muzzle velocity V_m we can find C/W_p from

$$C/W_p = (V_m - 272.8)/(2677.7 - 0.955\ V_m)\ . \qquad (12)$$

The second assumption we make is that we have a sabot weight predictor from which, given flight projectile weight, the in-bore projectile weight W_p can be computed. One of the relationships used in this study was developed by B. Burns[6] and is a fit of certain non-dimensional groups to existing saboted projectiles. The functional form of this relationship is

$$\frac{W_P - W_{SP}}{W_{SP}} = F\left(\frac{b^2 - D^2_{SP}}{D^2_{SP}},\ C/W_p,\ \frac{\bar{\rho} \cdot P_x}{\hat{\rho} \cdot \sigma_{ys}}\right) \qquad (13)$$

where σ_{ys} is the yield strength of the sabot material, $\hat{\rho}$ an average density of the flight vehicle, and $\bar{\rho}$ the density of the sabot material. Other relationships may be assumed and in some cases we have used a state-of-the-art figure[7], such as

$$W_{Sabot}/W_P \simeq 0.33$$

in order to calculate the in-bore projectile weight.

With these relations we obtain the charge weight C, from which, and the loading density, we calculate the chamber volume

$$CV = C/\Delta\ . \qquad (14)$$

The Pidduck-Kent constant, δ, is then computed from the relationships reported by Grollman and Baer[8]. The Pidduck-Kent constant enters into our calculation of the expansion ratio and the projectile travel. The expansion ratio is computed from an inversed Mayer-Hart[9] calculation, where the velocity, chamber volume and charge-to-projectile weight ratio is given. The travel, XM, is then given by

$$XM = CV(EXR - 1)/(\Pi b^2/4) \tag{15}$$

If none of the above calculations violate imposed constraints then this completes the calculation of a feasible point for the optimization problem. Additional points are generated and rejected as described in the algorithm of the Box Complex method[2,10]. Constraints may be imposed on projectile weight, travel, chamber volume, muzzle velocity, penetration weight and charge-to-projectile weight ratio. When the above procedure converges to an optimum several other calculations are performed in the TOGA model. In particular, estimates of the pressure, velocity and travel-time profiles are computed and recoil momentum is estimated utilizing a method reported by Baker et al[11].

V. PRESSURE, VELOCITY AND TRAVEL CALCULATIONS

As a part of certain feasibility studies it is necessary that some estimate of pressure, velocity and travel be provided as a function of time. In TOGA, these are provided by utilizing the Leduc method[3]. Grollman[12] has recently "revisited" the Leduc method and has traced its history and application in a forthcoming Ballistic Research Laboratory Report. Leduc assumed that the velocity of the in-bore projectile satisfies the relation

$$V = ax/(d + x) \tag{16}$$

or letting $\bar{V} = V/a$ and $\bar{m} = 2x/d$ we have

$$\bar{V} = \bar{m}/(\bar{m} + 2) \tag{17}$$

From Newton's equation ($F = ma$) it follows that the pressure is given by

$$P_r = \frac{km_p}{A} \frac{4a^2}{d} \frac{\bar{m}}{(\bar{m} + 2)^3} \tag{18}$$

and if we let km_p = mass of the projectile plus ½ times charge mass we have

$$P_r = \frac{(m_p + Cm/2)}{A} \frac{4a^2}{d} \frac{\bar{m}}{(\bar{m} + 2)^3} \tag{19}$$

and it can be shown that P_r attains its maximum at $\bar{m} = 1$, that is,

$$P_x = \frac{(m_p + Cm/2)}{A} \frac{4a^2}{27d} \tag{20}$$

Defining $\bar{P} = P_r/P_x$ we have

$$\bar{P} = \frac{27\bar{m}}{(\bar{m} + 2)^3} \tag{21}$$

and with a change of variable we find, from (17) that

$$\frac{d\bar{m}}{d\bar{t}} = \bar{m}/(\bar{m} + 2) \tag{22}$$

and where $\bar{t} = \frac{2a}{d} t$ (23)

Integration of (22) for $\bar{t}$ as a function of $\bar{m}$ poses a little difficulty as $m \to o$ for, from (22),

$$\bar{t} = \bar{m} + 2 \cdot \ln(\bar{m}) - 1, \tag{24}$$

however, we avert this problem by noting that at $\bar{m} = 1$ the maximum breech pressure is attained, and at full travel we have $\bar{m} = 2 \cdot XM/d = \bar{m}_m$. This information allows us to compute the time elapsed between $\bar{m} = 1$ and $\bar{m} = 2 \cdot XM/d$. Next we assume that total travel time is given by

$$t_{total} = XM/(Vm/2) \quad \text{or}$$

$$\bar{t}_T = 2\bar{m}_m/Vm \tag{25}$$

Subtracting the time elapsed, from $\bar{m} = 1$ to when $\bar{m} = 2XM/d$, from (25), then, from (24), we can calculate the value of $\bar{m}$ (<1) corresponding to the remaining time. A simple translation then can be used to define $\bar{t} = o$ at this value of $\bar{m}$. Thus from (17), (21) and the definition of $\bar{m}$ we can readily obtain pressure, velocity and travel as a function of time. The constants a and d are determined from (16), (20) since we know the muzzle velocity, total travel and maximum breech pressure.

VI. EXAMPLES

For given values of maximum breech pressure, loading density, bore diameter and penetrator length-to-diameter ratio, the maximum target thickness which can be penetrated, subject to constraints on total travel and charge-to-mass ratio, has been computed. The output in this case yields, in addition to maximum target plate thickness perforated, values of travel, charge weight, charge-to-projectile-weight ratio, in-bore projectile weight, chamber volume, flight projectile weight and diameter, penetrator diameter, striking velocity, expansion ratio and muzzle velocity. The variation of each parameter may be examined in relation to changes in the components of β, but here, we shall be contented with demonstrating the variation of P with respect to L/D and one other component of β. None of the presented calculations are related to existing known guns and, for each, the penetrator is depleted uranium with spherical nose and sub-caliber grooves.

In figure 1 the maximum thickness of target plate perforated, hereafter called maximum penetration, is plotted versus penetrator L/D for various bore diameters.

In figure 2, maximum penetration depth is plotted versus L/D, for various values of total travel, for a 75mm bore diameter gun.

In figure 3, the effect of maximum breech pressure on maximum penetration versus L/D is shown for a 75mm gun with "tube length" or travel constrained to be 168". Similarly in figure 4 the effect of loading density on P versus L/D is shown for a 70mm gun. Finally, in figure 5, the results from the Leduc calculations are shown for 75mm bore gun with total travel confined to 60 times the bore diameter. The plot shows pressure, scaled by maximum breech pressure, scaled velocity and scaled travel as a function of scaled time.

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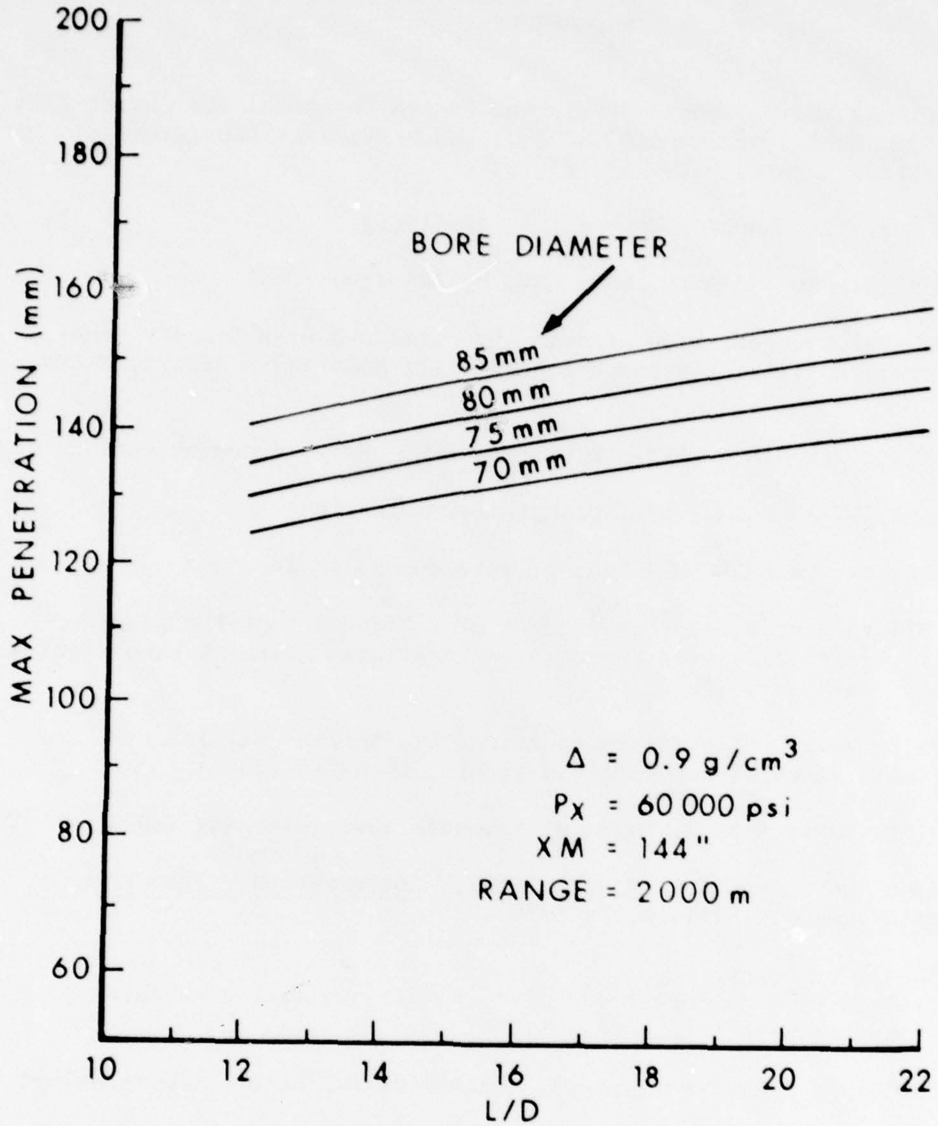
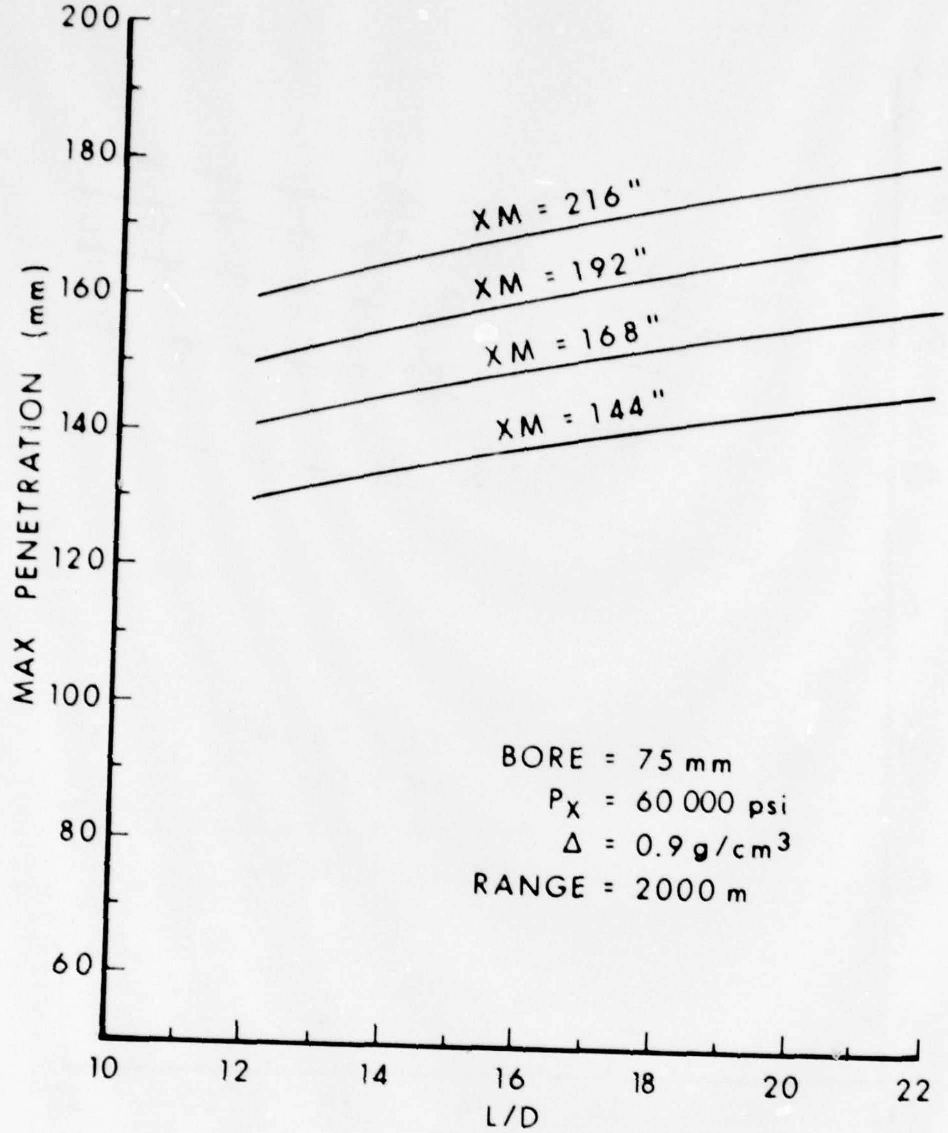

MAX PENETRATION (mm)
200
180
160
140
120
100
80
60
BORE DIAMETER
85 mm
80 mm
75 mm
70 mm
Δ = 0.9 g/cm3
PX = 60 000 psi
XM = 144"
RANGE = 2000 m
10
12
14
16
18
20
22
L/D



MAX PENETRATION (mm)
200
180
160
140
120
100
80
60
XM = 216"
XM = 192"
XM = 168"
XM = 144"
BORE = 75 mm
P_X = 60 000 psi
Δ = 0.9 g/cm^3
RANGE = 2000 m
10
12
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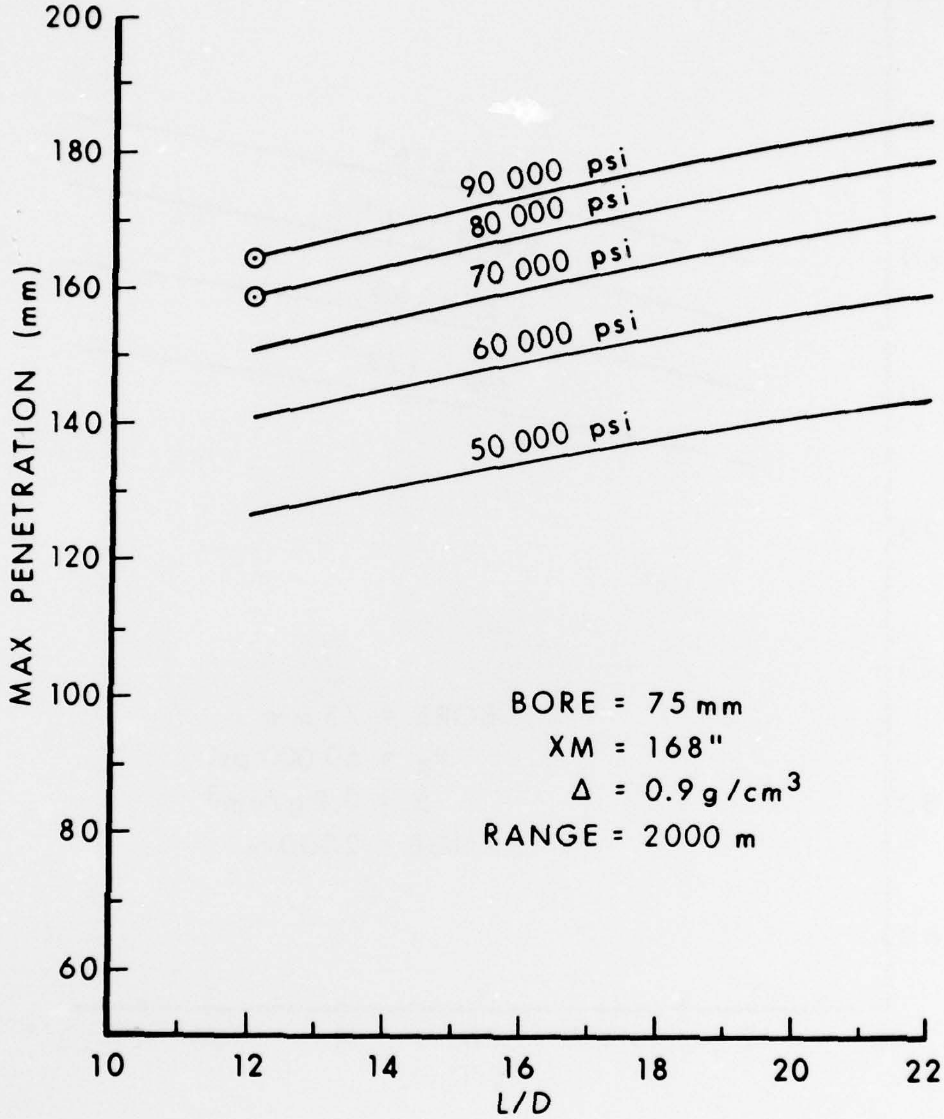
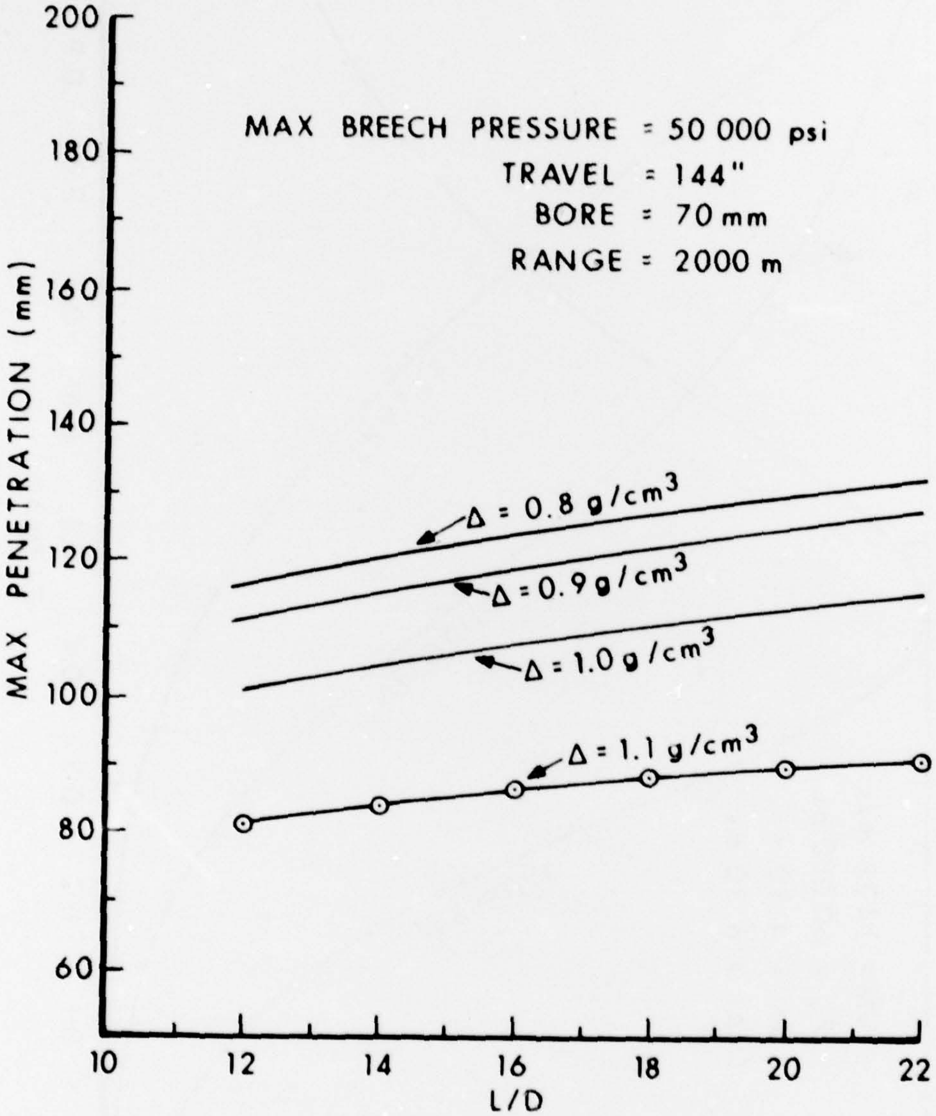
16
18
20
22
L/D


MAX PENETRATION (mm)
200
180
160
140
120
100
80
60
90 000 psi
80 000 psi
70 000 psi
60 000 psi
50 000 psi
BORE = 75 mm
XM = 168"
Δ = 0.9 g/cm³
RANGE = 2000 m
10
12
14
16
18
20
22
L/D


MAX BREECH PRESSURE = 50 000 psi
TRAVEL = 144"
BORE = 70 mm
RANGE = 2000 m
MAX PENETRATION (mm)
L/D
Δ = 0.8 g/cm3
Δ = 0.9 g/cm3
Δ = 1.0 g/cm3
Δ = 1.1 g/cm3
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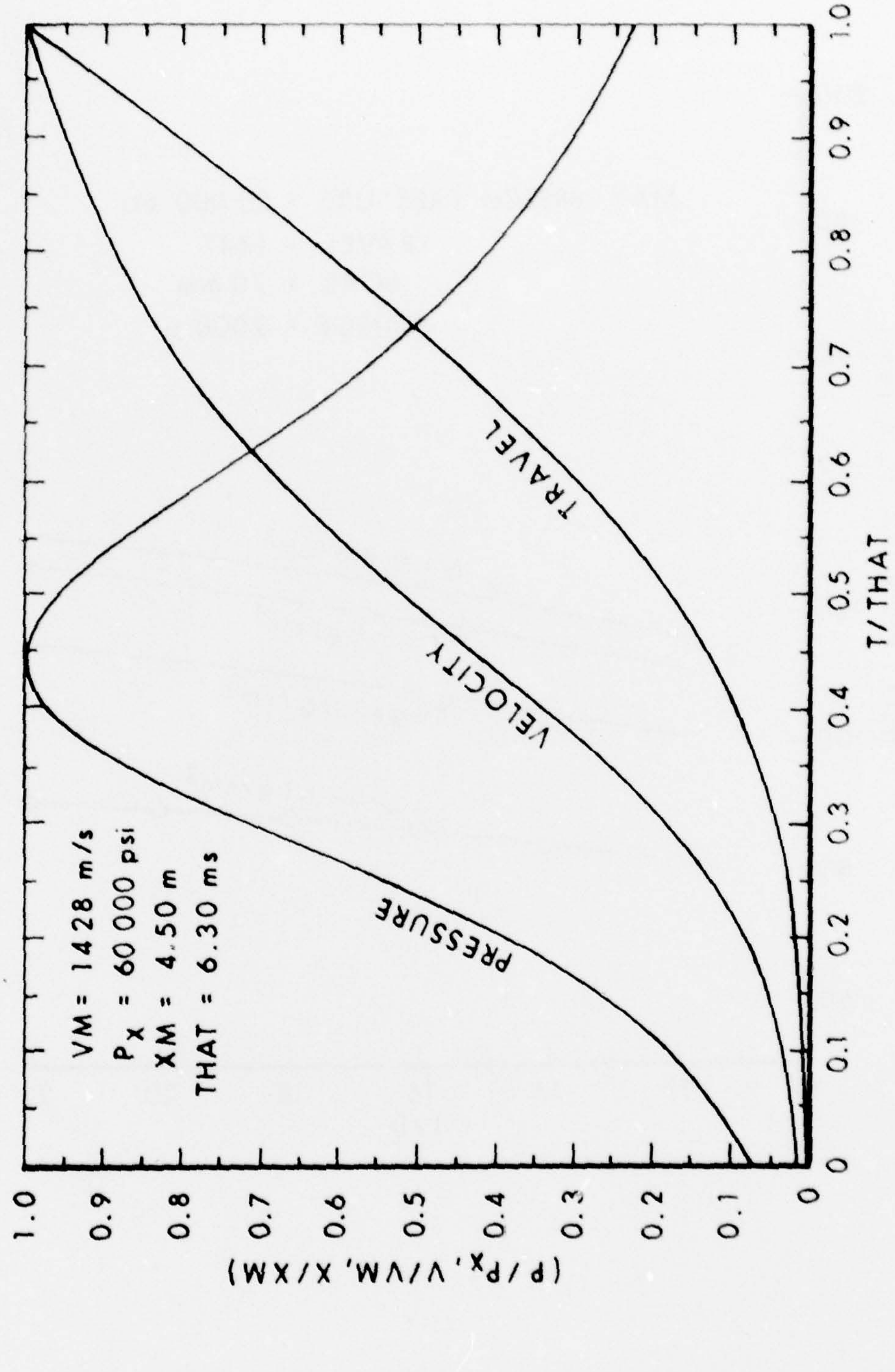
VM = 1428 m/s
P_X = 60 000 psi
XM = 4.50 m
THAT = 6.30 ms
PRESSURE
VELOCITY
TRAVEL
T/THAT
(P/P_X, V/VM, X/XM)
0
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ABSTRACT

TITLE: A Review of Data Estimates and Subjective Probability

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ABSTRACT:

This paper addresses subjective probabilities and estimates of uncertain events in the future. The discussion is concerned with estimator's ability to provide estimates. Tversky and Kahneman have found that three heuristics describe the factors which determine an individual's estimate. The basic conclusion one can reach is that individual or group subjective estimates are still a measure of belief.

A REVIEW OF DATA ESTIMATES AND SUBJECTIVE PROBABILITY

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1. INTRODUCTION

This paper addresses an aspect of decision making where subjective estimates are required on uncertain future events. The need for estimates arises when a problem has been recognized by the decision maker and the alternatives that are considered feasible must be evaluated. To evaluate the problem or venture under consideration requires either point estimates or probability distributions of possible outcomes. The predicting of outcomes of future events is not limited to decision making alone. Predicting hardware reliability, marketing income, or political elections can require estimates of future outcomes.

Typically, the situation in industrial or governmental decision making, where the possible outcomes of a venture are required, is when a decision is to be made whether to proceed or chose another venture. Rather than one venture this analysis may involve a series of ventures and alternatives. An example of a major venture would be the establishment of a new manufacturing plant. The outcome of an activity will only be known with certainty after the event has occurred. Thus, a problem facing one is making sound and reasonable subjective estimates of these outcomes.

If one had an activity that was inexpensive and repetitious, then an experiment could be conducted that ran a number of sample activities and determined a distribution of possible outcomes. However, in cases where a venture will only be performed once, then the distribution of possible outcomes must be determined subjectively. In obtaining these subjective estimates from experts most techniques try to ensure that each expert considers all known factors on the activity and gives an unbiased estimate.

2. DISCUSSION

2.1 Subjective Probability. To consider data estimates some discussion is required of subjective probabilities. In reviewing the variety of articles in the literature concerning subjective probability estimates, some definitions are required of probability. Savage gives three classes of interpretation of probability [10].

a. Objective probability is derived from a repetitive event that agrees reasonably close with the mathematical concept of probability for such an independently repeated random event.

b. A personalistic or subjective probability is a measure of confidence an individual has in truth of a particular proposition or event.

c. A necessary probability is measure of the extent one set of propositions, out of logical necessity and independent from opinion, confirms the truth of another.

A more classical definition of objective probability can be obtained from any text on probability, such as:

The probability of an event occurring is equal to the ratio between the number of successful outcomes, s, of the event to the total number of outcomes, n, that is

$$P\{E\} = \lim_{n \to \infty} \frac{s}{n}$$

A mathematical definition of probability is based on three conditions for a real valued set function P and a sample space A* satisfying,

a. $P(A) \geq 0$ for any event $A \varepsilon A^*$ for which $P(A)$ is defined.

b. $P(A_1 \cup A_2 \cup A_3 \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$ whenever A_1, A_2, A_3, ... are mutually exclusive events (i.e., $A_i \cap A_j = 0$, for all $i \neq j$)

c. $P(A^*) = 1$

A quite lengthy philosophical discussion of probability as a measure of belief is given by Venn. Probability can be defined as either objective or subjective, because it is in the long run of a series of events that the probability is objective rather than subjective. For example, in the toss of a fair coin the results of a small number of thosses, say ten, do not confirm the long run expectation. The result of these ten tosses is a measure of belief of the long run expectation. (This example is normally considered objective rather than subjective.) As the number of tosses increase the more this measure of belief would begin to agree with the mathematical expectation for this event. Thus, when the relative frequency is sufficiently large one has an objective probability [13].

There are two strong philosophical views of probability; they are objective verses subjective. In most situation involving uncertainty, any estimate or judgment of outcome will be subjective. Therefore, Bayesian statistical procedures are normally used [3].

Bayesian methods are based on the work of the Reverend Thomas Bayes which was published posthumously in 1763. The problem considered by Bayes was drawing conclusions about the binomial parameter based on the result of x successes in n Bernoulli trials. The parameter p is itself considered a random variable that has been chosen by a prior random process such that the probability that p is in some range between zero and one is given by dp. The joint probability of obtaining a value of p and observing x successes in n trials is given by the product

$$\binom{n}{x} p^x (1 - p)^{n-x} dp$$

and the probability distribution of p, given the sample results, is

$$f(p; x) = \frac{p^x (1-p)^{n-x} dp}{\int_0^1 p^x (1-p)^{n-x} dp}$$

The results of Bayes have been taken by a number of eminent mathematicians and developed further [3].

In applications of Bayes' Theorem, the following is used:

$$P(A/B) = P(A) \frac{P(B/A)}{P(B)}$$

where P(A) is the probability of A occurring.

P(B) is the probability of B occurring.

P(B/A) is the probability of B given that A has occurred.

P(A/B) is the probability of A given that B has occurred.

The ratio P(B/A) / P(B) gives what is known as the measure of relevance. If the ratio is one, the event B is independent of A. In using this theorem Easterling gives two examples which show where the controversy arises in Bayesian and classical statistics. In the first example of acceptance sampling from sequentially produced lots, Bayesian's updating of the prior knowledge with the new data is quite acceptable. The classical statistician would use this new data to make inferences concerning the distribution parameter θ; where θ is from an assumed distribution f (θ). In the other example the parameter θ is considered an unknown constant and f (θ) is our belief about what its value might be. This second example incorporates our personal (or subjective) probability into any statistical inferences and thus raises the controversy. However, it should be noted that any probability can be considered subjective since one must assume some distribution to model against the data [3].

In the decision making process one does not have the luxury of conducting experiments on what these possible outcomes can be and thereby have data upon which to build a probability distribution. The decision problem is similar to estimating the result of a single experiment. In decision making one is usually evaluating alternatives in which outcomes are at best uncertain. The degree of uncertainty in an outcome is usually expressed as some probability estimate. This value is based on an individual (hopefully more than one individual is queried) estimate and is subjective. However, this fact does not prevent one from using classical statistical method, but only remember that all results are based on this subjective estimate.

2.2 Biases in Estimating Subjective Probabilities. In a situation where outcomes are uncertain, estimates of outcomes are solicited from those individuals who are cognizant of the factors which influence these outcomes. Recent studies have been conducted to determine the type and extent biases entered into subjective estimates. The majority of the material in this section on methods by which individuals arrive at an estimate is based on an article by Tversky and Kahneman [12].

Even though these individuals are experts, this does not keep them from having biases and feelings about the areas in which they are questioned. In recent studies using experts and then native subjects, both groups were found to use the same type of heuristics in their predictions.

There are three heuristics used in determining probabilities and predicting values:

a. Representativeness

b. Availability

c. Adjustment and Anchoring

Within each of these heuristics, biases exist which modify one's judgement in predicting and assessing.

The representativeness heuristic may be defined as a method by which an individual in answering a quest[illegible], "What is the probability A came from a process B?" judges how well A resembles B. Thus an individual is not using data about A, but a comparison of A to B. With training in estimating, both the expert and novice can reduce the errors inherent in this heuristic and the other heuristic discussed later.

A number of factors have been found within the representativeness heuristic which affect judgements.

a. Insensitivity to prior probability of outcomes.

b. Insensitivity to sample size.

c. Misconceptions of chance.

d. Insensitivity to predictability.

e. The illusion of validity.

f. Misconceptions of regression.

In considering a question on A coming from B, the size or population A represents should enter into one's judgements. If prior information is available this data should be weighted by the estimator. In experiments where the subjects were told the make up of the underlying populations (30 percent A), the probability estimates were still based on the degree A represented B (estimated 50 percent). This effect is quite pronounced in that worthless information about A was used rather than disregarding it. When no information on A was given, the basic data on the overall population was used.

The size of the sample as opposed to the population size is ignored in assessing probabilities. As the size of the sample increases, the closer the sample statistics will approach the population values. In tests on various individuals the same estimates were given for all sample sizes.

The misconception of change is a significant problem in that even trained individuals in probability and statistics suffered from this factor. It has been found that people believe that a sequence of events generated at random will exhibit the same characteristics regardless of length. For example, in the tosses of a fair coin, H-T-T-H-T-H is considered more likely than T-H-H-H-H-H. Also, this belief leads to the gambler's fallacy. Consider a roulette wheel, after a long series of red most people would expect a black is now due because this will give a more representative sequence. However, these are independent events, thus red or black is equally likely each turn of the wheel. A related problem is that in believing small samples should represent the long term expectation, researches were found to put too much faith in small samples and to extrapolate too much.

Insensitivity to predictability is when the data concerning an outcome is used based on how good the information depicts the outcome. If the description of outcome A is very good, then the probability given A is high; if it is poor, then the probability assigned is low.

The illusion of validity is another strong factor in people. An individual will estimate an outcome based on how well that represents the input with minimum consideration for those items which limit predictive accuracy. Highly consistent patterns in the input data give individuals more confidence in their prediction. Two distributions can have the same mean even if one distribution has twice the range of the other, but an individual's confidence will be greater with the distribution which is more consistent.

The misconception of regression is also very common in that most processes tend to the mean of the process rather than to the extremes. In most cases one should expect a value near the average rather than the extremes.

The heuristic of availability is one in which an estimator will assess the probability of an event by the ease of which he can recall instances of the event. The more common an event is to an individual, the easier by which he can recall it, thus will associate a high probability to that event. The converse is also true.

a. Biases due to the retrievability of instances

b. Biases due to the effectiveness of a search set

c. Biases of imaginability

d. Illusory correlation

The ease at which one can recall instances of an event influences an estimator's judgement of event frequence. In an experiment using lists of famous people's names, subjects in one group were given a list on which the men were more famous than the women and the other group, the subjects were given a list on which the women were more famous. The subjects judged the list containing famous men as having more men than women on it when it did not, and the converse for the other group having the list containing famous women. In addition, if an event has recently occurred to the estimator, than it is easier to recall similar events, thus giving a higher probability to the frequency of occurrence.

The ease at which one can mentally search for like events will also affect the probability estimate. If in asking which of two events can occur most often, such as the occurrence of r as the first letter or the third letter of a word. It is easier to think of words beginning with r than in the third position. The result of the experiment is that subjects estimate it is the first most often when in fact the third position is correct.

Some questions asked of an estimator depend on his ability to imagine the events occurring. The estimating of sequences, mathematical relationship, and infrequent events requires an individual to mentally generate a series of events according to some rule. Usually population characteristics, such as a bell-shaped distribution, are not associated with generating these sequences.

Finally the illusory correlation biases entered into one's judgement when trying to estimate the frequence of two events co-occurring. Subjects were found to overestimate the frequence of co-occurrence of associated events. The degree of association seemed to be the cause of this effect.

The last heuristic that was reported was adjustment and anchoring. This effect would occur most often in conducting surveys. The initial value given is usally taken by the estimator as a reference point for all other predictions. The divergence from this point is usually not sufficient to cover the range of values. This effect also occurs if incomplete data or calculations are used. The biases that were four are given below:

a. Insufficient adjustment.

b. Biases in the evaluation of conjunctive and disjunctive events.

c. Anchoring in the assessment of subjective probability distributions.

Insufficient adjustment from the initial point can be seen in an example of computations. For example 8x7x6x5x4x3x2x1 and 1x2x3x4x5x6x7x8. These two sequences were presented to two different groups. Group one was asked to estimate the value of the first product within five seconds, the second group the second sequence. The first group estimated using the computations from the high end and had a higher estimate of the total product of 8! than the second group. Neither group recognized that these sequences were eight factorial.

The biases in conjunctive and disjunctive events occur because of the inability to relate the serial and parallel effect of events. In reliability the probability of success in a series is less than the lease reliable, and in a parallel configuration the probability of success is higher than any single item. In estimating this bias causes an overestimation for conjunctive events and an underestimation for disjunctive events.

In assessing probability distribution, the manner in which the questioning begins will determine the distribution given. If questioning begins with estimating the mid point (50 percent point), a more accurate distribution is obtained than when questioning begins at the extremes. Beginning at the extremes results in too tight a distribution.

In a study performed by Hahn, different biases entered into his results [4]. A questionnaire on an intended symposium and the respondent's probability of attendance were compared with the actual attendance. There was a significant difference; of the 436 respondents who indicated yes, only 82 individuals actually attended. The dicision was made to have the symposium and 463 people attended, but the data collected was wrong. The type biases that showed up when the nonattendees were interviewed were varied and different from those cited by Tversky and Kahneman.

These respondents were in an actual industrial environment which caused them to weigh their responses in light of management and benefits to themselves. Many individuals believed that the symposium was a good idea and should be held. Therefore, they answered that they would attend when they knew that they would not. Also the questionnaire did not give any incentive to provide good estimates.

In high incentive situations good estimates are usually obtained. Analyzing the results of horse race data collected from Aqueduct and Belmont Park in 1970 Hoerl and Fallin show for the χ^2 statistic at the 0.94 confidence level that the subjective probabilities are accepted as the correct theoretical frequencies of wins [5]. Thus, it is concluded that the group consensus can evaluate an actual situation and provide reliable subjective probabilities.

2.3 Combining Subjective Probabilities. Even with all the inherent defects of estimating by individuals or groups, the use of experts for predicting future uncertain events is currently the only method used for one time events. The standard practice, it seems, is to obtain more than one expert's estimate or opinion and form a consensus. The most common method of obtaining a consensus is to use the mean:

$$x = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{1}$$

where n is the number of experts and x_i is the $i^{\underline{th}}$ expert's estimate.

In the case of a "yes" or "no" answer then

$$x = \left[\frac{1}{n} \sum_{i=1}^{n} x_i + \frac{1}{2}\right] \tag{2}$$

where x = 0 or 1

This formula is the solution of the simple majority vote [1].

In some practical situations the question arises as to the expert's competency. In these cases (1) and (2) can be modified by including a ranking c_i, of each expert's estimate:

$$x = \frac{1}{n} \sum_{i=1}^{n} c_i \, x_i \tag{3}$$

$$x = \left[\frac{1}{n} \sum_{i=1}^{n} c_i \, x_i + \frac{1}{2}\right] \tag{4}$$

where $\sum_{i=1}^{n} c_i = 1$, and $c_i \geq 0$, $i = 1, \ldots n$

However, in using a ranking system more problems can arise because of who is establishing the ranking. If the experts rank each other then the c_i can be established by using a modification of (3).

$$c_i = \sum_{i=1}^{n} a_{ij} \, c_j, \quad j = 1, \ldots, n \tag{5}$$

where a_{ij} is the weight expert j assigns to expert i.

The numbers a_{ij} must satisfy

$a_{ij} \geq 0, i, \ j = 1, \ldots, n$

and $\sum_{i=1}^{n} a_{ij} = 1, \quad j = 1, \ldots, n$

However, this system of equations is linearly dependent and its determinant is zero. To solve this problem requires the use of decomposition of a matrix. The matrix is decomposable to the form

$$\begin{Vmatrix} A_1 & 0 \\ B & A_2 \end{Vmatrix}$$

Using this fact allows one to find a unique solution for the c_i's. For a complete discussion the reader is referred to [1]. A simple solution is recommended so that the determinant is nonzero. The expert does not rank himself, that is $a_{ii} = 0$. The researcher rates each expert and then normalizes the row:

$$a^*_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}}, \text{ where } a_{ii} \text{ is assigned by the researcher.}$$

In those cases where a probability distribution is obtained from each expert then the technique of convoluting the distributions can be used. Assuming that the probability distribution or probability density functions (pdf) are independent then a joint pdf can be defined as:

$$f\,(x_1, x_2, x_3,) = f_1(x_1).f_2\,(x_2).f_3\,(x_3)\ldots.$$

In convolution one desires the random variable s, defined by

$$s = x_1 + x_2 + x_3 + \ldots$$

For the continuous case the pdf of s is found by repeated application of first finding the pdf of $s_2 = x_1 + x_2$, then $s_3 = s_2 + x_3$ and so on. The pdf of s_2 is determined by:

$$g(s) = \int_{-\infty}^{+\infty} f_1\,(x_1)\,f_2\,(s-x_1)\,dx_1$$

The end result will be a consensus pdf for all experts [11].

2.4 Cognitive Psychology of Man. The psychologist view of man is from a different prospectus and is described by Hogarth [6]. Cognitive psychology is an area that considers the study of perception, problem solving, judgemental processes, thinking, concept formulation, and human information processing in general. In these various subgroups, the psychologist is searching for how man performs these activities and what are his limitations.

Two basic premises have been arrived at - one that man has only a limited information processing capacity; yet, the other is that he is an adaptive creature. He can determine a number of possible strategies when dealing with a complex task with his limited processing capability.

This limited comprehension capability can be seen in experiments using numbers. Using a string of digits, the average person can recall only about seven digits. A few people can recall ten digits. When faced with a complex problem of assessing many factors, it is a difficult task for man. But his adaptive capabilities enable him to handle these complex problems as a series of subproblems by using artificial aids, such as computers or pencil and paper, and to assimilate the various factors and reach a conclusion.

Psychologists have found in situations of uncertainty that most people will avoid the situation either by ignoring or rationalizing around the uncertainty. This shortcoming is carried over into assessing probabilities in that one is required to make estimates on an uncertain event and his natural instinct is to avoid such a situation.

When faced with uncertain events, man tends to organize the unknown factors associated with this event into groupings or categories. Grouping or categorizing is seen in most all pursuits and is characterized by using such things as symmetry, closure, proximity, and similarity. However, this organizing causes man to look for patterns in events when they are not there. It has been found that man can detect slight changes in dynamic processes much better than in a stationary process. The conjecture of why this is probable is that man is an adaptive creature that has been forced to learn to live in a dynamic world. This conjecture can also be seen in one's ability to predict outcomes of a series of events more accurately than a single event.

Psychologists have found that the customs of the western society do not encourage one to develop a degree of precision in judging uncertainty. Within this society, custom dictates that one is sure of oneself. and to state any decision as certain. Thus one pretends to be totally sure of an outcome rather than expressing one's belief in an outcome. If required to make probability estimates on an uncertain event, society confined the individual's ability to provide good estimates. In laboratory studies, it has been found that most subject's probability thinking is conservative for the most part.

Another psychological consideration is the meaningfulness of the task to the estimator. For the task to be meaningful to the individual it should concern a subject with which he is knowledgeable. The time frame for which the estimate is made has a significant effect on the prediction accuracy. Most long range forecasts have been found to be inaccurate. Lastly, it has been found that a sequential process of questioning normally yields a more accurate estimate such as the Delphi technique.

The recommendations that cognitive psychologists propose for improving estimator's abilities is to explore the strategies and simplify methods that are used and to build from this point. Basically, the question is "how" people evaluate uncertainties rather than "how well." Experimental

results confirm that training programs improve the estimates when the estimators have been given not only the concepts of probability and statistics but also assessment techniques. For most people probability thinking is a new way of thinking.

3. CONCLUSIONS

In the decision making process any estimate of possible outcomes is based upon an individual's judgement. Thus any probability estimate is a degree of belief an individual has in the event in question. There are two basic schools of thought in probability - the classical or objective school and the subjective or Bayesian school. Since there are fundamental differences of philosophy between these two groups, there will always exist some disagreement on techniques.

In reviewing the literature, a number of different methods are proposed on soliciting these estimates. One technique that has come into a degree of prominence is the Delphi technique. However a recent report by Sachman with the Rand Corporation was very critical of the Delphi technique. One of his basic conclusions was the Delphi was an unscientific and unverified method in theory and practice. A number of statistical problems exist because of the manner in which data is solicited. The iterative process in Delphi destroys the independence in the probability distributions. Thus convolution could not be used as stated earlier for developing a consensus distribution [9].

The overall conclusions one reaches from reviewing the literature is that subjective probabilities, or estimates, are a means of obtaining knowledge on an uncertain event for which minimum information is known. There will always be a philosophical debate over the goodness of any estiamtes made and it is not expected that the question will ever be resolved. [2, 7, 8, 14].

Though man has a limited information processing capability, he can predict outcomes with a reasonable degree of accuracy when no information is available. The techniques that have been discussed are means by which these extrapolations can be manipulated and a composite view of a number of individuals can be obtained. Yet in the end this consensus is still a subjective estimate.

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TITLE: Management of Change

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ABSTRACT: Each year, Army units must react to frequent personnel and equipment authorization changes. The frequency and volume of these changes are the end products of a number of largely independent management processes including such critical ones as personnel, materiel, and TOE development. The interaction or lack of interaction of these processes can create a great deal of frustration and unnecessary equipment/personnel turbulence for many units. In order to minimize the collective impact of these unsynchronized management processes, each individual process was modeled using network theory and later linked to the other processes using an advanced network analytic tool, OPTIMA 1100. Application of network theory and utilization of a sophisticated multi-network analyzer have permitted investigation of alternative proposals for rescheduling the management processes, thus creating a more harmonious arrangement and minimizing turbulence for field units.

SUBJECT: Management of Change (MOC)

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AGENCY: USA Concepts Analysis Agency

I. Introduction

Each year, Army units experience continued personnel and equipment authorization changes. The frequency and volume of the changes are the end products of a number of largely independent management processes including such critical ones as personnel, materiel, and unit Table of Organization and Equipment (TOE) developments. The interaction, or lack of interaction, of the management processes creates a great deal of frustration and unnecessary equipment/personnel turbulence for many units. In order to address Army-wide turbulence resulting from authorization changes, a quantitative methodology was developed to analyze the related activities, events, and time-dependencies of selected key management processes.

A. Management of Army authorization change is supported by a complex of functional processes. Importantly, the authorization management processes identified by the study group span a number of formal, official Army management systems. Preliminary investigative work in the MOC study identified and illuminated the key processes for detailed analysis--with principal emphasis on reduction of turbulence caused by authorization changes. As a further consequence of the study group's preliminary analysis of the overall Army authorization change environment, quantitative variables such as frequency of change, time and the schedule factors in accomplishing changes, and volume of change transactions surfaced as important contributors to turbulence.

B. The principal analytic constituents and activities of the MOC methodology are shown in Figure 1. MOC methodology development was directed to the formulation of qualitative and quantitative analytic approaches for addressing management issues and key problem variables observed in the authorization change environment. At the same time, a specific requirement was identified for a detailed logical structure in which to view, examine, and assess the key processes.

II. Functional and Qualitative Considerations of Methodology Development

Investigative and problem definition work constituted the initial methodological venture to explore the MOC problem space--the Army authorization management change environment. Investigative and definitional procedures included field visits, personal interviews, document reviews and data collection tasks incident to MOC study objectives. Concurrently, construction of a central data set was initiated to support subsequent methodology development and analysis. Management systems, related key management processes and specific procedural activities associated with authorization changes Army-wide were reviewed in detail to identify important functional and operational aspects of the Army's current change procedures. Symptoms of turbulence (e.g., late completion of TAADS updates) were identified along with certain qualitative causes

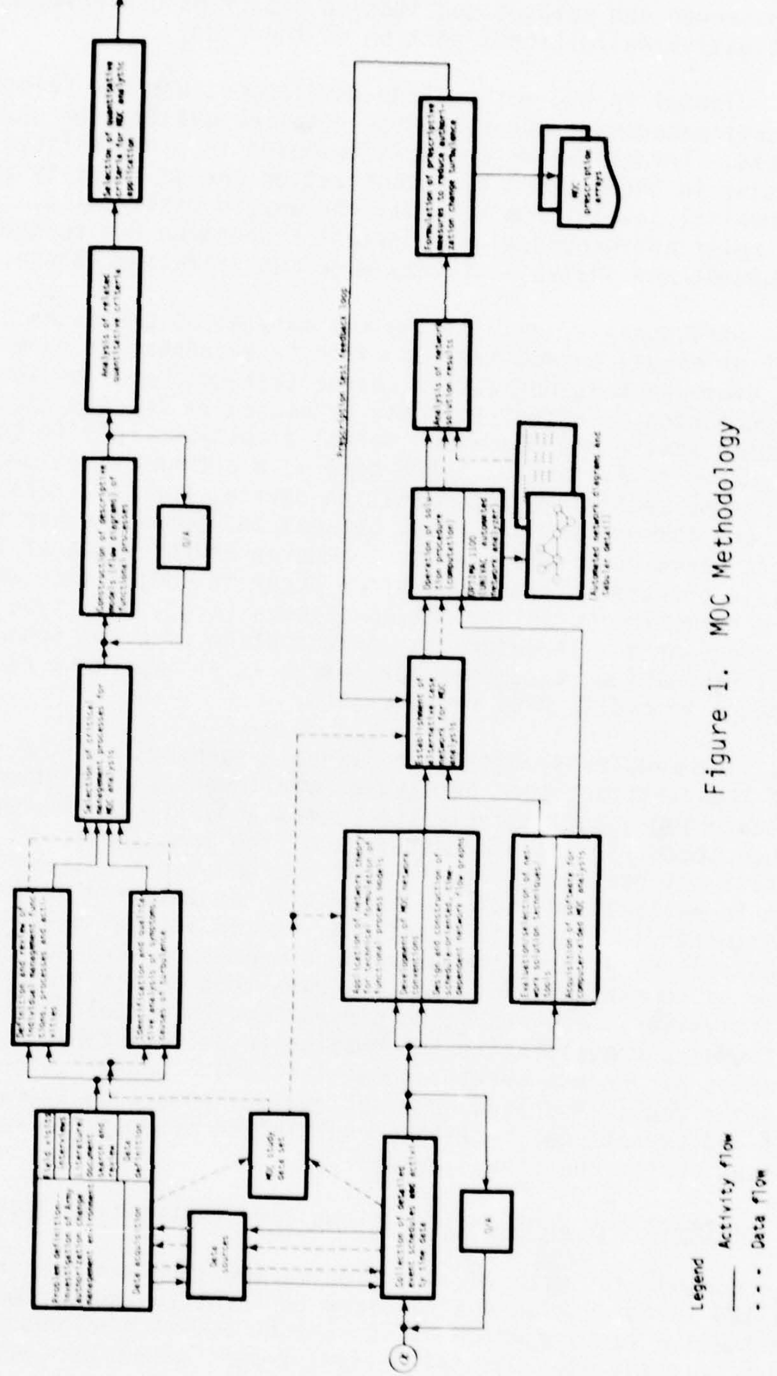


Figure 1. MOC Methodology

(e.g., the issuance of conflicting change guidance). Several problems were observed and related qualitative prescriptions developed during this investigative/definitional portion of the study.

A. Pivotal in MOC methodology development was the selection of specific key management processes for detailed qualitative and quantitative analysis. Investigative analysis revealed the criticality of certain processes in the pattern of authorization change activity spanning Army hierarchical levels from HQDA all the way to small units. Therefore, their selection provided an important foundation for further methodology development and analysis of Army-wide authorization change.

B. Structurally, each of the key management processes consists of a number of events and activities which interconnect to form procedural paths over which authorization change transactions flow to be acted upon and documented. Central to these processes is The Army Authorization Documents System (TAADS) which serves a multiple role in the authorization change environment. TAADS acts as a communication medium, transaction change mechanism, documentation device, and data bank. Interfaces and common linkages exist between and among the key management processes, based on common events, schedules and/or types of activities. Multiple process interactions often occur in conjunction with PPBS milestones resulting in authorization changes (e.g., unit type conversions) across the Army. Therefore, synchronization of event schedules for individual or combined management processes is an important requirement for adherence to cyclic PPBS requirements.

C. The complexity of the individual processes and the richness with which they interact pose particular challenges in detecting, identifying, and measuring quantitatively the causes and effects of change-driven turbulence (occurring as a consequence of the processes). In this regard, a critical methodological requirement emerged for a systematic structure in which to analyze and evaluate in detail the selected processes, their constituents and their interactions. As an initial step in building such an analytic structure, the management processes were modeled descriptively by construction of flow diagrams that depicted component events and activities. The resulting diagram models established a baseline of functional and qualitative information (e.g., existence and interrelationships of process events and activities). Construction of the descriptive models involved numerous quality assurance iterations with both staff and operational sections resulting in progressive refinement of information for the flow diagrams.

III. Methodology and Considerations for Quantitative Analysis

In conjunction with the descriptive modeling work described above, a detailed investigation was required to identify the various types of quantitative factors which would support a more vigorous analysis--frequency of activities/events; activity/event schedules; activity performance time (duration) requirements; number of interactions; volume of workload transactions were all considered. This investigation highlighted the importance and pervasiveness of time and schedule interdependencies among the critical components of all processes. Based upon these

findings, time and schedule considerations became predominant as quantitative measurement criteria for MOC methodology. Other change-related factors such as frequency and volume (workload) were treated in MOC analysis; however, time and schedule factors were selected as principal quantitative measurement and analysis criteria.

A. Scheduling considerations centered on frequency (e.g., times per year) of occurrence for events or activities and on calendar date (or process milestone date) of activity start or completion. The analytic focus on time criteria therefore involves determining the required time to perform each given activity within a selected process. The ability of Army organizations to meet activity or event schedules is largely dependent on the time allowed in the authorization change management processes. This is a particularly critical consideration since missed schedules contribute to the atmosphere of turbulence.

B. Consistent with time and schedule dependencies, findings concerning MOC study data activities involved collection of detailed event (schedule) data and activity performance (time) data. Data sources included functional organizations and personnel throughout the Army qualified to provide schedule and time estimates for activities within the key management processes based upon actual experience. Activity duration estimates represented expected (elapsed) time values in the current Army authorization management environment. Schedule data consisted of a mix of officially regulated event dates and actually experienced event dates associated with the management processes. Quantitative data acquisition procedures required quality assurance iterations for data refinement and verification.

IV. Use of Network Theory

The key Army management processes incorporated the time and schedule variables which cause much of the authorization change-related turbulence. Technically, these processes exhibited specific activity or event orientations and time-dependent properties which rendered them amenable to rigorous analytical treatment based on the principles of network theory. Building on the functionally oriented flow diagram models for each of the selected management processes, technically oriented network formulations were derived. These network formulations provided a highly structured,descriptive and quantitative means to analyze, illuminate, and assess authorization change problem variables and interactions. Most importantly, for MOC analysis, an operational network methodology offered a capability to postulate and evaluate, quantitatively, the consequences of alternative activity and event schedules (e.g., earlier start of selected events), and differing activity performance times (e.g., reduced time to perform authorization management activities of a given type). The following subparagraphs discuss major considerations in transforming the functional information developed in the investigative and definitional portions of the study into technical network constructs to support the quantitative analysis upon which the principal results and findings are based.

A. Networks. A network (or linear graph) is a mathematical abstraction from the real world in which certain points (or nodes) are connected by lines (or arcs). Generally, the network concept includes a flow of materiel through the nodes and arcs. The functional management processes selected for MOC analysis exhibit properties which are generally relatable to a generic class of network models called activity networks. As previously indicated, authorization change schedules, time durations, and frequencies are contributory factors in turbulence. To provide specific insights into the causal relationships, time analysis of activity networks for the selected management processes is a reasonable method of providing direct quantitative information concerning time delays, intra- and inter-process synchronization problems, and sequencing or scheduling difficulties which lead to turbulence in Army organizations. Moreover, such analysis provides a basis for nominating and testing--in technical network form--alternative process times and schedules for ultimate functional application in reducing turbulence.

B. Terminology. Time/schedule activity network modeling of functional processes (such as those selected for MOC analysis) required terminology, symbology, and conventions which establish a basic discipline for network formulations.

1. In targeting the MOC network formulation to Army authorization change management processes, several applicable terms and definitions are tabulated in Table 1.

2. Fundamental requirements for construction of time-dependent activity/event schedule networks include:

a. Specification of activities and events which constitutes each network. For MOC networks, the original functional flow diagrams provided a ready basis for satisfying this requirement.

b. Definition of linkages of events and activities to reflect interdependencies among events.

c. Estimation of time required for each activity (if feasible, include statement of uncertainty).

To extend such fundamental requirements to MOC network construction, Table 2 contains a list of building blocks to accentuate the component elements and actions in network composition.

3. In addition to the foregoing considerations of network building, the following specific composition rules apply for the MOC study.

a. All activity paths leading to an event must be completed before that event can occur.

b. No activity can start until its originating event has occurred.

c. Each event is unique and cannot supersede itself.

Table 1. Terms and Definitions

Term	Symbol	Definition	Example in MOC Context
Activity	xx (ARC)	Represents work being done. Has an associated time duration from start to finish.	3 — ODCSOPS prep force in SACS — 4. Activity 3-4 starts at event 3 and stops at event 4.
Event	# (Node)	The beginning or end of one or more activities. An objective, an accomplishment or start point.	
Network	(Web)	An ordered sequence of activities and events which represent a functional process.	Network diagrams of key management processes.
Time	(t_e)	The basic quantifier for measuring MOC activities. Refers to most likely time, i.e., most frequently occurring time to accomplish an activity.	3 — (4) ODCSOPS prep force in SACS — 4. The time estimate for activity 4-5 is 4 days.
Milestone	N/A	Network events of major importance with a specific date constraint.	17 — TRADOC Publishes CCT — 21. Event 21 must be completed 5 Mar & 5 Sep.
Process	N/A	A series of events and activities. MOC networks describe each specific process.	Key management processes described in MOC.
Environment	N/A	The collection of processes in MOC which combine to form the authorization management environment.	The TAADS Documentation network forms the core for linking the other processes to form the total environment.

Table 2. Network Building Blocks

- Collection of existing information on current management processes (functional and temporal).
- Selection and identification of milestone events and activity designations.
- Sequencing of interim events and activities and establishment of interrelations so that a network is developed to depict a logical progression to completion of a process.
- Detailed refinement of time estimates required to complete the activities defined by starting and ending events.
- Correlation of information on processes in order to formulate interprocess linkages.

4. Network diagram conventions for MOC include the following:

a. Activities (in each management process) are represented by network arcs (line connections between events).

b. Events are represented by networks nodes (numbered circles).

c. Network flow proceeds from left to right (no arrowheads required).

d. Condensed descriptive narrative is annotated for each activity:

1. Organizational responsibilities are indicated.

2. Nature of activity is described.

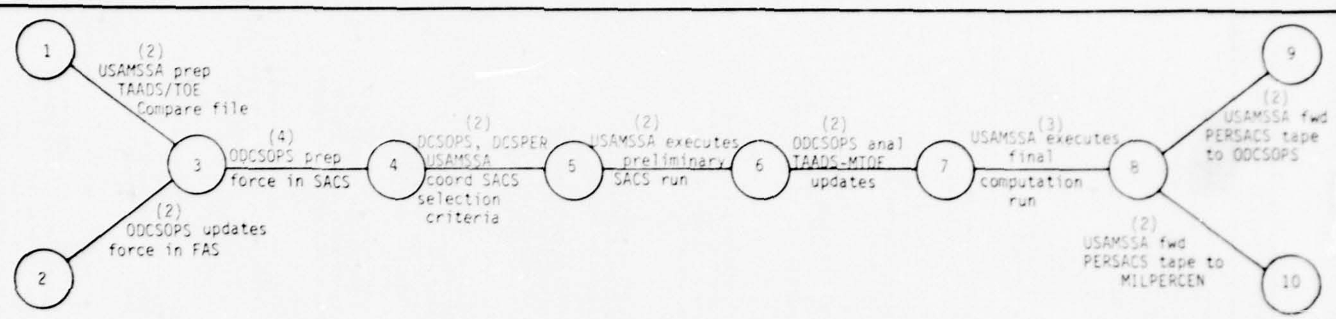
e. Numeric time entries define the number of days required to perform an indicated activity.

f. An example of a network constructed for MOC is found at Figure 2.

C. Computational Requirements. Construction of detailed activity networks in accordance with the rules and conventions described above provided a rigorous and disciplined set of models of the key authorization management processes. To apply these network models in the analysis of the management processes which they represent, operational and computational capabilities are required to develop and provide quantitative data on the effects of current and alternative process sequences and schedules, activity time durations, and frequencies. Table 3 specifies the types of computational capabilities required to support the MOC network analysis.

D. Network Analysis. The analytic requirements and corresponding operational and computational capabilities defined above indicate the specific orientation, scope, and complexity of the quantitative approaches needed to address causal factors in change-driven turbulence. Of particular importance are responsive computational and operational techniques for use in formulating explicit, prescriptive alternatives to current Army authorization management processes. The activity or event orientation of the MOC networks and the time-dependent scheduling and synchronization problems inherent in authorization change management, signal the need for the specific types of analysis prescribed in Table 3. A discussion of network analysis concepts and techniques which offer the requisite capabilities for MOC analysis is offered below.

1. Generally, developments in network theoretic problem solving concepts and associated computational methods have kept pace with the need for knowledge and application techniques in the implementation and use of network-based methodologies. Relevant to the MOC study, many sequencing and scheduling investigations can be analyzed as problems in network (and graph) theory. Further, problems of the nature, scope, and scale associated with Army authorization change processes are



a/For definition of acronyms, abbreviations and short terms, see Glossary.

b/Numbers in parentheses indicate number of working days normally required.

Figure 2. The PERSACS Network

Table 13. MOC Study Analytic Requirements and Capabilities

MOC Analytic Requirement	Operational/Computational Capability	Comments
• **Detailed Activity Time Analysis** •• **Assessment of alternate activity durations to reduce turbulence.** •• **Determination of total (longest) time to completion of process (network).** •• Identification of particular activities on which process duration depends.	• **Responsive (e.g., once per day) facility to define, test and evaluate alternative networks with different time durations for selected activities.** •• Ability to supply information on process duration time. •• Ability to provide data to identify and trace individual activities on longest completion path.	• **Each activity in processes (networks) has common quantification base--time in days.**
• Scheduling Analysis	• Facility to relate individual events to specific calendar dates. • Ability to define, test and evaluate alternative networks with set milestone dates for certain events. • Ability to set a single start date, finish date or interim event completion date and compute changes to entire schedule of events.	• Beginning or ending dates for authorization management activities and processes are often officially regulated.
• Analysis of Event Sequencing	• Ability to specify sequential dependencies within a process (network) to represent progression from network start to finish. • Responsive facility to modify event sequences and to test and evaluate results. • Ability to supply detailed information on alternative event sequencing effects on process accomplishment.	• Within and among functional processes, activity performance depends on completion of one or more prior ones.
• Synchronization and Timing Analysis	• Ability to identify synchronization requirements (e.g., simultaneous events; timing of critical completion dates) within and among processes (networks). • Responsive facility to define, test and evaluate alternative synchronization schemes through network modification and analysis.	• When events/activities occur simultaneously or are common to more than one authorization process, timing analysis is particularly important to assess interprocess event synchronization.
• Multiple Process Interaction Analysis (includes types of analysis listed above)	• Ability (and capacity) to combine all processes in network form with all associated time and relational properties treated explicitly and in individual detail. • Responsive facility to modify, test, compute results, and evaluate alternative network configurations representing interprocess linkages.	• Natural consequence of need for analysis of overall Authorization Management Environment. • Involves assimilation and solution of large scale network problem which is manageable within computer applications state-of-the-art.
• Observation, Inspection of Process Structures, and Component Times in Graphical Form	• Rapid facility (e.g., one per day) to compose (draw) network diagram for each alternative derived from types of analyses and related operations and computations described above.	• Involves automatic plotting capability which can accomodate large scale network diagrams.

computationally feasible with currently available solution methods and operational tools.

2. Of significance in the MOC study is the ability of such tools to supply information about the length of time involved in a management process (network) and about the particular activities on which the process duration depends. Operational tools for complex, large-scale network problem solution are characteristically embodied in computer-based applications programs or utility software packages which are documented and readily available in the automatic data processing (ADP) marketplace. On an annual basis, the repetitive occurence of the various networks, given the unique frequencies of each, resulted in over 2,000 activities. The number of activities and the analytic requirements of the nature prescribed in Table 3 define the specific properties and capabilities of the operational tools needed for MOC network analysis. Some major features of the automated capability acquired to provide computer-aided support for MOC network analysis are:

a. Readily achieved operational status on the CAA computing system.

b. User-oriented formats to facilitate digitization of the network data.

c. Automated operational procedures and computational algorithms which satisfy specific MOC network analysis needs and provide detailed quantitative data on alternative problem approaches. (Reference Table 3).

d. Computer-based graphics capabilities which produce network diagrams rapidly and automatically based on alternative solutions derived from MOC prescriptive analyses.

3. The OPTIMA 1100 Project Management System was selected as the principal operational tool for performing the automated network analysis. OPTIMA 1100 is a system of modular routines which permit detailed time, resource, and cost analysis of large, complex network groupings. OPTIMA can be operated on any UNIVAC Series 1100 computer, to include the CAA UNIVAC Model 1108. For the MOC study, the OPTIMA 1100 time analysis feature provided the capability to support the necessary scheduling, sequencing and synchronization analysis of the network diagrams. An example of the output provided by OPTIMA (Figure 3) corresponds directly with the manually developed and drawn network shown at Figure 2.

V. Application of the Methodology

The preceding discussion covered major considerations, issues and approaches concerning the development of the MOC study methodology. The resulting technical network models of selected key authorization management processes, along with related solution techniques and operational tools, form the analytic base for application of the methodology to develop prescriptive alternatives to the current Army authorization change environment. Central to the MOC study methodology application and the derivation of study results was the use of manual quantitative analysis

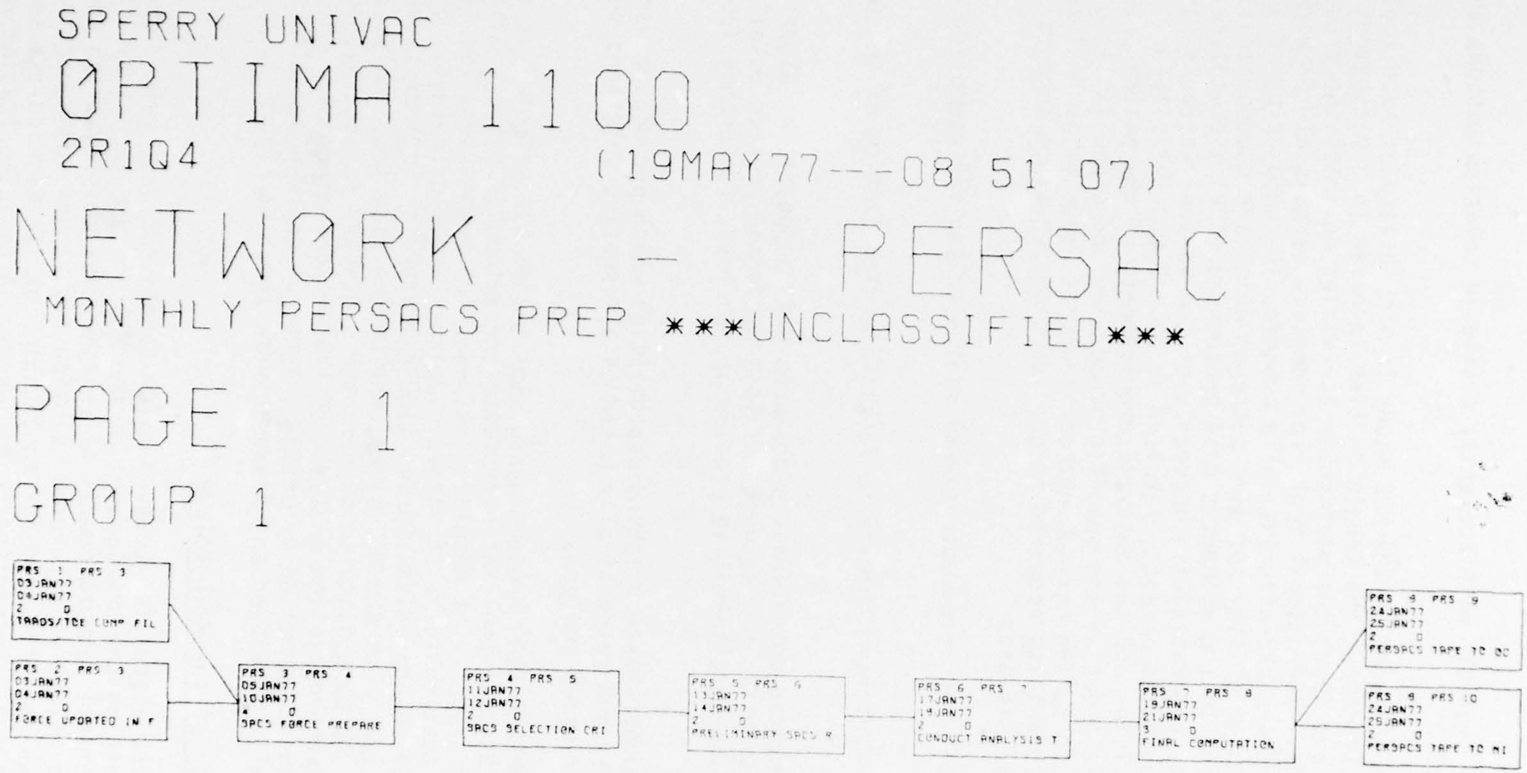


Figure 3. OPTIMA Drawing

procedures in concert with a computer-based network solution tool acquired to aid the analytic effort. The actual application of the methodology has resulted in a unique set of compatible descriptive network diagrams for viewing the authorization management processes and viable alternative schedules which should reduce the turbulence inherent in the current authorization environment.

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TITLE: A Method for Validating Missile System Simulation Models

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ABSTRACT: A method for deciding whether a missile system simulation model is sufficiently accurate for a specific purpose is presented. The procedure uses time series transformations to generate ensemble statistics for the simulation error associated with missile system performance parameters of interest. Well known formal decision theory methods can then be used to determine whether the model is adequate. While not specifically addressed, the procedure is generally applicable to models other than missile system simulations.

A METHOD FOR VALIDATING MISSILE SYSTEM SIMULATION MODELS

By

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Introduction

The ever increasing cost of actual missile flight firings imposes severe restrictions on the number of such tests that can be conducted during the development of any new system. Similar constraints are imposed on the experimental testing of modifications to existing missile systems. As a consequence, it is necessary to rely more heavily on mathematical models which simulate the performance of the real missile system. This requirement for increased reliance on simulation has stimulated efforts to develop methodologies to validate these models.

Although there has been an increase in activity directed toward computer simulation validation, a universally acceptable approach has not yet come forth. Indeed, Naylor and Finger's statement (2) that "the problem of verifying or validating computer models remains today perhaps the most elusive of all the unsolved methodological problems associated with computer simulation techniques" appears to be as true today as it was ten years ago. There seem to be two problems contributing to this situation. The first is the lack of a criterion to use in judging the validity of simulation models. The second is the nature of the process output, which typically consists of a sequence of correlated variables evolving over time. This second problem contributes to the first, since the usual statistical techniques for independent random variables are not directly usable. In this paper we describe a procedure for use on missile system simulations that circumvents these problems.

The definition of a criterion for stating that a simulation is valid is linked directly to the definition of validation. For the purposes of this paper, a simulation will be considered valid if the difference between the process output and the simulation output is small enough. What constitutes "small enough" depends on the use to which the simulation results will be put. For instance, if the cost of a wrong inference using the simulation output is small, then a large error can probably be tolerated, and vice versa. Setting the tolerance for validation is a decision problem that depends on the intended use of the simulation results, and as such, it can be handled by any of the well known decision models that are available today once the nature of the simulation error is defined. The remainder of this paper describes a method for comparing simulation and flight test results and illustrates this method by a simple example.

Problem Statement

The ultimate performance of any missile system is embodied in its

position in space as a function of time. For weapons, the point on this trajectory relative to the target at the time of warhead event constitutes the weapon's miss distance, an extremely important characteristic. While the trajectory is the ultimate measure of performance, there are many other performance variables that influence the trajectory and may also be of interest. For purposes of illustration, we confine ourselves to a single variable which could typically be one of the components of the position in space vector. On any missile flight, the actual value of this variable is usually measured experimentally during the course of the test. The actually observed value is governed by the characteristics of the atmosphere through which the missile flies, the physical characteristics of the missile itself, and perhaps even the characteristics of the target, and is further confounded by errors in the measurement process itself. These influences on the missile are random in nature; hence, their magnitude may be treated as a random variable. Figure 1 illustrates this situation.

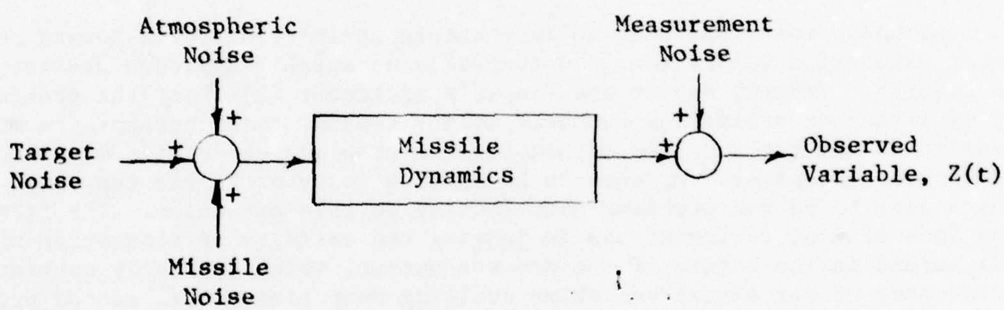


Figure 1. Missile Flight Test Block Diagram.

The observed variable, $Z(t)$, is a continuous random variable since the inputs to the process are random variables. However, $Z(t)$ is usually observed only at a fixed time interval. It is therefore convenient to consider instead the sampled variable which is a discrete random variable, Z_t, evolving over time, commonly called a discrete time series.

One of the major problems associated with statistical analysis of flight test results is that each of the points on the observed time series is highly correlated with its neighbors by virtue of the inertia in the system. Hence, the methods of statistical analysis that are based on the assumption of independence between these points cannot validly be used. The implication of this circumstance is that the observed time series is a sample, of size one, of all the possible time series that could have been realized from this process. In general, the observed time series is a stochastic process

having a joint probability density function $f_{Z_{t_1} Z_{t_2} \ldots Z_{t_n}}(z_{t_1} z_{t_2} \ldots z_{t_n})$.

Each realization of the process can be considered independent of all other realizations, and the moments can be estimated as a function of time by conducting a number of tests under the same conditions. This requires multiple flight tests at each point in the missile flight test matrix, a technique that is usually prohibited by cost constraints.

On the other hand simulation of a missile flight using a computer model is a relatively inexpensive process, even for the most complex of missiles. By incorporating random noise models of the atmospheric disturbances, the target noise, the missile noise, and the measurement noise into the simulation, it can be run in monte carlo fashion as many times as needed at the same test conditions and estimates of any necessary moments can easily and relatively inexpensively be calculated as a function of time using well known techniques. The problem is, how does one go about determining if there is any difference between the set of time series generated by the computer simulation and the time series observed from the flight test when only one realization of the latter is available.

Methodology

To facilitate a technique for comparing time series from simulation and test, we assume that the marginal distribution of the flight test time series at any time, i, defined as

$$f_{Z_i}(z_i) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_{Z_1 \ldots Z_{i-1}, Z_i, Z_{i+1} \ldots Z_n}(z_1, \ldots z_{i-1}, z_i, z_{i+1} \ldots z_n)\, dz_1 \ldots dz_{i-1}\, dz_{i+1} \ldots dz_n, \tag{1}$$

is gaussian. This assumption is justified by the fact that the top level system performance variables, such as trajectory components, are functions of a large number of random variables. Hence, the central limit theorem will tend to make the assumption valid. Accordingly, the marginal distribution at any time is completely characterized by the mean and variance of the distribution at that time, and the time histories of the mean and the variance completely characterize the process.

To find estimates of the ensemble mean and the variance time histories associated with the simulation model output is a simple matter. The simulation model is simply run repeatedly for the same test condition. Each run constitutes a random, independent realization of the flight test as portrayed by the simulation model. The mean and the standard deviation at each sample time can easily be estimated using well known procedures.

What is needed is a way to determine the mean and variance of the

actual process as a function of time. It is well known that almost every nonseasonal time series may be adequately represented by a model of the form

$$\phi(B)\nabla^d(Z_t-\mu_Z) = \theta(B)a_t \tag{2}$$

where $\phi(B) = 1-\phi_1 B-\phi_2 B^2 -\ldots- \phi_p B^p$

and $\theta(B) = 1-\theta_1 B-\theta_2 B^2 -\ldots- \theta_q B^q$

are polynomials in the backshift operator B, $BZ_t = Z_{t-1}$, $B^jZ_t = Z_{t-j}$, and ∇^d is the difference operator, $\nabla^d = (1-B)^d$, where d is the order of the difference. Then $\nabla Z_t = (1-B)\ Z_t = Z_t - Z_{t-1}$. a_t is a random shock from a discrete white noise process with zero mean and variance σ_a^2. μ_Z is the mean level of the time series and may be estimated by $\overline{Z} = \frac{1}{N}\Sigma\ Z_t$ where N is the number of points in the observed time series. The differencing operation is necessary only if the observed time series is not covariance stationary. In this case, the required order of differencing, d, is that necessary to achieve stationarity, which is determined by the behavior of the autocorrelation function.

A model of the form of equation (2) is known as an autoregressive, integrated, moving average (ARIMA) model of order (p, d, q). Methods for fitting such models to time series are fully described in Box and Jenkins (1). There are also numerous articles in the literature describing such procedures. Note that since the a_t are gaussian, then each of the Z_t is also gaussian.

Suppose we let $w_t = \nabla^d Z_t$. Then ∇^d can be regarded as a transformation that takes a nonstationary series into a stationary series, which can be represented by a model of the form

$$\phi(B)\ w_t = \theta(B)\ a_t. \tag{3}$$

Since the w_t series is stationary, all of its stochastic properties are described by its mean and its autocovariance function. These same properties are also captured in equation (3), and hence in equation (2). Therefore, since a properly fitted ARIMA model of order (p, d, q) embodies all of the stochastic properties of the flight test time series it can be used to represent the test data. The time series fitting process results in a transformation of the test data into an alternate form that captures its information content. Furthermore, this representation can easily be used in a simulation model to generate sample realizations of the time series process. The missile system simulation model and the ARIMA model are independent

models of the process in question. If they both represent the same stochastic process, then the mean time history and the variance time history from each model represent independent estimates of these parameters, and their equivalence can easily be tested at each time by well known statistical inference techniques.

Example

To illustrate the approach described above, an experiment using a first order linear filter was conducted. This filter was driven by a unit step function with an additive white noise step process superimposed. The initial condition was taken as zero. The output of the filter was sampled and corrupted by a white noise pulse process. Figure 2 is a block diagram of this process. The filter constant was arbitrarily taken to be 0.7, and the input and output noise standard deviations were taken as 0.1. The noise free differential equation of this filter was solved using a Runge-Kutta integration scheme and the result was checked against the well known analytical solution to make certain that the integration step size was not introducing error. The differential equation was then solved using the Runge-Kutta routine with the input and measurement noises superimposed. Fifty independent time series realizations were generated in this fashion, the ensemble mean and standard deviation time histories were estimated from these realizations and are displayed in Figure 3. Each realization had one hundred and twenty data points. (This part of the experiment corresponds to exercising the missile system simulation model). At random, one of the fifty realizations was chosen to represent the flight test data series. This series is depicted in Figure 4. Since it is known that this time series comes from the filter process, a properly fitted ARIMA model should yield the same mean and standard deviation time histories as the Runge-Kutta solution.

The key elements in identifying the proper ARIMA model to represent a time series are the autocorrelation function and the partial autocorrelation function. The most satisfactory estimate of the k^{th} lag autocorrelation, ρ_k, is

$$r_k = \frac{C_k}{C_0} \tag{4}$$

where

$$C_k = \frac{1}{N} \sum_{t=1}^{N-k} (Z_t - \overline{Z})(Z_{t+k} - \overline{Z}), \quad k=0,\ 1,\ 2,\ \ldots\ K. \tag{5}$$

The estimated partial autocorrelation function at lag k, ϕ_{kk}, can be calculated from the well known Yule-Walker equations,

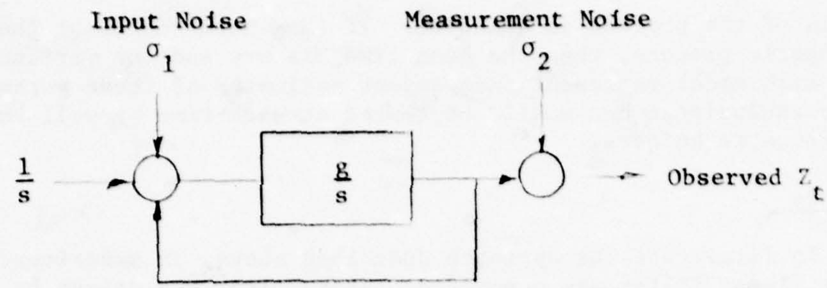


Figure 2. Filter Block Diagram.

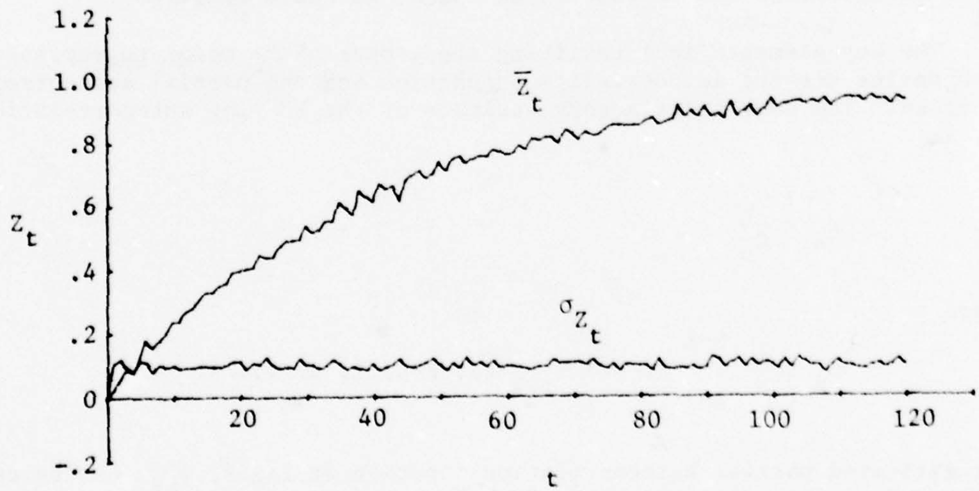


Figure 3. Filter Mean and Standard Deviation Histories.

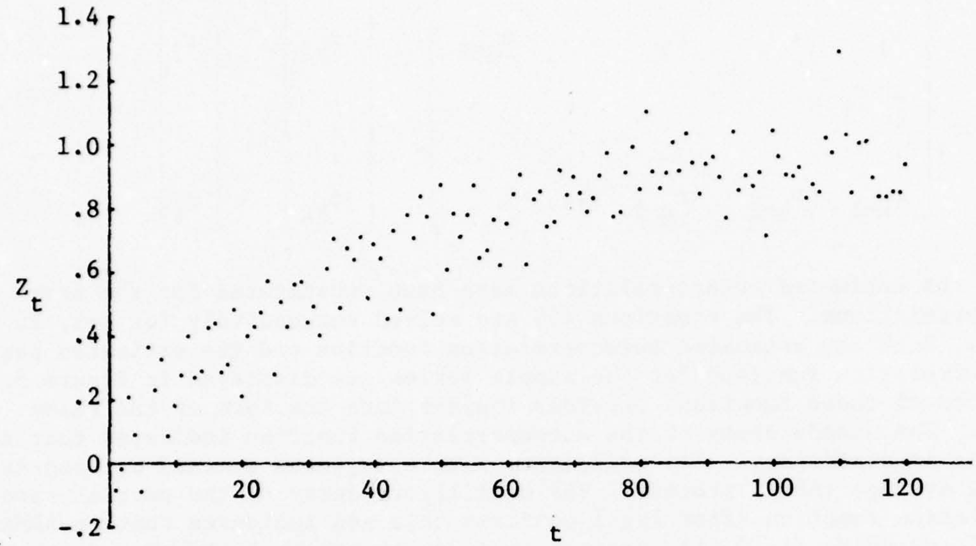


Figure 4. Random Realization of Filter Output.

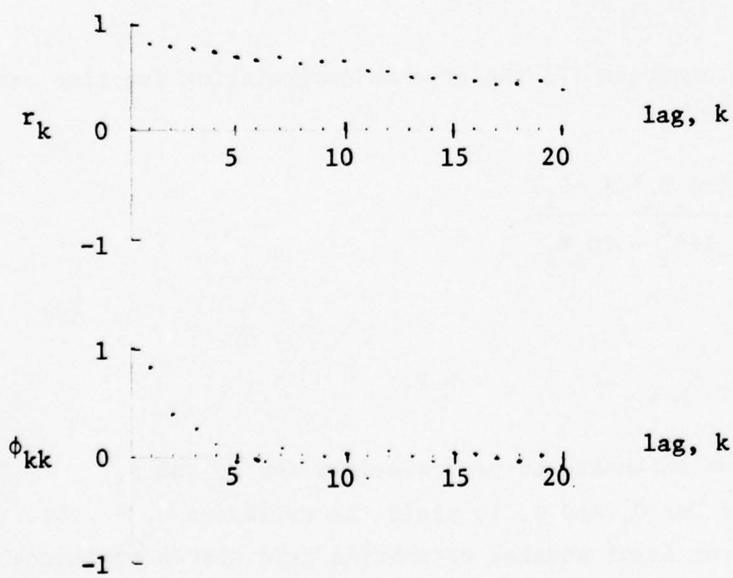


Figure 5. Sample Autocorrelation and Partial Autocorrelation Functions.

$$
\begin{bmatrix} 1 & r_1 & r_2 & \cdots & r_{k-1} \\ r_1 & 1 & r_1 & \cdots & r_{k-2} \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ r_{k-1} & r_{k-2} & r_{k-3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \phi_{k1} \\ \phi_{k2} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{kk} \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ \\ \\ \\ r_k \end{bmatrix} \qquad (6),
$$

where the estimated autocorrelations have been substituted for the true autocorrelations. The equations (6) are solved successively for k=1, 2, ... k. Both the estimated autocorrelation function and the estimated partial autocorrelation function for the sample series are displayed in Figure 5. The form of these functions provides insight into the form of the ARIMA model. The steady decay of the autocorrelation function indicates that the process is stationary. Its ocillitory nature suggests a mixed autoregressive, moving average (ARMA) process. The oscillitory decay of the partial autocorrelation function after lag 1 confirms this and indicates that an ARMA (1, 1), or ARIMA (1, 0, 1), process is a strong possibility. Hence, the model

$$
(1-\phi, B)\ (Z_t-\overline{Z}) = (1-\theta, B)\ a_t \qquad (7)
$$

was tried.

For a model such as equation (7) the true autocorrelation function can be shown to be (1, p. 77)

$$
\rho_1 = \frac{(1-\phi_1\theta_1)(\phi_1-\theta_1)}{1+\theta_1^2 - 2\phi_1\theta_1}
$$

$$
\rho_k = \phi_1\ \rho_{k-1} \qquad , \ k \geq 2. \qquad (8)
$$

Substituting the first two estimated autocorrelations for ρ_1 and ρ_2, equations (8) were solved for θ_1 and ϕ_1 to yield the estimates $\hat{\phi}_1 = .98$; $\hat{\theta}_1 = .7$. Using a nonlinear least squares estimation grid search procedure with these values as a starting point, and estimating the mean $\overline{Z}$ as well, the parameters of this model were estimated as

$$\hat{\bar{Z}} = .96$$

$$\hat{\phi}_1 = .967$$

$$\hat{\theta}_1 = .92$$

$$\hat{\sigma}_a = .1058$$

The fit of the model was then checked by testing to see if there was any reason to suspect that the residual sequence, a_t, was not gaussian. None was found.

In order to use equation (7) to simulate sample realizations of the test process, some initial conditions are necessary to start the recursion. Although in this experimental case these conditions were known, if the time series represented some missile system performance variable, this would not be true. It can be shown that for any ARMA model there is a dual model of the form

$$\phi(F)\ Z_t = \theta(F)\ e_t \tag{9}$$

where F is the forward shift operator, $FZ_t = Z_{t+1}$ and $\sigma_a = \sigma_e$. Clearly, $F = B^{-1}$. The model of equation (9) is known as the backward model. It can be demonstrated that both the forward and the backward models have identical autocovariance functions; hence, they both represent the same stochastic process equally as well. Generally, the backward form of the model can be used to calculate enough points to start simulating the recursion model, equation (7). However, in this case the root of the autoregressive portion of the model, $\hat{\phi}_1^{-1}$, is close to unity. Thus, the process is close to non-stationarity, and it is known that such processes can make long excursions from the mean. Under those conditions, the backward model is not as good as usual. To circumvent this problem, equation (7) was simply solved for the initial condition using the first point of the observed series and choosing a_t and a_{t-1} at random from the appropriate distribution. E.g., expanding equation (7),

$$Z_0 = [Z_1 - \bar{Z}\ (1-\phi)\ - a_1 + \theta a_0]/\phi \tag{10}$$

was used to obtain the necessary starting value for the forward recursion

$$Z_t = \bar{Z}\ (1-\phi) + \phi Z_{t-1} + a_t - \theta a_{t-1} \tag{11}.$$

Choosing the values of the a's from the appropriate normal distribution,

the recursion relationship (11) was solved to obtain 50 sample realizations of the "observed" process, and the mean and standard deviation time histories were calculated. These are presented in Figure 6. Figure 7 shows the mean time history superimposed on the sample observation from Figure 4. Figure 8 shows the mean and standard deviation time histories of both the Runge-Kutta (missile system simulation) and the time series solution. Figure 8 indicates good agreement except for the first twenty or so points. However, standard tests of the null hypotheses of equivalent means and equivalent variances at each time increment show that the null hypothesis must be rejected at about 10% of the points with a significance level of .01. The lack of agreement may be due to small errors in the time series model parameters resulting from the resolution inherent in the grid search routine used. Nevertheless, it is clear that the concept has a great deal of promise.

Discussion and Conclusions

A proposed procedure for making statistical comparisons of missile system simulation results and flight test results has been presented. It involves using a time series model to capture the stochastic properties of flight test results. Statistical comparisons can then be made on ensemble means and variances using the usual normal theory which is familiar to most managers. This is a strong point of the procedure. The example presented above demonstrates that the procedure works in general; however, statistical comparisons indicate a higher rejection rate than was hoped for. A substantial portion of the rejected points can be attributed to the transient introduced by the time series start up procedure. The reason for the remaining areas is the subject of additional investigations. Despite these problem areas, it is concluded that this is a promising approach.

References

1. Box, G. E. P., and Jenkins, G. M.; Time Series Analysis, Forecasting, and Control; Holden-Day, San Francisco, 1970.

2. Naylor, Thomas H. and Finger, J. M.; "Verification of Computer Simulation Models"; Management Science, Vol. 14, No. 2; Oct 1967.

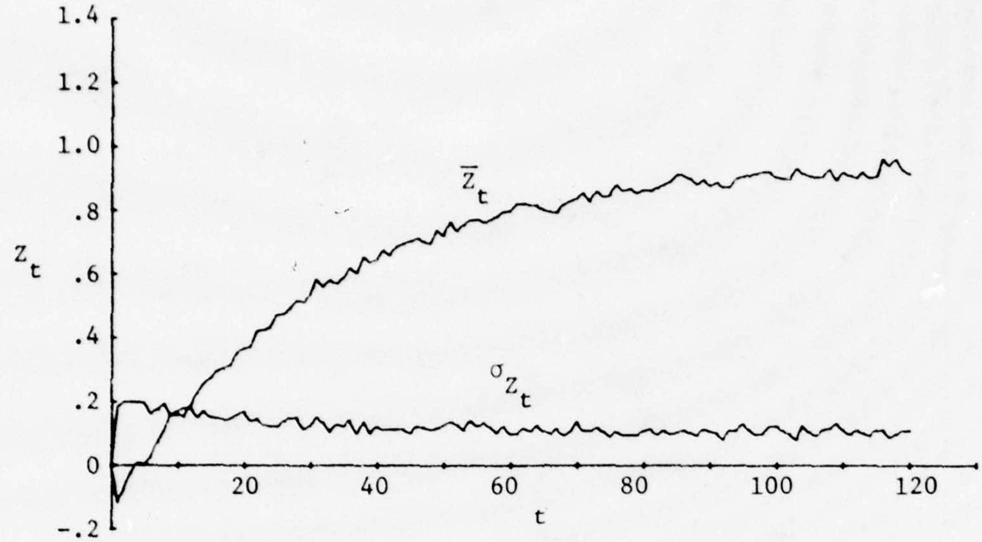


Figure 6. Time Series Mean and Standard Deviation Histories.

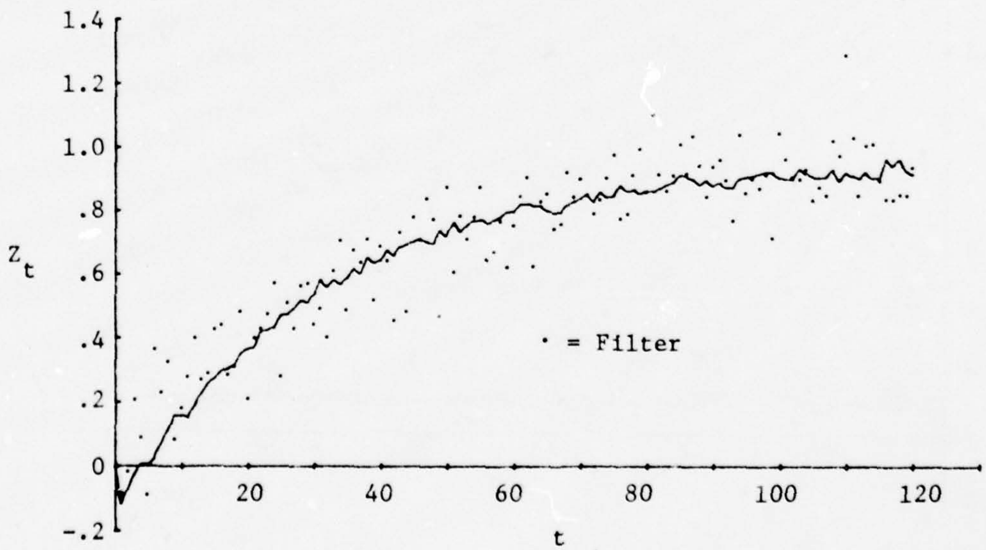


Figure 7. Comparison of Filter Realization and Time Series Mean.

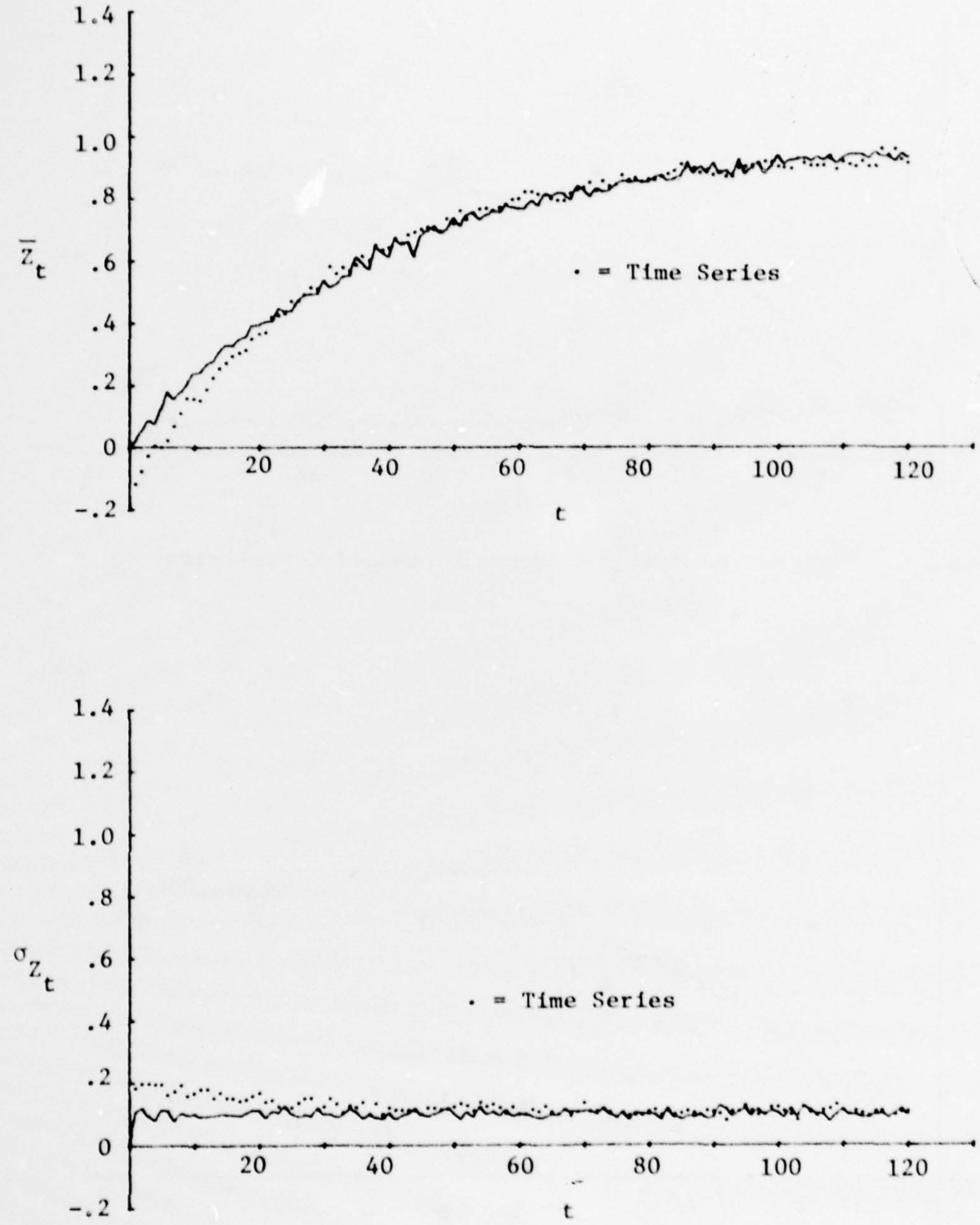


Figure 8. Comparison of Filter and Time Series Mean and Standard Deviation Histories.

OPERATIONS RESEARCH IN OPERATIONAL TESTING

Dr. James W. Dees

US ARMY AVIATION BOARD

What is there about operational testing that differentiates it from other operations research? Let's first take a look at what operational testing is

SLIDE 1

The object of operational testing is to determine if equipment will serve its intended purpose, while supporting the overall mission when operated by the intended user under the expected environmental conditions. Thus the requirements given in Slide 1--

a. That the user be the test subject.

b. That a realistic tactical situation be examined.

c. That realistic measures of performance be obtained.

The emphasis of operational testing then, is on man/systems simulation in as realistic an environment as possible.

SLIDE 1 OFF

Operational testing was separated from developmental testing by the Army because developmental testing is limited to an engineering test of the equipment against specifications and contract requirements. We have learned from many painful occasions that equipment can meet its specifications and still not be useful in the field. If the specifications don't define useful equipment, useless equipment can meet the specifications. Obviously then operational testing which is undertaken in order to check the usefulness of equipment will be different from developmental testing which is undertaken to assure that the equipment meets specifications.

In operational testing we deal with the operation of the equipment by the expected range of military personnel under field conditions. Since both the military personnel and the field conditions are variables with great diversity, sampling techniques and statistics will be used extensively in operational testing. Developmental testing seeks to verify that equipment will do what it is designed to do. Hence it confirms design qualifications. Operational testing seeks to determine how well the design is suited to the needs of combat troops in a simulated combat environment. Hence it confirms design utilization. In addition, most operational tests seek not only to define which of two or more items of equipment perform best in the field, but also to provide a measure of performance.

SLIDE 2

Therefore operational testing tends to emphasize definitive rather than comparative measures.

SLIDE 2 OFF

Choice of Measures: As in all operations research, the choice of measures in operational testing is crucial to the success of the test. Choosing your measures is an art, not a science, but a few guides are available.

SLIDE 3

These steps are not peculiar to operational testing. However, in operational testing you generally try to simulate "real conditions" more closely than you would in other areas of operations research.

SLIDE 3 OFF

Much but not all of the data required in operational tests must come from questionnaires addressed to the user. Attention to the level of data dictated by the design of the questionnaire will often reap large dividends in the usefulness of the questionnaire. S. S. Stevens years ago introduced the concept of levels of data.

SLIDE 4

It is especially important that this data classification be considered during the design of questionnaires. Many of you are familiar with this data classification model. For those of you who are not, I will provide a few brief examples.

SLIDE 5

This is nominal data or nose counting. It is frequency data allowing only the most basic statistical manipulations.

SLIDE 6

For example, here all you can say is that the OH-58 requires the most maintenance and the OH-6 the least in the opinion of 100 crewmen.

SLIDE 7

On the other hand, you could have asked for ordinal data, as in this example, rather than nominal.

SLIDE 8

The results would then have looked something like this: Here you would have seen that;

a. The OH-58 was identified as requiring the most maintenance by 35% of the crewmen (the highest percentage).

b. The AH-1 was identified as requiring the most maintenance by only 25% of the crewmen.

c. However, 50% of the crewmen identified the AH-1 as requiring the second most maintenance.

Therefore, with this level of data, you can conclude that the OH-58 and the AH-1 constitute a class by themselves in maintenance problems with the UH-1 and OH-6 requiring noticeably less maintenance than either of the other two aircraft. Please remember that these are synthetic data and do not represent actual results of questionnaires or tests.

SLIDE 9

In order to shift from ordinal to interval data, it is necessary that you restructure the questions from generalities to specifics. It therefore takes more questions and also more knowledge, both on the part of the questionnaire designer and the individuals answering the questionnaire. If you can tolerate the additional questionnaire size; if you have sufficient knowledge to write the questions; and if your subject population has sufficient knowledge to answer at this level; then the amount of information gained from your questionnaire will be enhanced considerably.

SLIDE 10

With interval data, you can obtain specific performance information on the items under test. Whether you're working with a questionnaire or other aspects of test design, you generally pay a penalty in additional test size and/or instrumentation for moving up the data scale. As a test designer, whether or not this penalty is worth paying must always be considered.

SLIDE 11

The difference between interval and ratio data in questionnaires is usually due to opportunity rather than construction. In taking these types of measures we seldom have any control over whether or not the scale of measurement uses a real or an arbitrary zero point. However, we should be sensitive to this point because proportions are meaningful only for ratio data.

SLIDE 12

This data classification system is especially useful in the preparation of questionnaires, but is also relevant to other test measures. Generally, the highest level of measurement practical should be used.

SLIDE 12 OFF

Probably the most distinguishing element of operational testing is its reliance upon traditional population statistics. All of the tools of operations research are appropriate and will occasionally be used in operational testing. However, computer simulation and war gaming are generally incompatible with the fundamental mission of operational testing, which is to test actual equipment in an actual mission environment. They will, therefore, seldom be used.

On the other hand, the testing of actual equipment in an actual environment is a test of a variable in a variable setting operated by various people. This is a classic definition of an area where sampling, and therefore population statistics, are appropriate. Thus, analysis of variance, chi square, "t" tests, etc. dominate the test designs. Correlational techniques on the other hand do not often lend themselves as well as statistical tests to making decisions among choices of equipment. However, correlational and regression techniques will be used to display relationships between equipment and performance parameters. This will often be crucial to the extrapolation of test data to untestable situations on equipment. For example, in a Force Development Tactics Evaluation, correlation and regression techniques might be used to extrapolate results from simulated to actual threat equipment based upon regressions of performance parameters.

In summary, the object of operational testing is to determine if equipment will serve its intended purpose while supporting the overall military mission when operated by the intended user under the expected environmental conditions. This imposes a number of restrictions on test design. These restrictions by and large emphasize the use of population statistics at the expense of gaming and computer simulation. Correlational techniques will be used primarily to extrapolate the data to untestable situations or equipment. Questionnaires are an essential part of operational testing. Using S. S. Stevens' model, the level of data obtained from the questionnaire should be as high as the length of the questionnaire and the knowledge of both the questionnaire constructor and respondent permit. Similarly the highest practical level of data should be obtained from all aspects of the test considering the penalties of test size and instrumentation sophistication which will be required.

WHAT IS OPERATIONAL TESTING?

A. USER AS TEST SUBJECT

B. REALISTIC TACTICAL SITUATION

C. REALISTIC MEASURES OF PERFORMANCE

SLIDE 1

COMPARATIVE VS DEFINITIVE MEASURES

COMPARATIVE = OBTAINING SUFFICIENT DATA TO ALLOW A SOUND CHOICE BETWEEN TWO OR MORE ALTERNATIVES.

DEFINITIVE = OBTAINING SUFFICIENT DATA TO ALLOW A DEFINITION OF THE PERFORMANCE OF THE EQUIPMENT.

SLIDE 2

GUIDES FOR CHOOSING MEASURES FOR OPERATIONAL TESTS

I. <u>DEFINE MISSION GOALS</u>

e.g. DESTROY THE ENEMY

STAY ALIVE

2. <u>DEFINE CRITICAL PERFOMANCE PARAMETERS</u>

e.g. SHOOT THE ENEMY BEFORE HE CAN SHOOT YOU

3. <u>TRANSLATE CRITICAL PERFORMANCE PARAMETERS INTO TEST MEASURES</u>

e.g. ELAPSED TIME TO FIRST HIT

SLIDE 3

TYPES OF DATA

NOMINAL = NOSE COUNTING

ORDINAL = RANK ORDER

INTERVAL = KNOWN AND CONSISTENT DIFFERENCE BETWEEN UNITS

RATIO = TRUE ZERO POINT

SLIDE 4

NOMINAL DATA EXAMPLE

QUESTIONAIRE ITEM:

WHICH TYPE OF AIRCRAFT DO YOU BELIEVE REQUIRES THE MOST MAINTENANCE?

- AH-1 ☐
- UH-1 ☐
- OH-58 ☐
- OH-6 ☐

SLIDE 5

RESULTS ON NOMINAL QUESTIONAIRE ITEM TO 100 CREWMAN (SYNTHETIC DATA):

TYPE AIRCRAFT	MOST MAINTENANCE
AH-1	25
UH-1	30
OH-58	35
OH-6	10

SLIDE 6

ORDINAL DATA EXAMPLE

ALMOST THE SAME QUESTIONAIRE ITEM:

RANK ORDER THE FOLLOWING AIRCRAFT ACCORDING TO MAINTENANCE REQUIREMENTS. A RANK OF 1 INDICATES MOST MAINTENANCE.

- AH-1 ☐
- UH-1 ☐
- OH-58 ☐
- OH-6 ☐

SLIDE 7

RESULTS OF RANKING QUESTIONAIRE ITEM GIVEN TO 100 CREWMEN (SYNTHETIC DATA):

	RANK			
ACFT	1	2	3	4
AH-1	25	50	20	5
UH-1	30	10	30	30
OH-58	35	10	30	25
OH-6	10	30	20	40

SLIDE 8

RELATIVE VALUE OF DIFFERENT TYPES OF DATA

INTERVAL: ALLOWS COMPARISON AND DEFINITIONAL MEASUREMENT.

E.T., (FROM EXAMPLE DATA)

	OLD TRANSMISSION	NEW TRANSMISSION
MEAN	326° F	400° F
STANDARD DEVIATION	25° F	20° F
NUMBER-CASES	15	12

T = 8.54

DF = 25

CL = .9999

DIFFERENCE BETWEEN MEAN IN TEMPERATURE TOLERANCE = 74° F.

DIFFERENCE IN FREQUENCY OF OCCURRENCE NOT SIGNIFICANT.

SLIDE 9

INTERVAL DATA EXAMPLE

WHAT WAS THE TEMPERATURE (F^{o}) OF THE TRANSMISSION AT THE TIME OF FAILURE.

RESULTS:

	OLD TRANSMISSION	NEW TRANSMISSION
MEAN	326^{o} F	400^{o} F
STANDARD DEVIATION	25^{o} F	20^{o} F
NUMBER FAILURES	15	12

SLIDE 10

RATIO DATA EXAMPLE

HOW FAR WERE YOU FROM THE SOURCE WHEN YOU WERE DETECTED BY RADAR?

RESULTS:

	AH-1G	AH-1S
MEAN	1500 METERS	1800 METERS
STANDARD DEVIATION	100 METERS	75 METERS
NUMBER DETECTIONS	15	15

SLIDE 11

RELATIVE VALUE OF DIFFERENT TYPES OF DATA

NOMINAL: ALLOWS ONLY SIMPLE COMPARATIVE MEASUREMENT

E.G., (FROM EXAMPLE DATA)

THE OH-58 REQUIRES THE MOST MAINTENANCE, THE OH-6 THE LEAST, ETC. IN THE OPINION OF 100 CREWMEN.

SLIDE 12

TITLE: An Analysis Technique for Helicopter Aim Error Measurements

AUTHORS: Dr. Robert S. Bennett, Falcon Research and Development Company
Mr. William T. Pibil, US Army Materiel Systems Analysis Activity

ABSTRACT: This paper develops a technique for estimating the mean time required for a helicopter to meet the aim requirements for launching an autonomous homing missile. The situation considered is that in which a pilot, operating the helicopter near hover, attempts to keep the longitudinal axis of the aircraft aimed toward an airborne target. It is assumed that a launch can take place given that the angular error between the helicopter axis and the line of sight to the target remains within a prescribed limit for a period of time sufficient for operator response.

This launch condition may be described in terms of the "zero crossing" problem often encountered in signal analysis, where a random function of time occasionally crosses a given threshold value. Statistical techniques for determining the frequency with which the random function remains below a specific threshold for a given period have been published, and are used here in analyzing the helicopter aiming requirements. The mean time required for meeting missile launch conditions is derived in terms of published zero-crossing statistics, through the use of the power spectrum and the moments of the aim error distribution. Input data are obtained from flight test data.

An example is given for a particular flight test sequence. The flight test data were extensive enough to permit a direct empirical determination of the mean aiming time, and this was used to evaluate the accuracy of the zero-crossing technique. Advantages of the zero-crossing technique include a significant reduction in the quantity of flight test data required for determining the mean aiming time for a given helicopter and target combination, with consequent saving in test time and expense for an equivalent result.

Knowledge of the mean aiming time is of importance in determining the suitability of a given missile system to helicopter launch, since the ability of the pilot to launch his missile is far more dependent on aiming accuracy than it is on the target signal intensity. Therefore this technique provides far more information for missile selection for helicopter armament than does the simple lock-on boundary often used for ground launch.

TITLE: An Analysis Technique for Helicopter Aim Error Measurements

AUTHORS: Dr. Robert S. Bennett, Falcon Research and Development Company
Mr. William T. Pibil, US Army Materiel Systems Analysis Activity

Introduction: Before an infrared-seeking or similar autonomous missile can be launched, the missile must be oriented such that the angular error between the line of sight to the target and the missile axis is within a prescribed limit. Furthermore, this angular error must remain within the limit for a period of time sufficient for operator response. In most cases, the error tolerance increases, and the operator response time decreases, as the signal on which the missile is to home becomes stronger. A parameter of particular importance is the time required for meeting the aiming conditions, since excessive aiming time may preclude a successful engagement. In the special case of an infrared missile launched from an airborne helicopter, the mean aiming time required depends on the signal-to-noise ratio at the missile detector, the skill of the operator of the missile system, and the ability of the pilot to accurately keep his aircraft aimed at the target.

This paper is concerned with a method for predicting the mean aiming time for an infrared-seeking missile launched from a helicopter against a fixed-wing target aircraft, as shown in Figure 1. The missile is assumed to have an initial aim error tolerance which is related to the signal-to-noise ratio in the manner indicated by Figure 2. The prediction method is based on some published zero-crossing statistics, and requires as input the power spectrum and moments of the aim error distribution for the helicopter in question. These inputs may be obtained from a relatively simple flight test of short duration.

A particular example is given, in which a UH-1M helicopter was used to track a target aircraft crossing the path of the helicopter at an angular rate of about five degrees per second. Aiming error was measured with a television-type optical tracker. This device produced analog output voltages, proportional to the elevation and azimuth angles between the helicopter longitudinal axis and the line of sight to the target. Tapes of these analogue voltages were subsequently used in determining the power spectrum and the moments of the aim error distribution. These data were then used as input for the analysis method to be described here. In addition, the mean aiming time was empirically determined from the time history of the aim point. This is a more time-consuming procedure, and requires flight tests of longer duration than the method described in this paper, but since sufficiently long flights were in fact made, it provided a means for checking the zero-crossing method.

Description of the Method: A representative time history of the helicopter aim error is shown in Figure 3. Analysis of this aim error, both in the case of the UH-1M flights and in previous tests involving helicopters aiming at ground targets, indicates that azimuth error accounts for nearly all of the total angular error. This is not entirely unexpected, considering the geometry of the experiments; a target crossing the path of a helicopter at approximately the same altitude. In addition, these

angular errors have been found to closely follow a Gaussian probability density in nearly all cases.

If the maximum allowable error, e_{max}, for which a missile launch is possible is indicated by lines at e_{max} on Figure 3, the result, shown in Figure 4, strongly suggests the classical "zero crossing" problem from signal analysis, The term "zero crossing" is something of a misnomer, since the classical problem is concerned with the statistics describing the occasional crossings of <u>any</u> fixed level of a random variable.

A particularly useful study has been made of some statistical properties of zero crossings of Gaussian processes. These properties were published in Technical Report AF-109 of the Carlyle Barton Laboratory of the Johns Hopkins University. ("Statistical Properties of Noise Pulses", A. J. Rainal, June 1964, U.S.Air Force Contract AF 33(657)-11029). This report considers the statistics of noise pulses which are produced when Gaussian noise is gated in such a way that no output is obtained if the noise waveform is below a fixed threshold, and the waveform is passed through whenever its amplitude is above the threshold. Among the statistics available from this report are the average durations of these noise pulses, and the average time between pulses.

In the analysis of helicopter aim errors, a similar situation exists in that the missile can be launched provided the angular error remains between two thresholds for a given period of time. Thus the data in AF-109 is applicable here.

In AF-109, data are given from which the time between crossings of a fixed threshold by a Gaussian process can be determined, as well as the duration that the process remains above the threshold. From these data, the graph shown in Figure 5 was plotted. This is a normalized graph, with the abscissa showing the threshold value in units of standard deviations of the Gaussian process, and the ordinate in units of the mean time between sign changes of the Gaussian process. In order to use Figure 5, given that the angular errors are Gaussian, all that is needed is the power spectrum and the standard deviation and the mean time between sign changes of the Gaussian process could be used, but the mean time between sign changes can be obtained from the power spectrum, and this is often more available.

If an "event" is defined as having occured when the error remains below e_{max} for a time equal to the operator response time, the average time between events can be read from Figure 5. That this can be done will be illustrated by the following example.

In the UH-1M experiments, it was found that the standard deviation of the aim error was 1.10 degrees. Error threshold values of 0.4, 0.8, 1.2, 1.6, and 1.8 degrees will be considered as representative of various requirements, for different signal to noise ratios, for some class of infrared missiles.

Now, consider the case where the operator response time is 0.5 seconds and the 0.8 degree threshold is being considered. An "event" is defined as having occurred whenever the error remains below 0.8 degrees for 0.5 seconds. When the duration of an interval during which the error remains below 0.8 degrees is 1.6 seconds, for example, the determination is made that three events have occurred. If the error remains below 0.8 degrees for 0.4 seconds, no event is recorded.

Now, the five threshold angles under consideration correspond to abscissa values of 0.364, 0.727, 1.091, 1.455, and 1.636 on the Figure 5. The corresponding values of the ordinate, for time intervals above and below the threshold, are shown in Table 1. Because of the symmetry of Figure 5, only the values for positive thresholds are shown.

Table 1. Normalized Zero Crossing Intervals

Abscissa	Normalized Time Above Threshold	Normalized Time Below Threshold
0.364	.380	0.69
0.727	.300	1.00
1.091	.251	1.46
1.455	.213	2.32
1.636	.195	3.20

To convert the normalized times in Table 1 to true times, it is necessary to multiply the normalized values by the mean time between sign changes of the Gaussian process. For example, in the UH-1M tests the mean time between sign changes was 4.325 seconds. This value was obtained by considering the power spectrum of the error, which was found to have a "shape" roughly proportional to the inverse of the frequency, for frequencies between 0.05 and 0.4 cycles per second.

Now, for a power spectral density W(f) of the form:

$$W(f) = \begin{cases} f^{-1} \text{ for } 0 < f < f_c \\ 0 \text{ otherwise} \end{cases}$$

and Gaussian distribution with mean value zero, as is the case with the "bias-free" data, the average number of sign changes of the error per second is given by:

$$N_I = \frac{f_c}{\sqrt{3}}$$

Since the "cutoff" frequency, f_c, for the aim error data is about 0.4 cycles, the average rate of sign change is:

$$N_I = 0.4/1.73 = 0.2312 \text{ per second,}$$
$$\text{or } 1/N_I = 4.325$$

Multiplying by this value, the entries in Table 1 become those shown in Table 2.

Table 2. Zero Crossing for Aim Error Data

Threshold in Degrees	Average Time Above Threshold in Seconds	Average Time Below Threshold in Seconds
0.4	1.644	2.984
0.8	1.298	4.325
1.2	1.086	6.315
1.6	0.921	10.035
1.8	0.843	13.841

Now, it must be remembered that the angular "window" within which a missile can be launched actually consists of two symmetric thresholds, one leading the target and the other lagging the target. To account for this, Table 2 must be modified. The Average Time Above Threshold column will remain as it is—the time above 0.4 degrees, for example, is precisely the same as the average time below - 0.4 degrees by the symmetry of Figure 5. However, when the error is below 0.4 degrees, part of the time interval may be time when the error is below - 0.4 degrees, and thus out of the window. On the average, the error will fall outside the window on the negative side once for every time it falls outside on the positive side. This means that the times in the Average Time Below Threshold column should be reduced by the times in the Average Time Above Threshold column, and the result divided by two. This has been done in Table 3, in which the pair of thresholds are designated as "windows".

Next, consider a hypothetical flight of duration 100 seconds. It is obvious that the number of intervals during which the aim error is within a window can differ from the number of intervals during which it is outside the window by no more than one. In this analysis the number of intervals inside will be taken to be exactly the same as the number of intervals outside. The total time for an average excursion consisting of one interval outside a given window and one interval inside will be the sum of the two values in Table 3

Table 3. Window Intervals for Aim Error Data

Window in Degrees	Average Time Outside Window	Average Time Within Window
0.4	1.644	0.670
0.8	1.298	1.514
1.2	1.086	2.615
1.6	0.921	4.557
1.8	0.843	6.499

for the window of interest. Thus the number of such excursions during the 100 second flight is:

Window	Number of Excursions
0.4	100/2.314 = 43.22
0.8	100/2.812 = 35.56
1.2	100/3.701 = 27.02
1.6	100/5.478 = 18.25
1.8	100/7.342 = 13.62

Now, for the five windows, the number and mean duration of the intervals during which the error is within the window are known for the 100 second flight. Also, the total time spent outside the window is known. These are shown in Table 4.

Table 4. Aim Error Statistics for a 100 Second Flight

Window	Number of Intervals Within Window "N"	Average Duration Interval Within Window "D"	Total Time Outside Window "T"
± 0.4	43.22	0.670	71.04
± 0.8	35.56	1.514	46.16
± 1.2	27.02	2.615	29.34
± 1.6	18.25	4.557	16.83
± 1.8	13.62	6.499	11.48

In order to obtain the mean waiting time between "events", as was done in the empirical analysis, the number of events in the 100 second flight must be calculated. This could be done by multiplying the number of intervals within a window by the average duration of the interval and dividing by the operator response time, were it not for the fact that some intervals are much shorter than average, and may not be long enough for an event. This is a particularly bothersome problem when the narrow window and long response time combinations are considered.

The distribution of interval durations about the mean follows, approximately, a Poisson distribution. This fact can be used in determining the percentage of intervals which should be added to the time outside the window in calculating the mean waiting time between events. This percentage can be expressed as a function of the ratio of operator response time to the average duration within the window.

Response Time / Average Duration of Interval R/D	Percentage of Intervals Containing no Event "P"
0.1	1
0.5	20
1.0	50
2	80
10	99

The total number of events, for a given window and response time, is therefore

$$\text{Events} = \frac{ND}{R} \quad (1 - \frac{P}{100})$$

where R is the operator response time

The total waiting time between events is

$$\text{Wait} = T + \frac{PD}{100}$$

The mean waiting time between events, which is the final result is then:

$$\text{Mean Waiting Time: } \frac{\text{Wait}}{\text{Events}}$$

For example, in the case where a window of $\pm$ 1.2 degrees, and an operator response time of 0.6 seconds is considered,

N = 27.02	D = 2.615	T = 29.34
R = 0.6	R/D = .229	P = 6.9

$$\text{Events} = \frac{27 \times 2.615}{0.6} \; (1 - \frac{6.9}{100}) = 109.6$$

$$\text{Wait} = 29.34 + (2.615 \times \frac{6.9}{100}) = 31.2$$

Mean Waiting Time = .28 seconds.

The complete set of mean waiting times, for fice windows and ten operator response times, is shown in Table 5. The full time required for missile launch is the sum of the mean waiting times in Table 5, plus the operator response time. This full time is shown in Figure 6.

Table 5. Mean Waiting Time Between Events

Response Time (Seconds)	Maximum Allowed Angular Error in Degrees				
	0.4	0.8	1.2	1.6	1.8
0.1	.25	.09	.04	.02	.01
0.2	.55	.18	.08	.04	.03
0.3	.88	.27	.13	.06	.04
0.4	1.33	.38	.17	.08	.05
0.5	1.81	.49	.22	.10	.07
0.6	2.64	.61	.28	.13	.08
0.7	3.45	.74	.32	.15	.09
0.8	4.48	.87	.38	.17	.11
0.9	7.84	1.05	.43	.20	.12
1.0	8.51	1.24	.49	.22	.14

For comparison, Figure 7 shows the full time required for missile launch as determined by direct measurement from a large number of test flights of the UH-1M. It will be noted that although the direct measurement required much more data than that required by the zero crossing technique, the results are in close agreement.

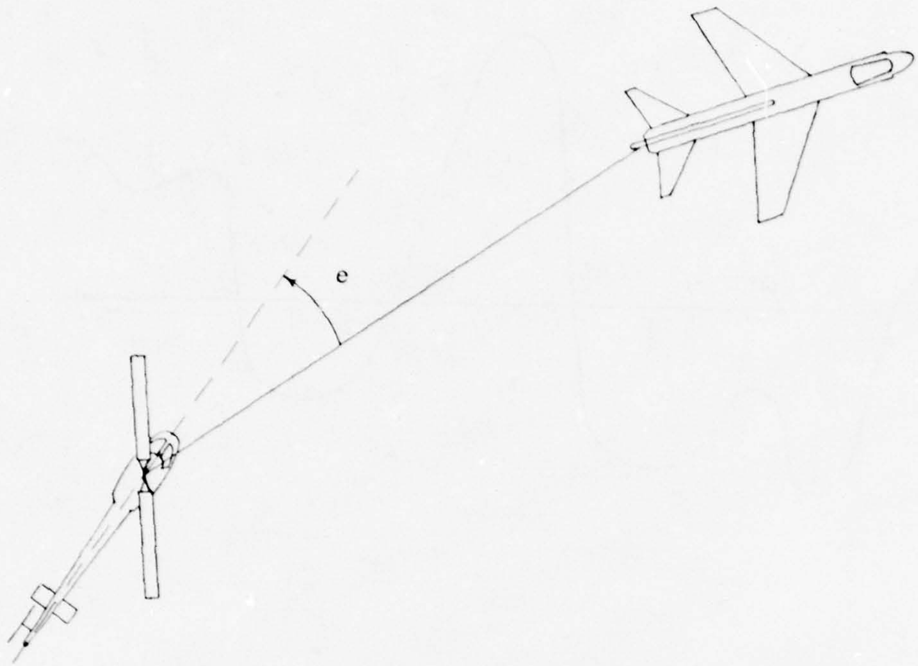


FIGURE 1. AIM ERROR

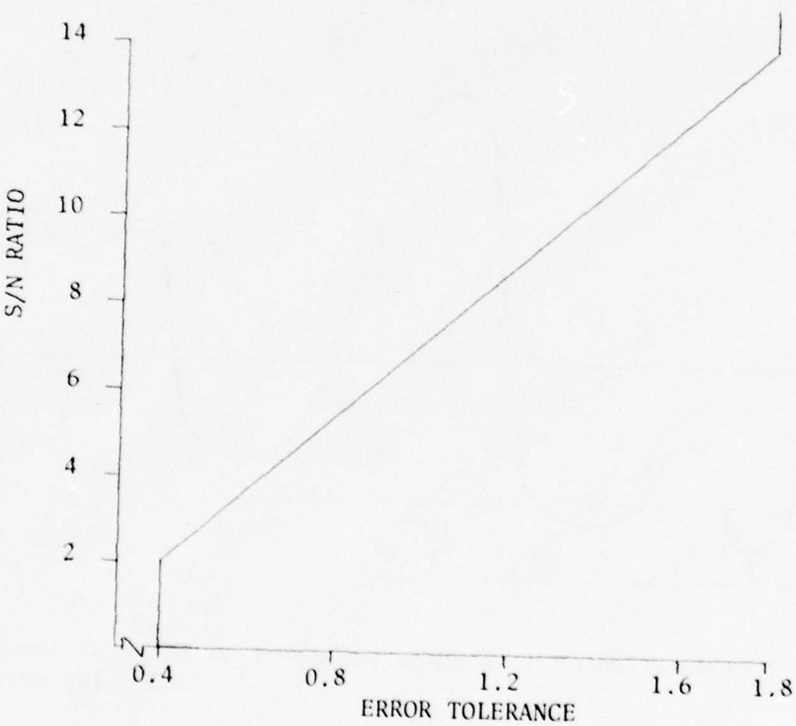


FIGURE 2. ERROR TOLERANCE VS S/N

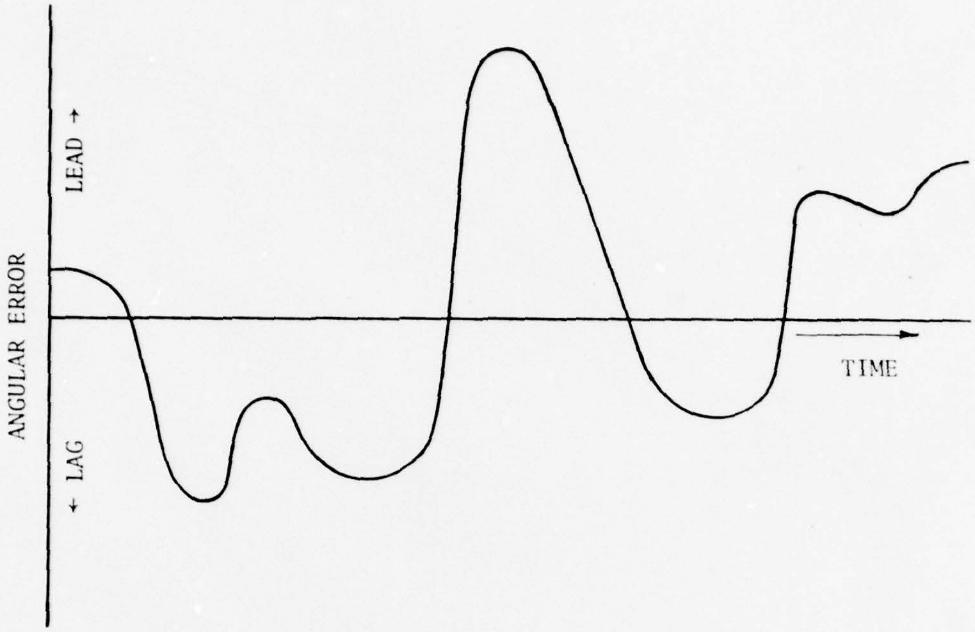
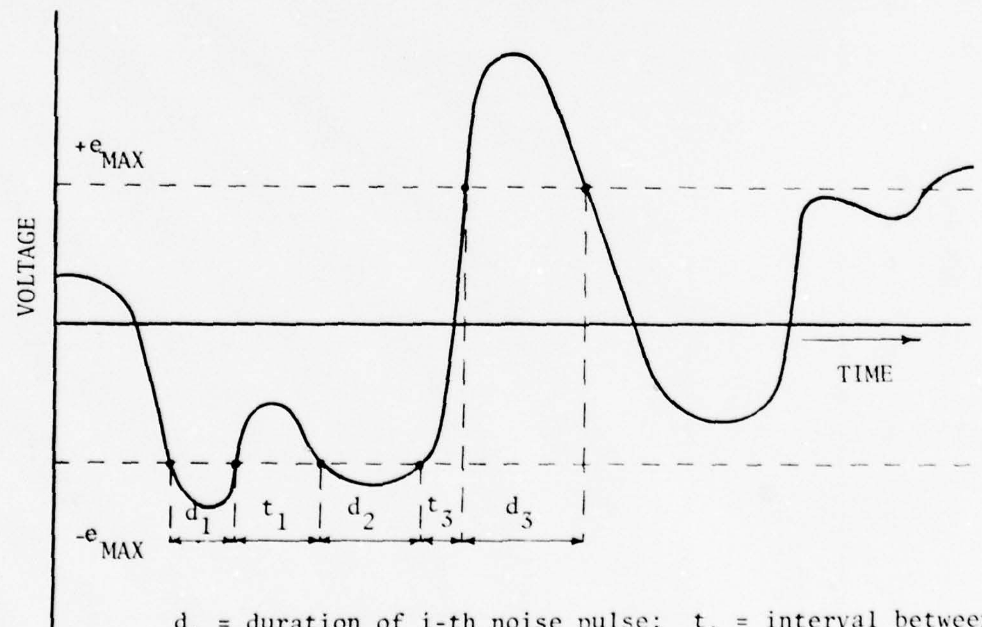


FIGURE 3. HELICOPTER AIM ERROR VS TIME



d_i = duration of i-th noise pulse; t_i = interval between i-th and i+1st noise pulse

FIGURE 4. CLASSICAL ZERO CROSSING PROBLEM

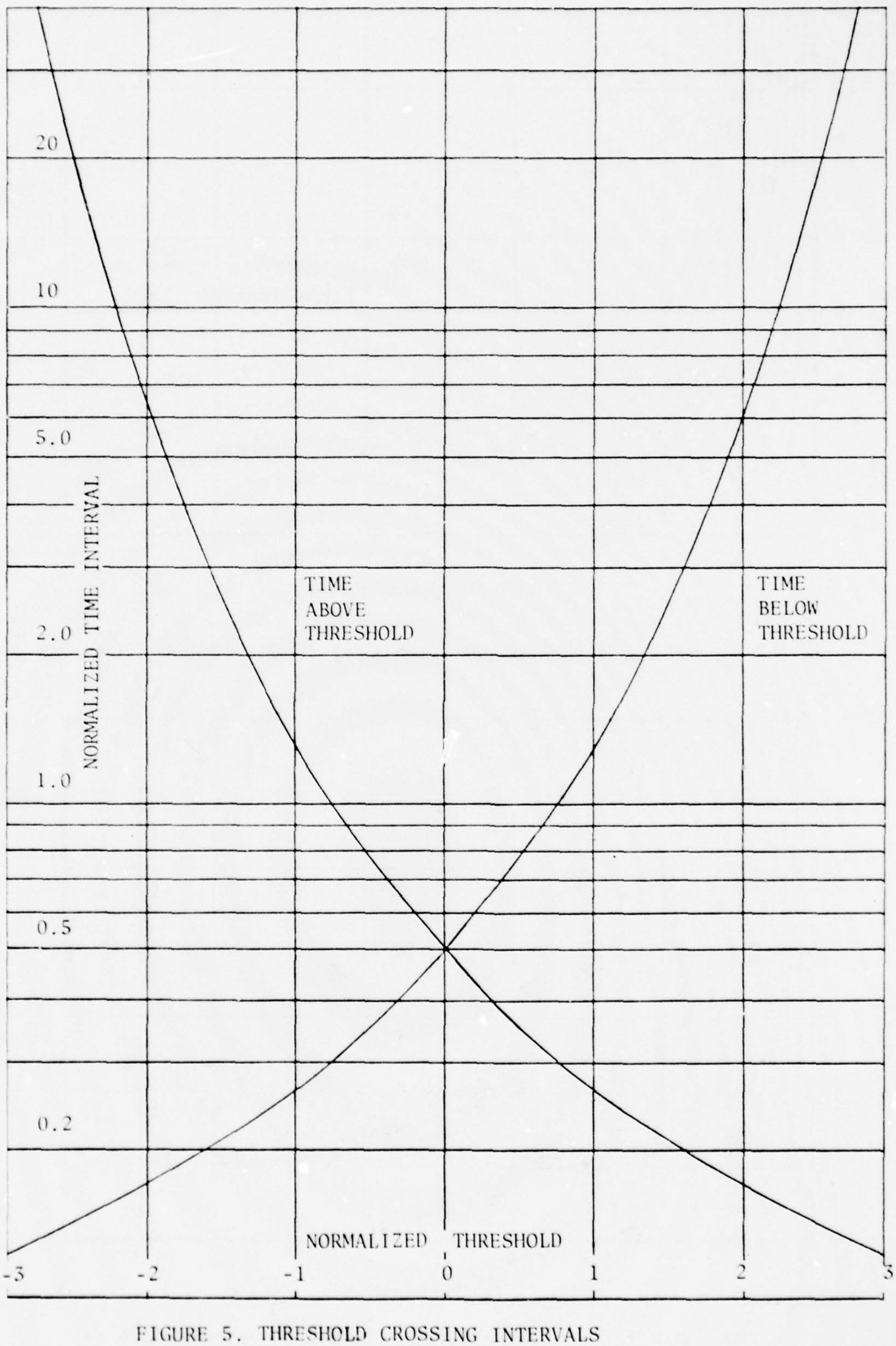


FIGURE 5. THRESHOLD CROSSING INTERVALS

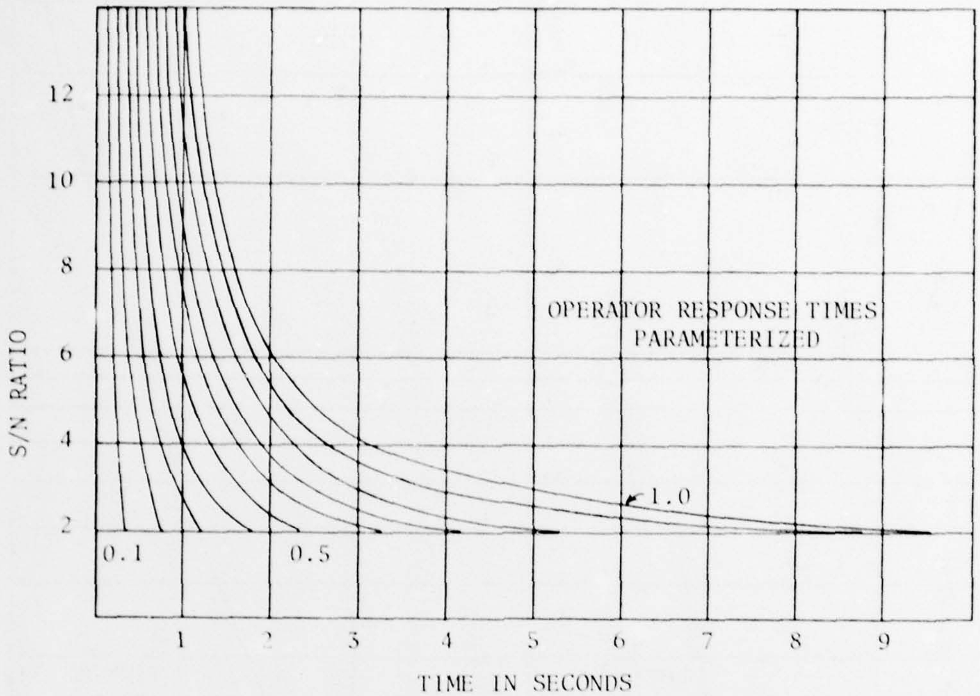


FIGURE 6 MEAN AIMING TIME CALCULATED BY ZERO CROSSING TECHNIQUE

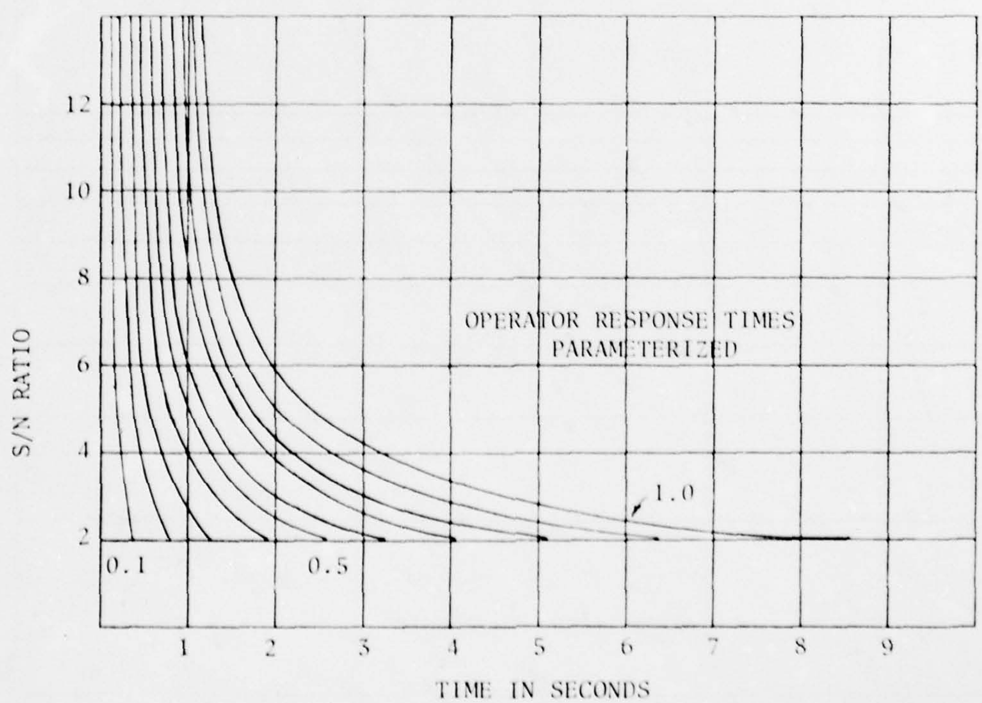


FIGURE 7. MEAN AIMING TIME OBTAINED BY DIRECT MEASUREMENT

TITLE: Decision Risk Analysis - AVSCOM Management Tool

AUTHOR: Mr. Paul B. Shapiro
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ABSTRACT: This new management tool uses a proven Army technical tool: Decision Risk Analysis (DRA). DRA is used to model major on-going Army programs. The model is then modified on a continuing basis to conform to changes in the program. Program cost, schedule and performance can be looked at simultaneously. Continuous exercising of DRA results in a flow of timely information to program decision makers. Program alternatives can be looked at far in advance; interrelationships of various program activities and various program alternatives can be studied. Program control is improved through the continuous tracking of the program. The objective of continuous use of DRAs is to provide program management with better insight into their programs leading to better-run programs accomplished in a more timely manner, at less risk, at lower cost. The tool has equal application to development programs and to readiness programs.

Decision Risk Analysis - AVSCOM Management Tool

Mr. Paul B. Shapiro

US Army Troop Support and Aviation Materiel Readiness Command

The purpose of this presentation is to inform you of a new management tracking tool.

The US Army Aviation Systems Command (AVSCOM), and the US Army Troop Support Command (TROSCOM), both in St. Louis, have been reorganized into two new, separate commands, the US Army Troop Support and Aviation Materiel Readiness Command (TSARCOM) and the US Army Aviation Research and Development Command (AVRADCOM), effective 1 July 1977.

The TSARCOM Systems Analysis Office (SAO) is presently starting to support a Readiness Project Office using this new management tool. The groundwork of the management tool, however, evolved in AVSCOM prior to the reorganization. This presentation will cover the work done at AVSCOM.

The purpose of this new management tool is to supply decision makers with a better insight into their programs, either in summary form, or in any desired level of detail. This tool has been demonstrated in the Advanced Attack Helicopter Program Management Office (AAH PMO), which is presently in development. A recent investigation showed that approximately four times as much money is presently being spent on aviation readiness than is being spent on aviation development. This, coupled with the fact that the tool is applicable to both readiness and development, led to the opinion that the tool should be even more useful in supporting readiness programs.

Future use of this tool should contribute to better programs accomplished in a more timely manner, at less cost. Below, we will first examine the background of the tool and then discuss Decision Risk Analysis (DRA).

Analysis has evolved within the Army and changed considerably during the past ten years. Program managers and office heads were often in the position of having to make timely decisions under conditions where very little information was available. Decisions were often made on the basis of intuitive judgments with personal feelings playing a significant part in shaping these decisions.

Small, manually manipulated networks evolved with the intent to supply decision makers with more information, but these networks had limited capability, or only looked at small parts of programs, or examined vastly over-simplified models of these programs. It is controversial whether these over-simplified small networks did more harm than good.

With the popularization of the use of modern computers, large networks came into use; but these large networks tended only to be used at major

milestones of large programs because of the slow pace of generating the networks, gathering the data and producing and analyzing the answers. It is only during the last year that large, automated networks have been used that are flexible to rapid changes in large programs. The main usefulness of these large, automated networks is the continuous supply of information to decision makers of large programs during times of rapid, major changes.

We will now address the more recent past and the treatment of risk. Until the last year or two, the number of risk assessment studies far out-weighed the number of risk analyses. Risk estimates resulted in tags of High, Medium or Low risk. Because of the fact that different programs can accept different levels of risk and also because High, Medium and Low have different connotations to different people, risk assessments sometimes led to differences of opinion on what the results of the study really meant.

Risk analysis, with its attendant numerical estimates, led to better communication of the results of studies. People understood that a 10% risk meant that the chance was one in ten that a program would fail.

DRA, in the DARCOM community, has slowly but surely replaced both risk analysis and risk assessment, except in areas such as performance analysis. DRA is a highly structured method that is sophisticated and powerful. DRA is capable of examining a very large program, essentially without any loss of detail, given the time, money and necessary dedicated effort. The usefulness of DRAs has now increased to the point where a major Army contractor has requested use of this Army tool to track a major Army program. It can truly be said that recent Army efforts in this area represent the state of the art.

Before discussing DRA itself, most recent AVSCOM DRAs will be briefly discussed. In 1975, DRAs still mostly used small networks and were only sporadically exercised. The major DRA in 1975 was an examination of a large number of small programs being set up by the Aircraft Survivability Equipment Program Manager (ASE PM). By mid-1976, AVSCOM had moved on to large DRA networks. These networks were being used for major milestones only. An example of this is a network containing 317 arcs and nodes used in support of the AAH PM Development Concept Paper (DCP). The AAH PM was phasing into advanced development, and as a result of some major funding constraints, had to examine eight major program options in detail. A result of the necessity to rapidly accommodate new information and report the main results to the PM with turn-around times as short as 24 hours in some cases, forced automated processing of new information, and rapid modification of large existing programs. Both analysis of the results and verification of network accuracy had to be accomplished in an extremely short time.

Large DRA programs historically had turn-around times that ranged from three to four months. When support of the AAH PM started, initial program set up and turn-around time of what turned out to be Option 1, was 30 days. Turn-around time decreased as further options had to be looked at.

Several things were necessary for the type of operation discussed above. These were: 24 hour a day, 7 days a week availability of an IBM 360-65 computer, high computer priority, availability of technical people that did not have to be trained during the exercise, availability of necessary technical personnel when needed, and sufficient funding for necessary overtime.

The team conducting the DRA itself never exceeded three people. Access, however, to people in the AAH PMO, to people at the AAH Source Selection Evaluation Board (SSEB), and to people in other PMs such as Hellfire, and various experts in the Army, had to be assured through the support of the AAH PMO.

The above rapid response to many alternatives demonstrated that DRA is capable of continually tracking a large program undergoing routine changes. This in turn led to an expressed desire by AAH PM personnel that the Systems Analysis Office (SAO) set up continual tracking of their program. In addition, a major change was also requested: the DRA used for continual tracking should have schedule, cost and performance incorporated into one network. In the past, with the exception of work done in Rock Island by Mr. Gerald Moeller and associates, cost and schedule have been separately tracked. In addition to separate tracking of cost and schedule, the work has usually been performed by separate offices.

Hughes Helicopter is the prime contractor for the AAH. The program manager at Hughes has requested that the AAH DRA model be made available to him so that Hughes can insert their own estimates of risk into the model and use the results to discuss the program with Army personnel.

We will now discuss DRA itself. DRA is a mature, proven tool throughout the DARCOM community. This tool is now being put to a new management use: continuing management visibility into on-going programs.

DRAs can supply information connected with two management functions: planning and control. Because of the nature of DRA, planning can be done far in advance of specific events. Interrelationships of various activities can be studied. Older computer programs utilized several parallel sequences of events to simulate programs. Our DRAs are networks; a change in one event can affect many other events in a network. In addition, we are using a specific computer program, VERT, that is capable of simultaneously processing cost, time and performance information. Considering cost, time and performance: each characteristic can be made to drive either or both of the other characteristics. There are other computer programs that look at cost, time, and possibly performance, but these characteristics are processed in series. One usually cannot drive the other.

Any part of the VERT network can be expanded to essentially any desired detail in order to focus on critical events. Alternatives can be explored either in series or parallel. Iterative use of a network can be used to establish priorities of management attention.

In the control area of management: periodic review and update of programs can surface trends that tend to flag bottlenecks in the program. As an example, if it is found that a small change in an estimated value changes the critical path from one series of events to another, a hidden bottleneck may surface and be able to be flagged for management attention as a potential trouble spot. Parametric or senstivity analysis can also lead to improvements of a program without having to go through major program changes.

The Army has used a number of computer programs to implement DRA. Computer programs have become more versatile and powerful with time. The computer program in most common use in the Army today, Venture Evaluation and Review Technique (VERT), represents the state of the art both in the Army and in industry. VERT was developed by Mr. Gerald Moeller of ARRCOM. This model is being used in a way that it is intended for: communication of results and recommendations to decision makers. What is new is that the model is being used to continually track the status of major programs instead of being used on a very infrequent basis.

Conducting a DRA classically includes six main functions. The six functions are: 1.) Defining the objective of the DRA, 2.) Development of a network, 3.) Obtaining input to the network, 4.) Running the computer program, 5.) Analyzing the output, and 6.) Communication with the decision makers on the program.

The objective of a DRA may be obvious or may take considerable discussion with various levels of management and working levels in the office being supported. After the objective has been defined, a network is developed. This network must be an adequate model of the program, problem or specific situation being examined. It is usually necessary to communicate what the network represents with working level people in the office being supported.

The third step in a DRA is the most critical. The limitation of accuracy of a DRA resides in the accuracy of the input numbers to the network itself. This input data may be obtained from the office being supported or may be obtained partially or wholly from people outside the office, outside the command, or even outside the Army, dependent upon the rules established as part of defining the objective of the DRA. The next step, running of the computer program, is the most routine step.

Analysts running the program analyze the output in detail. This analysis has two objectives and results: 1.) Verification that the network is error-free, and 2.) Understanding each result at each point in the network. The last step is reducing of the significant data and communication of results to working level personnel, and then to decision makers.

The results may be in extreme summary form such as one graph or listing, or may be in great detail, explaining results, reasons for each result, and massive back-up documentation.

The six steps above were discussed as if they happen in series. The normal occurrence, however, is that most of the steps are iterative. Results

of a DRA usually lead to efforts by personnel in the office being supported to either improve the program or examine a specific area of the program. The network is accordingly modified; results are obtained, analyzed and communicated.

A worthwhile DRA depends upon constant communication between the analysts conducting the DRA and the customer. Looking at some of the tasks in an iterative manner leads to better, more realistic input and more insight into the program itself, possibly leading to changes in the program.

Communication of DRA results has matured in AVSCOM. In-depth analysis of computer runs results in verified accuracy of the output, better understanding of the output, and sometimes identification of subtle problem areas that were previously unknown. DRAs are now reported in two phases: fast reaction, summarized output followed by full documentation of the DRA.

Full documentation gives complete traceability of the network itself and the input to the network. All input data is identified by source or sources and by date or dates obtained. Any future questions on the DRA by the Command itself, by DARCOM, DA or DOD, can be fully answered because of this complete traceability.

In conclusion, it is felt that DRAs presently contribute to better management insight. This "better insight" should contribute to better-run programs, run in a more timely manner, with less risk, at less cost.

ABSTRACT

TITLE: Weapon Slices--An Incremental Approach to Force Design

AUTHOR: Mr. Robert C. Spiker
US Army Concepts Analysis Agency

ABSTRACT: A methodology has been developed which can ascribe theater-level support requirements to individual major weapons in the combat elements of a theater force. The methodology uses the FASTALS Model to roundout a base case combat force. Then, by deletion of the weapons category of interest and its directly associated personnel and equipment, a reduced troop list is produced by FASTALS. The difference in troop lists is, for example, the "tank" slice. A method has been developed to permit definition not only of the tank slice but of an "XM1" slice. The slice methodology provided the incremental data used for the force structuring mathematical programing in the CONAF IV, CONAF V, and TRANSFORM studies. The TRANSFORM application using goal programing is presented in another paper of these Proceedings: "The Use of Goal Programing in the Theater Level Design of Forces," by COL Robert E. Robinson and LTC Edward E. Hildreth, Jr. The general methodology has been applied to determination of the shipping weight of a single weapon and its total required support. A costing methodology provides recurring and nonrecurring cost coefficients and constraints. The slice approach also has potential real world applications such as estimating the gross personnel effects of adding a helicopter company to Seventh Army or of converting an infantry battalion to a tank battalion.

SUBJECT: Weapon Slices--An Incremental Approach to Force Design

AUTHOR: Mr. Robert C. Spiker

AGENCY: US Army Concepts Analysis Agency

INTRODUCTION

A weapons slice, what is it? Until about two and a half years ago the terms "weapon slice" and "system slice" were not a part of the force designer's vocabulary. Until that time he had known a model called the battalion slice; he had described forces in terms of division slices. The weapon slice, however, would permit a new incremental approach to the design of forces. An application of the weapon slice methodology in a force structuring study is presented elsewhere in the Proceedings in a paper entitled: "The Use of Goal Programing in the Theater Level Design of Forces," by COL Robert E. Robinson and LTC Edward E. Hildreth, Jr.

BACKGROUND

The CONAF series of studies (CONAF is the acronym for Conceptual Design for the Army in the Field) have considered conceptual designs for the division forces as part of a US Theater Army operating in the central region of NATO in the far midrange time frame (about 12 years after the study period).

In CONAF IV, the methodology for conceptual force design was expanded to use a systems approach. Major materiel systems such as tanks were considered the force design variables. System slices were developed to include all combat personnel, support personnel and workloads that were associated with the changing size of a given materiel system. For a formal definition of the system slice the following will serve:

> A system slice is a weapon category in a force, defined in terms of all combat and support personnel, equipment and costs associated with the weapon category directly and indirectly which enable the slice weapons--and only those weapons--to function in combat.

A weapon slice is similarly defined except that the above qualifying terms are applied to a single weapon type, "the XM1 slice," rather than to a group of like weapons. An individual weapon slice, the slice associated with a single XM1 tank, is simply the weapon slice divided by the number of weapons in it. Before discussing the methodology of calculating weapons slices or examining the data that is obtained using this methodology, let us address the uses to which slices have been put.

a. In CONAF IV system slices were calculated which included the total number of weapons of a given type, for instance all the tanks or all

the artillery pieces. These system slices were used to calculate the coefficients and constraints which were used in the CONAF IV linear program methodology. For CONAF IV the approach was limited by the newness of the tool and the lack of understanding of the total capabilities inherent in slice methodology. Only six slices were defined: a tank slice, an armored personnel carrier (APC) slice, an artillery slice, a helicopter slice, a mortar slice, and an antitank guided missile slice. The theater population included in these six slices was only 40 percent of the total theater strength. The remainder of the theater was considered to be in a residual slice which was not useful for force design purposes.

b. The CONAF V study was directed at answering the question, "What can be varied to change the productivity of the force within resource constraints?" The most desirable approach would have included a capability to trade individual weapons for other weapons of the same type; e.g., to trade 10 new type tanks for 40 older but product improved tanks. CONAF IV could not do this since there was no way of differentiating the productivities of two different weapons which existed in the same system slice. CONAF V could not do this either. Progress was made, however, in developing the slice methodology in CONAF V. The number of slices was increased by two. An infantry slice was defined and an APC mounted TOW slice was manually separated from the overall APC slice. This was the beginning of the development of individual weapon slices. In addition, a methodology for defining separately Active and Reserve Component slices was developed. Costing procedures were improved so that costs of the Active and the Reserve Component slices for each system would reflect the differences in costs of the Active and Reserve forces. Through improved slicing techniques and the definition of the infantry slice, the residual slice was reduced to 35 percent of the total theater force.

c. For the TRANSFORM (Trade-Off Analysis Systems/Force Mix) Study,* the weapon slice methodology was developed and refined so that the use of individual weapon slices was possible. In addition, a nonweapons system slice, the Theater Level Support slice, was defined to include those personnel whose presence in theater is independent of the size of any weapon system, i.e., those personnel such as theater headquarters personnel who are there simply because the theater is there. An air defense systems slice was defined for TRANSFORM but it was "fenced" and its resources were not made available to the force design program. The costing methodology was further improved to provide not only recurring and nonrecurring costs separately, but also to provide two separate types of nonrecurring costs, modernization and activation. Base level or modernization costs refer to the estimated costs to achieve slice or force modernization by the base design year through currently planned equipment modernization or replacement; activation costs reflect the estimated cost to purchase newly activated slices or fractions thereof which are in excess of currently programed levels.

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*TRANSFORM applied the slice methodology and mathematical (goal) programing to evaluate the Division Restructuring Study (DRS) heavy division and the Aviation Requirements for the Combat Structure of the Army III (ARCSA III) Study.

d. The development of the weapon slice methodology and the concurrent improvements in slicing and costing techniques added new mathematical perspectives to the art of force design in the CONAF and TRANSFORM studies. The individual weapon slice is indeed available as a tool for the incremental design of forces and for many other applications only now beginning to be investigated.

DERIVATION OF SYSTEM SLICES

The exact size and other attributes of any slice will vary somewhat depending upon the base case from which the slice is extracted. Even using the same base case force the slice size will vary depending upon the time during the deployment of the force that the slice is defined, constraints that may be in effect at the chosen time, maturity of the theater, and other variables. Slices may be subdivided, as was done for TRANSFORM, into Active and Reserve Component slices. M-day and D-day slices may be calculated. Slice weights may be determined and analyzed in terms of the associated strategic lift required. Supply consumption or maintenance requirements may be evaluated. These and other quantities may be developed using the methodology and presented in any format that is useful to the force designer.

The first step in developing the system slices is to count the numbers of each type of weapon authorized the base case force at any time period of concern. This requires some military judgement to identify those weapons which contribute to the firepower potential of the force under normal combat conditions. For example, while most APC have some armament, many are used for maintenance tasks in Armor units or in Engineer units and normally do not engage in combat; these should not be included in the slice weapon count. Next the individual weapon types are grouped into weapon systems which are convenient for handling in the Force Analysis Simulation of Theater Administrative and Logistics Support (FASTALS) Model and which will secure a proportionate share of command, control and support personnel through normal functioning of the model. FASTALS rounds out an input combat troop list through development of associated work loads and application of appropriate allocation rules to provide a balanced troop list which includes all units in the theater.

For TRANSFORM the Active and Reserve Component system slice weapons categories and the numbers of individual weapons types in each are shown in Table 1. A major difference between this list and the systems slices used in CONAF V is the definition of the lightly armored tracked vehicle (LATV) slice. The LATV slice contains the infantry fighting vehicle/cavalry fighting vehicle (IFV/CFV), formerly the MICV/TBAT; the improved TOW vehicle (ITV); the armored reconnaissance airborne assault vehicle, (ARAAV), the M551, SHERIDAN; and the armored personnel carrier (APC), M113A1. As will be seen later, the APC was treated as an individual weapon slice in TRANSFORM and the other weapons in this slice were grouped together in a composite slice called the light antitank tracked vehicle (LATTV) slice.

Table 1. Weapons and Weapon Systems

Weapon type	Numbers of weapons		
	Active units	Reserve units	Total
Tanks			
XM1	1,773	0	1,773
M60A2	378	0	378
M60A3	1,493	2,087	3,580
Lightly Armored Tracked Vehicles (LATV)			
Improved TOW Vehicle (ITV)	1,996	1,116	3,112
IFV/CFV[a]	1,677	0	1,677
M551 Sheridan	933	207	1,140
M113A1 APC	4,411	3,220	7,631
Artillery			
XM 204-105T	126	378	504
XM 198-155T	288	414	702
M109A1-155SP	870	474	1,344
M110E2-8-in SP	412	320	732
Antitank Guided Missiles (ATGM)			
TOW (ground mount)	384	1,062	1,446
DRAGON	4,159	3,333	7,492
Helicopters			
AAH	234	0	234
AH1Q/S	185	270	455
AH1G	157	165	322
Mortars			
81mm	666	270	936
107mm	799	644	1,443
Light Weight Company Mortar (LWCM)	192	531	723
Air Defense			
Nike Hercules	96	0	96
HAWK	330	0	330
CHAPARREL	360	192	552
VULCAN	408	192	600
REDEYE	1,542	1,264	2,806
Infantry			
Infantry individual weapons	25,929	30,252	56,181

[a]Name changes distributed by ODCSOPS on 2 June 1977 changed: MICV/TBAT II (TOW BUSHMASTER ARMORED TURRET) to Infantry Fighting Vehicle (IFV), and MICV/Scout (also TBAT) to Cavalry Fighting Vehicle (CFV).

The division of the weapons and weapon systems into Active and Reserve Component parts was based on the following logic. Active Army and Reserve Component (RC) units differ particularly in two important attributes--readiness and costs. In TRANSFORM, the Transportation Model (TRANSMO) was able to deliver most Active Army combat units and a few RC brigades by D+34. The first RC division did not arrive until D+56. This provided a clear break-point in the FASTALS roundout for definition of the Active slice and the Reserve slice. In the FASTALS roundout, all units and equipment arriving and all workloads generated before D+39 (the end of the 6th FASTALS time period) were considered part of the Active slice; everything thereafter, part of the Reserve slice. It is recognized that more Reserve units, particularly service support companies and battalions, were counted as part of the Active slice than vice versa. It would have been impossible to identify these units at unit identification code (UIC) level in TRANSFORM, but using the 40th day dividing line gave a surprisingly good representation of the real world division of the Active Army - Reserve Component strengths. In TRANSFORM, 58 percent of the force was counted as Active Army; if based on present force composition, it might have been closer to 45 percent for a 90-day war.

The system slice approach envisions treating the whole slice as a variable of design. Development of the system slices in TRANSFORM started with the weapons as shown in Table 1. The next step in defining each system slice was to determine the logistic roundout associated with the slice. To do this the complete combat portion of the base case was identified, and its combat activity in an appropriate scenario was determined by CEM. CEM output such as combat activity were input to the FASTALS Model. This simulation computed workloads based on anticipated intensity of combat together with the buildup of the force over time. Logistic support units and personnel were developed as a product of these computations. From this process the total force for the TRANSFORM base case was defined.

To identify the system slice components of that total force a simple artifice was employed. All of the previously identified combat systems were removed for each slice one at a time. The net combat force was then input to the FASTALS model. The resulting roundout force had a lower combat population than the base case, had less workload involved and therefore less support units. For each slice case and for the base case the total force populations and costs were computed. The omitted slice results were compared with the base case results. The differences were attributed to and related to the system slice. It is assumed that as the slice size changes the derived workload demand, population and cost will change proportionately. The TRANSFORM base case differed little from the CONAF V base case in some weapon systems--helicopters, mortars, and infantry--and these CONAF V system slices were used for TRANSFORM.

As noted earlier in defining the system slices, it is not possible to associate all of the personnel and units in the theater with one and only one of the weapons systems. Some units, such as division headquarters, are there to command all the weapons systems in the division and cannot function as a partial unit. Other units, the theater headquarters, for example, are there because the theater is there and have no relationship with the size of a particular weapons system. The functions of some

units and personnel are not very sharply defined and, for TRANSFORM, these personnel were allowed to remain in the unassigned or variable residual slice. Nevertheless, the residual slice was reduced to only 18.6 percent of the force. *It is expected in the future to reduce this further* or, as a minimum, to make some part of it available to the force design process.

DERIVATION OF WEAPON SLICES

FASTALS is both a necessary and sufficient model in the slice methodology for development of system slices, but certain model characteristics make it less than satisfactory for direct calculation of most weapon slices. The model logic does not readily permit description of very small slices. FASTALS only assigns whole units which implies, therefore, the use of rounding rules (round up, round down, or round up over 0.5). Moving just across one of the rounding steps can cause a considerable distortion in the troop list for a small slice. A second major problem in using FASTALS directly for calculation of the large number of possible individual weapon slices is the workload associated with the preparation of the input data for each FASTALS simulation. As a minimum, manually input and D-day units must be adjusted, divisional and brigade strengths must be changed, and POMCUS recalculated for the scenario. For the masterfile, unit strengths, weights and requirements for automotive and aircraft maintenance must be calculated and entered, allocation rules changed, and rounding rules adjusted if necessary.

Personnel strengths and constraints are a major concern in any force design project. At the individual weapon level of force design, development of individual weapon personnel slices is a requirement. FASTALS is used to determine the personnel values for fairly large composite slices of closely related weapons, the system slices. These system slices are then divided into individual weapon slices based on known direct personnel requirements peculiar to the weapon. The remaining personnel in the system slice are allocated among the weapon slices on the basis of numbers of weapons or some similar basis for dividing these indirectly associated personnel. In this manner each individual weapon slice receives a fair proportion of command and control personnel, corps engineers, terminal service company support and other indirect supporting personnel. For TRANSFORM, this methodology was applied to system slices calculated at several specific times during the FASTALS simulation. The Active Army and Reserve Components (RC) weapon slices were calculated from system slices derived for D+39 and the end of the simulation. Table 2 lists the Active Army, RC, and end-of-war or doctrinally supported personnel slices for each system, weapon type and individual weapon in TRANSFORM.

a. All slices reflect some of the characteristics of the base case from which they were developed. In general the slice for a RC weapon is slightly larger than the slice for an Active weapon. This is true because the arrival of many of the lower priority units for a fully supported theater are deferred until late in the simulation. As an exception, Active attack helicopter slices are larger than reserve slices because of the large numbers of assault-type helicopters and supporting personnel in the Active slice.

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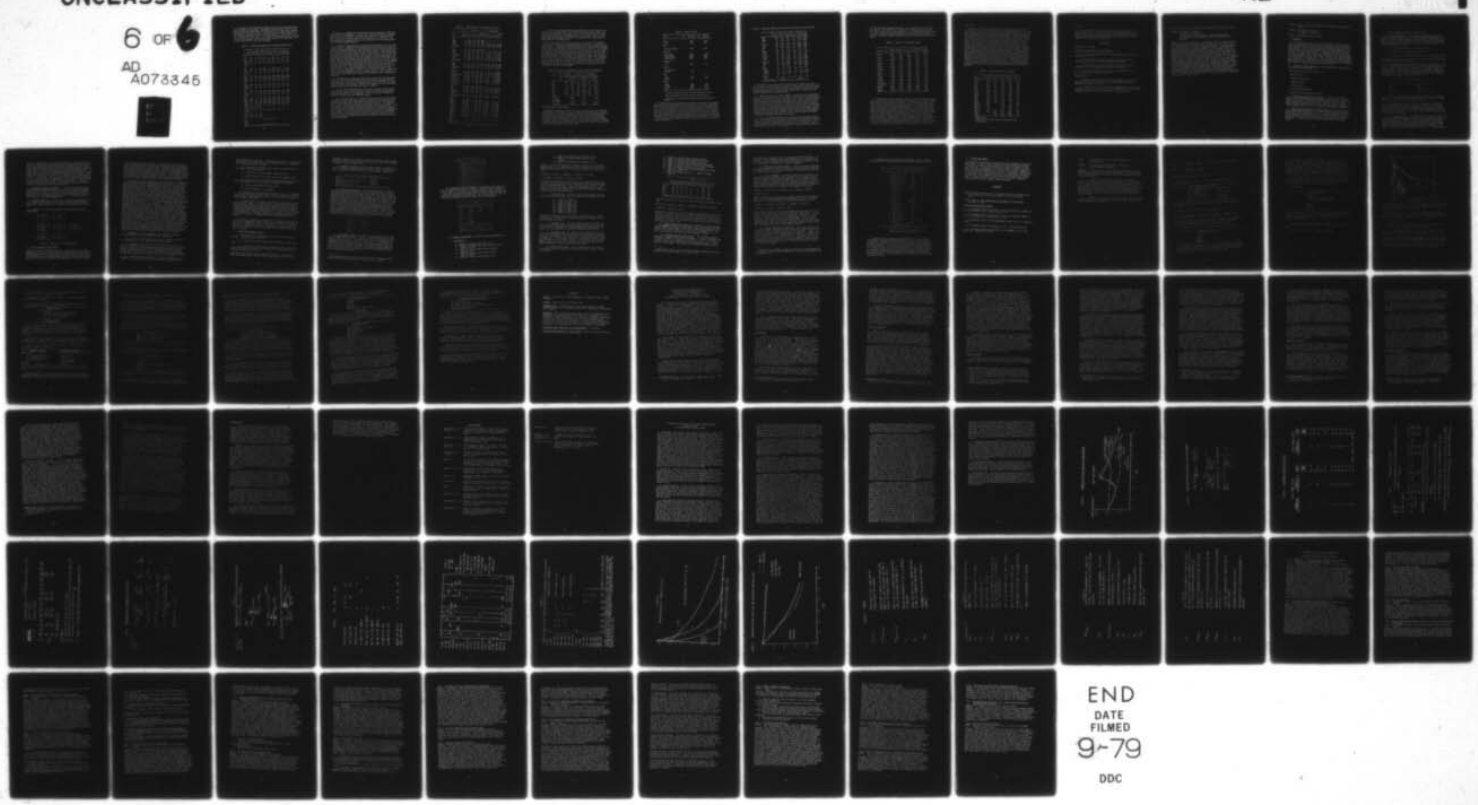

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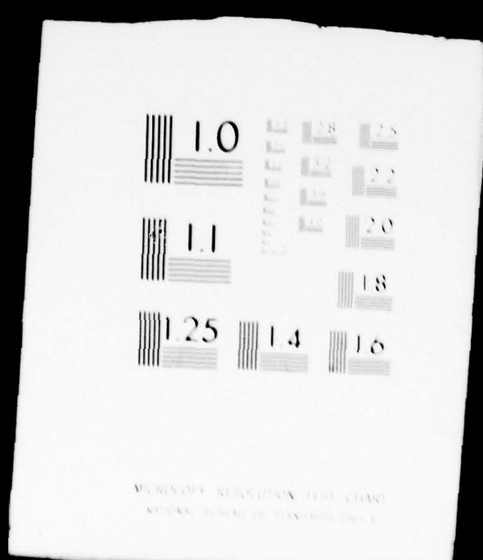

b. Not all of the individual weapon slice coefficients were used in the TRANSFORM goal programs. One that was of particular value for the Division Restructuring Study (DRS) force design was the light antitank tracked vehicle (LATTV) slice. This was not an individual weapon slice but rather a composite of three individual weapons - the IFV/CFV, the ITV, and the M551 light tank. It illustrates the powerful tool available in individual weapons slices which allows synthesis of composite slices at the convenience of the force designer.

Table 2. Individual Weapon Slice Personnel Coefficients

Slice	Number of weapons			Personnel per slice			Personnel per weapon		
	Active	Reserve	Fully supported	Active	Reserve	Fully supported	Active	Reserve	Fully supported
LATV									
IFV/CFV	1,677	0	1,677	15,973	0	17,761	9.525	0	10.590
ITV	1,996	1,116	3,112	19,011	14,182	32,959	9.525	12.707	10.590
M113A1	4,411	3,220	7,631	33,141	34,415	65,557	7.525	10.707	8.590
M551	933	207	1,140	10,752	3,044	14,358	11.525	14.707	12.590
Total or average	9,017	4,543	13,560	76,927	51,704	130,631	8.753	11.361	9.634
LATTV (LATV less M113)	4,606	1,323	5,929	45,736	17,226	65,074	9.930	13.020	10.972
Tanks									
XM 1	1,773	0	1,773	29,312	0	29,883	16.533	0	16.854
M60A2	378	0	378	6,249	0	6,371	16.533	0	16.854
M60A3	1,493	2,087	3,580	24,683	36,348	60,339	16.533	17.416	16.854
Total or average	3,644	2,087	5,731	60,245	36,348	96,593	16.533	17.416	16.854
Artillery									
XM 204	126	378	504	6,370	20,478	26,545	50.556	54.174	52.668
XM 198	288	414	702	14,925	23,079	37,835	51.624	55.745	53.896
M109A1	870	474	1,344	47,420	26,612	74,427	54.505	56.144	55.378
M110E2	412	320	732	27,572	21,654	49,502	66.923	65.292	67.625
Total or average	1,696	1,586	3,282	96,287	92,022	188,309	56.773	58.021	57.376
ATGM									
TOW	384	1,062	1,446	2,240	7,188	9,100	5.833	6.769	6.293
DRAGON	4,159	333	7,494	5,543	7,562	13,433	1.333	2.269	1.793
Total or average	4,543	4,395	8,938	7,783	14,750	22,533	1.706	3.356	2.521
Helicopters									
AAH	234	0	234	9,714	0	9,358	41.51	0	39.99
AH1Q	185	270	455	7,600	10,254	18,196	41.51	37.98	39.99
AH1S	157	165	322	6,517	6,267	12,878	41.51	37.98	39.99
Total or average	576	435	1,011	23,911	16,521	40,432	41.51	37.98	39.99
Mortars									
LWCM	192	531	723	2,630	8,877	10,892	13.696	16.717	15.065
81 Grd	36	0	36	493	0	544	13.696	0	15.115
81 SP	630	270	900	9,258	4,784	14,503	14.696	17.717	16.115
Total 81	666	270	936	9,751	4,784	15,047	14.641	17.717	16.076
107 Grd	48	236	284	729	4,299	4,719	15.196	18.217	16.616
107 SP	751	408	1,159	10,662	7,024	18,098	14.196	17.217	15.615
Total 107	799	644	1,443	11,391	11,323	22,817	14.257	17.583	15.612
Total or average	1,657	1,445	3,102	23,772	24,984	48,756	14.346	17.290	15.718
Infantry (individual combat weapons)									
Total or average	25,929	30,252	56,181	56,181	86,890	143,071	2.166	2.872	2.547
Air Defense									
Nike Herc	96	0	96						
HAWK	330	0	330						
CHAPARREL	360	192	552						
VULCAN	408	192	600						
REDEYE	1,542	1,264	2,806						
Total				34,633	22,520	57,153			
Theater level support									
Total				69,979	33,303	103,282			

Force cost is probably the second most severe constraint on force design--after force manpower. In many cases, available monetary resources may be even more important than available manpower in determining not only the size but also the type of force which can be supported. The slice methodology adapts nicely to the introduction of cost constraints into the force design process.

a. TRANSFORM introduced three significant improvements in slice costing procedures - separation of nonrecurring and recurring costs, the division of nonrecurring costs into modernization and activation costs, and the calculation of slice costs for individual weapons. Recurring costs (RC) are a positive cash flow indicator and are part of the affordability evaluation for any force design. Modernization or base level costs, low nonrecurring costs (NRC Lo), are the presently planned expenditures for major items of new or modernization equipment which are reflected in the projection and modernization of the Army from the base year to the design year. Activation costs, high nonrecurring costs (NRC Hi), are the nonrecurring costs associated with buying more of a weapons system than is currently planned. The methodology for determining these costs for system slices and cost related constraints is beyond the scope of this paper, but it is discussed in detail in the TRANSFORM report. Costs used in the slice methodology are not true budget costs but are related to them; tests have shown that no significant distortions are introduced by their use.

b. The costing of individual weapon slices uses much the same methodology as used in calculating the individual weapon personnel slices. The system slice costs are calculated directly as the difference in the costs of FASTALS troop lists. System slice, weapon slice and individual weapon costs used in TRANSFORM are shown in Table 3.

(1) The slice modernization or base level nonrecurring costs are approximately equal to the procurement costs of the new weapons multiplied by the numbers of new weapons. This is reflected in the NRC-Lo columns in Table 3 for the TRANSFORM forces.

(2) The Active NRC-Hi or activation costs are equal to the direct procurement costs of the total number of each type of weapon plus an indirect cost which covers activation and training costs. The indirect cost is the difference between the procurement costs of all weapons in the systems slice and the activation costs for the systems slices. It is prorated to the individual weapon types based on the number of weapons or, if there are differences in weapon crew sizes, based on the number of crew members. It is to be noted that the NRC-Hi costs for the Reserves reflect only the procurement of additional equipment since it is considered unlikely that new Reserve units are needed to satisfy force design requirements.

Table 3. TRANSFORM Individual Weapon Cost Data

Weapon	Active Army					Reserve Components				
	Number	Cost (millions)			Slice population	Number	Cost (millions)			Slice population
		WAC HI	WAC LO	AC			WAC HI	WAC LO	RC	
Tanks										
XM 1	1,773	2405.8	1676.6	587.07	29,312	0	0	0	0	0
Each	1	1.3583	0.9456	0.3311	16,533					
M60A2	378	1844.0	0.0	125.16	6,209	0	0	0	0	0
Each	1	1.2351	0.0	0.3311	16,533					
M60A3	1,493	402.9	1227.8	494.36	[illegible]	2,087	2086.7	1709.4	131.4	36,348
Each	1	1.0659	0.8190	0.3311	16,533	1	0.9999	0.8191	0.0630	17,416
Total slice	3,644	4650.1	2899.7	1206.6	62,245	2,087	2086.7	1709.4	131.4	36,348
Avg per tank	1	1.2761	0.7956	0.3311	16,533	1	0.9999	0.8191	0.0630	17,416
LATV										
IFV/CFV	1,677	1005.8	728.15	268.43	15,973	0	0	0	0	0
Each	1	0.5997	0.4342	0.1601	9,525					
ITV	1,996	620.2	289.62	319.49	19,011	1,116	231.0	162.1	42.54	14,182
Each	1	0.3107	0.1452	0.1601	9,525	1	0.2070	0.1452	0.0381	12,707
M551	933	618.0	0.0	180.69	10,752	207	108.5	0	9.13	3,044
Each	1	0.6623	0.0	0.1937	11,525	1	0.5240		0.0441	14,707
LATTV(IFV/CFV + ITV + M551)	4,606	2243.9	1018.0	768.61	45,736	1,323	339.5	162.1	51.67	17,226
Avg	1	0.4872	0.2211	0.1669	9,930	1	0.2566	0.1452	0.0391	13,020
M113A1	4,411	905.9	136.90	557.79	33,151	3,220	438.6	92.6	103.43	34,478
Each	1	0.2054	0.0950	0.1265	7,525	1	0.1362	0.0950	0.0321	10,707
Total LATV slice	9,017	3149.8	1155.1	1326.4	78,527	4,543	778.1	254.7	155.1	51,704
Avg per vehicle	1	0.3493	0.1281	0.1471	8,753	1	0.1713	0.0561	0.0341	11,381
ATGM										
TOW (grd)	384	42.7	21.2	31.4	2,240	1,062	101.3	38.9	18.96	7,188
Each	1	0.1113	0.0550	0.0818	5,833	1	0.0954	0.0366	0.0179	6,769
DRAGON	4,159	146.9	88.6	77.8	5,543	3,333	104.6	37.0	19.94	7,562
Each	1	0.0353	0.0213	0.0187	1,333	1	0.0314	0.0111	0.0060	2,269
Total ATGM slice	4,543	189.6	109.8	109.2	7,783	4,395	205.9	75.9	38.9	14,750
Avg/Launcher	1	0.0417	0.0242	0.0240	1,713	1	0.0468	0.0173	0.0089	3,356
Artillery										
XM 204 (105T)	126	150.78	18.32	117.75	6,464	378	243.4	55.0	77.80	20,478
Each	1	1.1966	0.1454	0.9345	51,304	1	0.6439	0.1455	0.2058	54,174
XM 198 (155T)	288	383.29	67.78	275.80	15,140	414	312.8	97.5	87.68	23,079
Each	1	1.3309	0.2353	0.9576	52,570	1	0.7555	0.2355	0.2119	55,745
XM 204 + XM 198 (towed)	414	534.07	86.10	393.55	21,604	792	556.2	152.5	165.48	43,557
Avg towed	1	1.2900	0.2080	0.9506	52,184	1	0.7023	0.1926	0.2089	54,996
M109A1 (155SP)	870	1431.76	0.0	825.58	46,803	474	487.1	0	101.10	26,612
Each	1	1.6457	0.0	0.9800	53,798	1	1.0275		0.2133	56,144
M110E2 (8" SP)	412	939.87	0.0	507.87	27,880	320	422.7	0	83.02	21,854
Each	1	2.2812	0.0	1.2327	67,670	1	1.3209		0.2542	66,923
M109A1 + M110E2 (SP)	1282	2371.63	0.0	1360.45	74,683	794	909.8	0	184.12	48,466
Avg SP	1	1.8499	0.0	1.0612	58,255	1	1.1458		0.2319	61,040
Total arty slice	1,696	2905.7	86.10	1754.0	96,287	1,586	1466.0	152.5	349.6	92,023
Avg per tube	1	1.7133		1.0342	56,773	1	0.9243	0.0962	0.2204	58,022
Helicopter (attack)										
AAH	234	2369.02	1969.25	294.5	9,714	0	0	0	0	0
Each	1	10.1240	8.4152	1.2584	41,512					
AH1Q/S	185	1261.30	945.16	232.8	7,680	270	755.4	651.8	45.25	10,254
Each	1	6.8178	5.1090	1.2584	41,512	1	2.7977	2.4138	0.1676	37,980
AH1G	157	982.31	606.22	197.5	6,517	165	369.0	192.4	27.65	6,267
Each	1	6.2567	3.8613	1.2583	41,512	1	2.2366	1.9406	0.1676	37,980
Total helo slice	576	4612.6	3520.2	724.8	23,911	435	1124.4	844.2	72.9	16,521
Avg per helo	1	8.0080	6.1115	1.2582	41,512	1	2.5848	1.9406	0.1676	37,980
Infantry										
Squad level fighters	25,929	778.4	0.0	793.5	56,191	30,252	778.4	0	235.0	86,690
Per fighter	1	.03002	0.0	.03060	2,166	1	0.0258	0	0.0078	2,872
Mortars										
81mm (Grd)	36	5.85	0	7.13	493	0	0	0	0	0
Each	1	0.1625		0.1981	13,696					
81mm (SP)	630	161.16	0	133.89	9,258	270	52.3	0	12.10	4,784
Each	1	0.2558		0.2125	[illegible]	1	0.1936		0.0448	17,717
Total 81mm	666	167.01	0	141.02	9,751	270	52.3	0	12.10	4,784
Avg/tube	1	0.2508		0.2118	[illegible]	1	0.1936		0.0448	17,717
107mm (Grd)	48	9.41	0	10.54	729	236	30.8	0	10.87	4,299
Each	1	0.1961		0.2197	15,197	1	0.1304		0.0461	18,217
107mm (SP)	751	224.42	0	154.20	10,662	408	97.9	0	17.77	7,026
Each	1	0.2988		0.2053	14,197	1	0.2400		0.0438	17,217
Total 107mm	799	233.83	0	164.74	11,391	644	128.7	0	28.64	11,323
Avg	1	0.2927		.2062	14,257	1	0.1998		0.0445	17,583
Mortars, Grd	84	15.26	0	17.67	1,222	236	30.8	0	10.87	4,299
Avg	1	0.1817		.2104	14,548	1	0.1304		0.0461	18,217
Mortars, SP	1381	385.58	0	288.09	19,922	678	150.2	0	29.87	11,808
Avg	1	0.2792		[illegible]	14,424	1	0.2215		[illegible]	17,416
LWCM	192	31.36	0.67	38.04	2,630	531	57.3	1.9	22.46	8,877
Each	1	0.1633	0.0035	0.1981	13,696	1	0.1080	[illegible]	[illegible]	16,717
Total mortars	1,657	432.2	0.67	343.8	[illegible]	1,445	235.3	1.9	63.2	[illegible]
Avg	1	0.2608		0.2075	14,346	1	0.1649	0.0013	0.0437	17,290
Air Defense										
Total		1931.1	0	666.9	34,633		286.0	0	77.4	22,520
Theater level support										
Total		2251.1	26.1	1372.2	[illegible]			26.1	115.1	[illegible]

(3) The recurring costs (RC) are calculated by allocating the recurring costs for the system slice among the weapon types in the slice based on the personnel per weapon slice. It is realized there is some difference in the recurring Operations and Maintenance, Army (OMA) account among weapon types, but this is not deemed to be significant for weapon types in the same slice.

While intertheater lift was not used as a constraint in TRANSFORM, minimization of shipping weight requirements was a goal in the force design process. System slice shipping weights for unit equipment deployed after D-day as calculated in FASTALS are listed in Table 4. The shipping weight requirements (weight slices) for unit equipment associated with each type of individual weapon were calculated and are listed in Table 5. The methodology is similar to that for personnel slices. In this case the direct weight is the weight of the weapon and identifiable support such as the prime mover or a dedicated ammunition truck. The remainder of the slice weight is then prorated to the weapon types in the slice based on the number of each type. For weapons in units with only one weapon type, the weight of the unit divided by the weapons per unit is considered to be the direct weight.

Table 4. Slice Unit Equipment Shipping Weight Requirements

Slice	Active D-day thru D+39		Reserve D+40 thru D+79		Total D-day thru D+79	
	Weapons	Weight (STON)	Weapons	Weight (STON)	Weapons	Weight (STON)
Tanks	1,169	109,328	2,087	178,424	3,256	288,352
LATV	3,117	94,183	4,543	148,884	7,660	243,067
Arty	736	92,142	1,586	183,764	2,322	275,906
Helo	267	20,110	435	28,127	702	48,237
Inf	13,267	36,416	30,252	109,686	43,519	146,102
ATGM	2,261	2,758	4,395	15,937	6,656	18,695
Mortars	760	18,708	1,445	36,545	2,205	55,253
Slice total		374,245		701,367		1,075,612
Residual and other slices		115,345		187,529		302,874
Theater total		489,590		888,896		1,378,486

ADDITIONAL WEAPON SLICE DATA

Personnel, equipment and cost describe, by definition, the essential characteristics of a weapon slice. Weight was also added as a descriptor in the previous section. In developing the slices and quantitatively describing them, much additional data becomes readily available with relatively little expenditure of effort. To illustrate, the following paragraphs describe some types of slice-related data developed for design or analytical purposes during TRANSFORM; the tables contain the TRANSFORM values, mostly at the system slice level.

Table 5. Weight Slices

Weapon	Slice weight per item	Weight item only
Tanks	STON	STON
M60A3	90.0	48.5
LATV		
All except IFV/CFV	31.0	12.0
Artillery		
M204 (105T)	95.5	14.5[a]
XM 198 (155T)	109.5	28.5
M109A1 (155SP)	126.1	45.1
M110E2 (8" SP)	143.5	62.5
Helicopters		
All	68.7	3.5
Infantry		
Per squad-level fighter	3.3	
ATGM		
TOW	6.2	4.3[b]
DRAGON	1.9	.0
Mortars		
LWCM	17.9	2.6[c]
107 Grd	22.0	6.7
107 SP	29.0	13.7
81SP	32.3	17.0

[a]Artillery weights include prime mover and battery ammo vehicles.

[b]TOW weight includes two 1/4 ton trucks and 1/4 ton trailers.

[c]Mortar weights include weapon transportation and platoon ammunition vehicles.

An important design variable and/or constraint in many force design applications concerns the overseas stationing of troops and equipment. One of the major constraints in TRANSFORM was the Europe manpower ceiling. To be in consonance with the rest of the program, the M-day slice strengths were required. These were calculated for individual weapon types in the same manner as for the personnel slices described earlier. To reflect the effects of Prepositioned Materiel Configured to Unit Sets (POMCUS), D-day slice personnel strengths were also needed. The numbers of each type of weapon and the personnel slices for M-day and D-day are listed in Table 6.

Table 6. TRANSFORM Weapon and Personnel Slice Data, M-day and D-day

	M-Day			D-Day		
	Wpns	Pers	Pers/Wpn	Wpns	Pers	Pers/Wpn
Tanks	1,560	14,562	9,335	2,475	38,623	15,605
XM 1	1,128	10,530	9,335	1,773	27,668	15,605
M60A2	324	3,024	9,335	378	5,899	15,605
M60A3	108	1,008	9,335	324	5,056	15,605
LATV	3,742	19,745	5,277	5,900	48,793	8,256
IFV/CFV	1,092	6,214	5,690	1,677	14,635	8,727
M113A1	1,341	4,949	3,690	2,201	14,805	6,727
ITV	742	4,222	5,690	1,222	10,664	8,727
M551	567	4,360	7,690	810	8,689	10,727
ATGM	1,454	1,708	1,175	2,282	3,375	1,479
TOW	0	0	0	0	0	0
DRAGON	1,454	1,708	1,175	2,282	3,375	1,479
Artillery	568	23,797	40,471	960	53,346	55,569
XM 204	0	0	0	0	0	0
XM 198	90	3,296	36,622	144	7,409	51,451
M109A1	342	12,698	37,129	540	28,039	51,924
M110E2	156	7,803	50,019	276	17,898	64,849
Heliocopters	189	4,361	23,074	309	10,831	35,052
AAH	108	2,492	23,074	180	6,309	35,052
AH1S	67	1,546	23,074	94	3,295	35,052
AH1G	14	323	23,074	35	1,227	35,052
Mortars	571	6,505	11,392	897	13,273	14,797
LWCM	0	0	0	0	0	0
81mm Grd	0	0	0	0	0	0
81mm SP	252	2,871	11,392	396	5,860	14,797
107mm Grd	0	0	0	0	0	0
107mm SP	319	3,634	11,392	501	7,413	14,797
Infantry Fighters	8,074	10,805	1,338	12,662	22,219	1,755
Total combat slices		81,483			190,460	

The balanced troop list produced using FASTALS includes all units in the theater. It reflects many workloads calculated within the model which are printed out in the computer report. As previously described, the differences between the base case and the reduced force roundouts are the values attributed to the system slices.

a. FASTALS develops the total theater and nondivisional populations and presents them as population workloads, time-phased and geographically located in the division, corps, and COMMZ areas. Other FASTALS personnel workloads which can be related to weapon slices include casualties, hospital patients by location, hospital patient returns-to-duty, replacements, and personnel by branch. It would even be possible to obtain personnel by MOS, if someone required it, through use of the Force Stratification Program at the TRADOC System Analysis Activity (TRASANA). Table 7 presents the TRANSFORM total theater, Active Army and Reserve Component strengths for the base case and each slice, and the end-of-simulation divisional population and theater casualties.

b. FASTALS-developed workloads include many in the logistic support area. Port and airfield operations, intratheater transportation, direct support and general support automotive and aircraft maintenance, and engineer construction requirements workloads are all available. These workloads can be divided into system slice or weapon slice workloads us-

ing the techniques already described. It is thus possible to predict, for example, the impact on port operations of substituting an attack helicopter company for a tank battalion. For illustrative purposes Table 8 includes weights of unit equipment and dry cargo shipped into the theater, and general support automotive and aircraft maintenance requirements.

Table 7. System Slice Personnel Data

Force/slice	Theater strength	Active strength	RC strength	Theater divisional strength	Total theater casualties
Base case	1,020,679	594,047	426,632	416,228	184,350
Tank	96,593	60,245	36,348	52,159	21,270
Percent	9.5	10.1	8.5	12.5	11.5
LATV	130,631	78,927	51,704	63,150	27,810
Percent	12.8	13.3	12.1	15.2	15.1
ATGM	22,533	7,783	14,750	16,317	3,430
Percent	2.2	1.3	3.6	3.9	1.9
Artillery	188,309	96,187	92,022	67,572	30,590
Percent	18.5	16.2	21.6	16.2	16.6
Helicopter	40,432	23,911	16,521	15,527	7,760
Percent	4.0	4.0	3.9	3.7	4.2
Mortar	48,756	23,772	24,984	34,320	9,520
Percent	4.8	40	5.9	8.2	5.2
Infantry	143,071	56,181	86,890	85,893	21,540
Percent	14.0	9.5	20.4	20.6	11.7
Air Defense	57,153	34,633	22,520	19,395	10,310
Percent	5.6	5.8	5.3	4.7	5.6
Theater level	103,282	69,979	33,303	1,770	13,780
Support percent	10.1	11.8	7.8	0.5	7.5
Total slice	830,760	451,718	379,042	356,098	146,010
Percent	81.4	76.0	88.8	85.6	79.2
Residual	189,919	142,329	47,590	60,130	38,340
Percent	18.6	24.0	11.2	14.4	20.8

c. The effects of force structure changes on the consumption of supplies are also predictable using the slice methodology. FASTALS supplies are divided into six general categories: (1) refrigerated cargo, (2) light supplies (rations, clothing, packaged POL, and other minor equipment), (3) bulk POL, (4) construction material, (5) ammunition of all types, and (6) major items and repair parts. These supplies are allocated on the basis of pounds per man per day based on the activity of a unit in a general area of the theater, or, in the case of combat units, on their combat posture. Supplies, depending on type and physical characteristics, are prestocked to reduce transportation requirements after initiation of hostilities. Force structure changes may modify prestock requirements and the approximate amount can also be predicted. System slice ammunition consumption has been included in Table 8 to show the type of consumption information that was available in TRANSFORM.

THE FUTURE

The slice methodology has introduced a new dimension to the force structuring process. The wide range of information that can be developed using the methodology and its potential application to problems, many not even considered at this time, would seem to portend continuing growth in its capabilities and broadening of its applications. In the near term, it will be used by CAA in a study called Conceptual AFCENT Force Design-1990 (CONAFOR-90) for which the tasking directive is now being coordinated. This study will require extension of the methodology to NATO forces, but probably not in the detail possible with US forces. For US forces a new base case will be developed--a not inconsiderable effort requiring use of at least the CEM, FASTALS and TRANSMO Models. New values for the present weapon slices will be calculated and weapon slices will be developed for a modernized Air Defense force. In addition it is expected that three helicopter system slices will be developed. These slices will better define the exact size of the attack helicopter slice and will focus attention on the function of utility helicopters and the number and cost of command and control aircraft. (Major parts of the latter two slices are now in the residual slice.) In the more distant future, increased automation of the slicing process can be expected, concurrently with the application of the methodology to widening groups of problems.

Table 8. System Slice Logistic Support

Force/slice	Dry cargo shipped - STON	Unit equip shipped STON	GS auto maint 1000 man-hours	GS acft maint 1000 man-hours	Ammo consumption 1000 STON
Base Case 88	3,483.08	1,378.49	2,903.78	1,240.38	994.33
Tank	580.59	268.35	573.68	26.42	80.98
Percent[a]	16.7	20.9	19.8	2.1	8.1
LATV	639.47	243.07	581.58	32.15	101.89
Percent	18.4	17.6	20.0	2.6	10.2
ATGM	61.94	18.70	16.79	1.86	14.56
Percent	1.8	1.4	0.6	0.1	1.5
Artillery	1,026.54	275.91	577.98	24.79	434.45
Percent	29.4	20.0	19.9	2.0	43.7
Helicopter	141.10	48.24	62.17	509.91	19.82
Percent	4.1	3.5	2.1	41.1	2.0
Mortar	176.88	55.25	110.61	5.64	36.97
Percent	5.1	4.0	3.8	0.5	3.7
Infantry	330.01	146.10	[illegible]	28.91	25.65
Percent	9.5	10.6	3.6	2.3	2.8
Air Defense	336.99	c	111.27	4.72	136.36
Percent	9.7		3.8	0.4	13.7
Theater level Support	445.81	c	247.66	99.94	70.03
percent	12.8		8.5	8.1	7.0
Total slice	3,739[b]	1,075.61	2,385.43	738.34	920.72
Percent	107.4	78.0	82.1	59.2	92.6
Residual		302.87	518.35	506.04	73.61
Percent		22.0	17.9	40.8	7.4

[a]Percent of base case.

[b]Exceeds 100 percent because of multiple counting of resupply shipped for stockage.

[c]Included with residual.

SUMMARY

The slice methodology, despite its infancy, has been shown to have great flexibility in its application to current force structuring problems. Weapon slices do provide a new and incremental approach to force design. Use of this methodology should be considered where detailed knowledge of force makeup is required.

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TITLE: The Use of Goal Programing in the Theater Level Design of Forces

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ABSTRACT: During the conduct of the Conceptual Design of the Army in the Field (CONAF) IV and V Studies, linear programing was utilized to optimize the design of the US force structure required in a European scenario circa 1987 and 1988. The linear programs for CONAF IV and V were constructed utilizing coefficients derived using an incremental slice methodology that measures support in terms of people and dollars required to place an individual weapons system on the battlefield in 1987/1988. The derivation of these coefficients is a subject of another paper entitled: "Weapons Slices--An Incremental Approach to Force Design": to be presented by Mr. Robert C. Spiker. The linear program utilized these slice coefficients as well as firepower potential and a measure of reaction power to an enemy breakthrough in the force design. Appropriate manpower and dollar constraints were utilized to constrain the mathematical model. This force design methodology was expanded for the Trade-Off Analysis Systems/Force Mix (TRANSFORM) Study, from the CONAF V model that optimized only one objective function, to a multiple objective optimization process (goal programing). This latter methodology will be expanded and improved for the projected study now envisioned to study force design for the Allied Forces, Central Europe in 1990.

SUBJECT: The Use of Goal Programming in the Theater Level Design of Forces

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AGENCY: USA Concepts Analysis Agency

I. Background

A. In the early 1970's a series of studies were initiated by the Army, in an attempt to identify viable alternatives in force design in the midrange period (10-12 years) that would optimize the use of available resources. These studies recognized that highly modernized, low-risk forces would not be affordable in the austere atmosphere of post Vietnam. The studies were entitled: Conceptual Design of the Army in the Field, or CONAF. CONAF force design methodology has evolved through the various editions of the CONAF study. The most recent edition, Phase V, was published in October 1976. A brief review of the CONAF methodology is included as an introduction to the techniques used and to identify improvements in the methodology that occurred in its most recent application: Trade-Off Analysis-Systems/Force Mix (TRANSFORM).

B. Outline of CONAF Methodology. The major facets of this force design process are as follows:

Partition Base Case into System Slices

Design forces at:

Weapons System Level

Battalion Level

Division Level

Using linear programing

Verify design using Theater Simulation

C. Selection of System Slice Variables. Starting in the CONAF IV Study, it was determined that the most descriptive level for investigation of a force was at the weapon system or family of weapons level. For CONAF V, eight weapons systems were used as design variables. They were: tanks, armored personnel carriers w/TOW, armored personnel carriers, antitank guided missiles, mortars, helicopters, artillery and infantry.

D. Partition of Base Case into System Slices. This phase of the design process required that the FY 77 force be modernized in accordance with DA plans and a modernized force, circa 1988, be defined. This force was designated as the Base Case 88 force. With the definition of the force, certain characteristics were described relative to the design variables. This description of the force became known as the system slices.

E. Use of a Mathematical Model to Design Forces.

(1) Resources Required. System slice coefficients generated for the Base Case 88 force were used to formulate linear relationships between the design variables and were used to establish constraints on manpower, dollars, materiel, strategic lift or other resources of interest. For CONAF V, resource constraints using system slice variables were formulated to recognize the limits on:

(a) Cash flow available to finance Base Case 88.

(b) Peacetime US troop population in Europe.

(c) The total slice related personnel of a fully supported force.

This method was also used to describe the requirements for the Active and Reserve components of the total force.

(2) Design Criteria. The CONAF methodology postulated that a force designed to match the various types of Pact firepower potential (FPP) at critical points in the scenario would be able to execute a forward defense and limit the Pact advance. Four types of firepower potentials were considered. Each described a weapons system potential against the appropriate type target in a particular combat posture. The four types of firepower considered are shown in Table 1.

Table 1. Firepower Potential

Type	Employed against
Hard	Armor
Medium	Light armor
Soft	Unarmored targets
Artillery	Artillery

The FPP scores selected were those attributed to FPP for a Pact attack against a US prepared defense as best describing the majority of engagements that could occur in the selected scenario.

(3) Predator versus Prey. The US was viewed as a predator who tried to match the Pact FPP on the battlefield (the prey). Using the Intelligence Threat Assessment Detachment (ITAD) data, the time-phased arrival of the Pact targets on the battlefield was determined. It then became the force designer's problem to optimally match that threat given the constraint of resources.

(4) Linear Programing. The mathematical model chosen for CONAF V was linear programing. Appropriate objective functions were derived that quantified the force's ability to react to Pact breakthrough tactics. These various objective functions were used separately in conjunction with the resource constraints and design criteria to optimally design forces. The linear program generated three types of solutions. The first was the weapons system solution, in terms of an individual weapon,

e.g., $X_1 = 3,942$ tanks. This solution provided an insight into the optimum mix of weapons systems that the resource constraints and design criteria of the mathematical model generated. The next step was to translate system slice coefficients from the weapons system model to battalion model. This translation process was done by assuming linear relationships, that is, if the hard firepower potential for one XM1 tank is .0640 points, then 54 tanks in a tank battalion would have 54 x .0640 = 3.456 points and similarly the 360 tanks in an armored division would have 360 x .0640 = 23.04 points. Thus the problem can be structured at three levels: (1) weapons system, (2) battalion, and (3) division. Force design solutions were obtained for CONAF V at all three levels. Appropriate insights obtained at the weapons system and battalion module level were utilized in the design of conceptual divisions.

F. Indexing Divisional Candidates. Since linear programing could optimize only one objective function at a time, considerable sorting of solutions was necessary as solutions to the various objective functions were obtained. A technique called decision making under uncertainty (DMUU) was used to narrow candidate divisions to a manageable number for the final step in the CONAF methodology.

G. Combat Simulation. The final step consisted of performing a combat simulation in the theater level Concepts Evaluation Model (CEM). Results were compared with Base Case 88 to verify force design efficiencies.

H. Summary. The figure below summarizes the CONAF force design methodology.

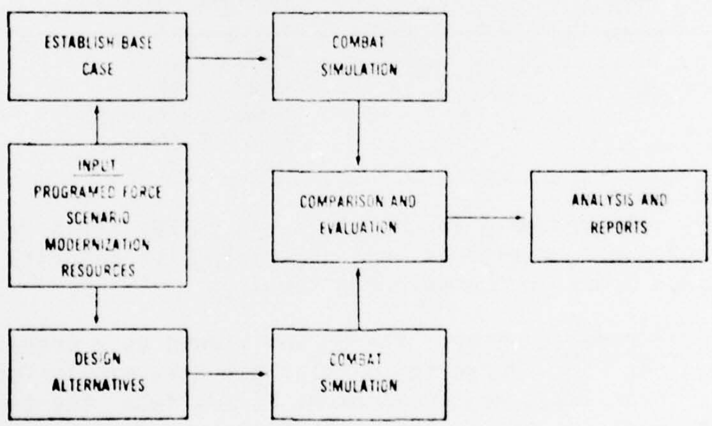

Figure 1. CONAF Force Design Methodology

II. Goal Program Design Methodology.

CONAF V force design methodology was used in the conduct of the recent TRANSFORM Study. The original impetus for TRANSFORM was to determine the implied trade-off that increased attack helicopter assets in the context of the new Division Restructuring Study (DRS) heavy division would cause. During the course of the TRANSFORM Study however, a decision on aviation

structure ended the need for further investigation of helicopter structure, and attention was focused on using the CONAF force design methodology to evaluate the DRS organization. TRANSFORM used a more powerful optimization technique, the goal program. The goal program allows the force designer to perform multiple optimization by establishing a series of objectives which are ordered into priorities. This technique enlarges the flexibility of the CONAF mathematical model to treat several objectives simultaneously without having to reformulate the problem each time the model is used or when no solution exists (infeasible solution).

A. Description of Goal Programing. Goal programing is a mathematical technique which optimizes several objectives at a time, in a priority ordered sequence. Linear programing may be viewed as a special case of goal programing where all constraints are priority one and the single objective is priority two. As an example, a force design problem may begin with various types of divisional and nondivisional units as candidate variables. The problem could be to discover that combination of variables which best meets European manpower limits in peace time and matches hard, medium, soft and artillery firepower potential to opposing targets at events in the scenario buildup. The objective could be to minimize the costs required to achieve firepower potential matching without exceeding manpower limits. As a linear programing problem, there are one objective and five types of constraints. The program may not be mathematically solvable by linear programing. In such cases, the stated problem is said to be infeasible. The military analyst must then devise some method to decide new values for the constraints which will both yield a feasible solution and "best" meet the requirements of the problem. Goal Programing will always provide a solution. Such solutions may underachieve some objectives. The problem stated above could be solved by using goal programing. The "constraints" could be treated as priority one objectives and minimizing the costs could be priority two. But goal programing could also reverse the priorities, weight objectives differentially within a priority level or spread out less important objectives to lower priority levels. Goal programing seeks to satisfy priority one level goals first, then adjusts the solution to best achieve priority two objectives without preempting the achievement of priority one goals. This process continues through all priority levels stated by the problem; seeking the best fit without violating higher priority achievement.

B. Requirements for Goal Programing. The following summarizes conditions needed to state a problem in goal program form:

(1) A series of objectives must be stated in mathematical terms. The mathematical model of TRANSFORM met this requirement.

(2) A numerical value must be established for each objective as a goal. In TRANSFORM firepower potential objective values were set by analyzing the threat and, in particular, selected events in the scenario buildup of the threat force. Cost constraints were set by using, separately, annual recurring and nonrecurring dollar cash flows which would be available to purchase and sustain the base case through end of FY 88.

(3) The objectives must be ranked in order of importance. For TRANSFORM, this ranking was inferred from guidelines established by the

tasking directive and the SAG. If the military analyst is indifferent to such a ranking, all objectives could be made priority one. In general the following ranking was followed:

(a) Stay within cost constraints limits (for both nonrecurring and annual recurring cash flows).

(b) Stay within European manpower stationing limits.

(c) Do not exceed current active and reserve manpower spaces.

(d) Meet force design criteria in terms of firepower potential as modified by suppression or requirements to concentrate firepower.

(e) Do not exceed materiel prestock limits.

(f) Minimize strategic lift tonnages.

(4) Objectives must be grouped into priority levels. In TRANSFORM all firepower matching objectives after D-day were grouped into a single priority level. Nonrecurring and annual recurring costs were grouped into a single priority level.

(5) Weights must be assigned to objectives grouped within the same priority level. Where it is not clear how to weight equally preferred objectives differentially (grouped within the same priority) they may be initially weighted equally and the results analyzed to discover where a difference in weights would change the solution. Several such solutions may be displayed for the military planner. If a given priority level set of objectives are all achieved, differential weighting will have no impact on the solution. In TRANSFORM differential weighting was not required.

(6) Finally, it must be decided whether an attempt will be made to minimize the negative deviation, minimize the positive deviation or equal the goal. In TRANSFORM, firepower potential was to be met or exceeded, within given resource goals or less, if possible. Stating a goal as an equality is inherently a severe requirement and can unnecessarily limit useful solutions.

C. TRANSFORM Mathematical Models.

(1) Two separate goal programs were utilized in the force design process. These two sets consisted of:

(a) A systems level model which yielded the weapons systems solution.

(b) A division level model which selected divisional level variables as solutions to the program and yielded the conventional force design and the DRS force design.

Both models contained the same achievement objectives. The initial systems level model yielded a solution that caused the priorities of the

mathematical model to be altered. The revised priorities for achieving objective levels were then utilized in the division level model for the conventional and DRS design forces.

(2) Composite System Used. The following composite systems were used as variables in weapons systems solutions: Note: a composite system is a weighted average of the mix of actual systems in the slice.

Table 2. Composite Systems Used in TRANSFORM

Tank	Mortar
Light Antitank Tracked Vehicle (LATTV)	Helicopter
Armored Personnel Carrier (APC)	Artillery
Antitank Guided Missile (ATGM) (Ground Mounted)	Infantry

(3) Establishing Priorities. Goal programing permits a solution optimizing to multiple objectives within an ordered set of priorities. The degree that the solution achieves a given priority level will not be preempted by any lower priority level objectives. This requires that the highest priority levels be the most important; priority level 1 must contain any requirements which are needed to implement the solution. In the mathematical model, this requires two variables for each system for each event: one incorporating the appropriate low nonrecurring cost coefficient and the second incorporating the high nonrecurring cost coefficient.* The high cost coefficient must be used when the number of systems selected exceeds currently programed levels. This fact must be clearly imbedded in the mathematical problem statement. This is achieved by setting as a priority one objective, a limit on the number of each system type that can be selected with the base level cost coefficient.

Table 3. Initial Priorities for TRANSFORM

1. Low nonrecurring cost constraints.	6. FOCUS limit.
2. Recurring and nonrecurring cost constraints.	7. Ability of force to concentrate firepower potential at M-day.
3. European manpower stationing limit.	8. Strategic lift tonnages.
4. Deployed force manpower constraints.	9. Crew size.
5. Design criteria-firepower potential D-day and D+19.	10. Firepower potential at M-day.

(4) Achievement Function. In addition to an optimal choice of number of weapons systems or types of units available to be chosen, the main output of the goal program is an achievement function which measures the inability of the solution to achieve its priorities. In comparing two achievement functions having the same objectives, the smaller number signifies the better achievement function. The relative measure of the optimality of a goal program is how well it minimizes the achievement function. Table 4 illustrates the composition of the achievement function for the TRANSFORM goal program.

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*High nonrecurring costs result from the necessity to increase procurement and activate new units when base case levels are exceeded.

Table 4. TRANSFORM Achievement Function

Priority	Achievement Function
1.	This priority forced the program to pay higher nonrecurring cost when number of weapons selected exceeded slice level from the base case.
2.	Amount that total of recurring and nonrecurring costs exceeded the base case totals.
3.	Number of total personnel stationed in Europe that exceeded the base case slice related Europe based force.
4.	Number of personnel that a fully supported force exceeded the base case fully supported active slice force.
5.	Shortfall in FPP relative to matching Pact unsuppressed FPP on D-day and D+19.
6.	Dollar amount that POMCUS cost exceeds the base case value.
7.	Shortfall in reaching desired level of reaction power to Pact breakthrough tactics on M-day.
8.	Tonnage to be lifted from CONUS.
9.	Number of weapons systems crew members required.
10.	Shortfall in FPP to match Pact on M-day.

(5) Structure of the Goal Program. The solution to the goal program considered as variables: (1) Type of weapons system or module (division level); (2) location of the weapons system or module selected and (3) the time at which the weapons system or division level unit could be counted as contributing to the battle. *Table 5 represents the unit, location and firepower contribution scheme of the goal program.*

Table 5. Units, Locations and Events Portrayed in the TRANSFORM Goal Program

Weapons System	Modules (Div Level)	Locations	Events
1. Tanks	1. Inf div	1. Europe	1. M-day[a]
2. LATTV	2. Mech div	2. POMCUS	2. D-day[b]
3. APC	3. Armd div	3. CONUS	3. D+19[c]
4. ATGM	4. ACR		
5. Mort	5. ACCB		
6. Hel	6. Arty gp		
7. Arty	7. DRS		
8. Inf	8. DRS+2 atk hel co		

[a]Europe based units were assumed to be capable of providing M-day FPP.

[b]POMCUS units were assumed to be capable of providing D-day FPP.

[c]CONUS based units were assumed to be capable of providing FPP by D+19.

(a) Weapons System Model. Stating the problem mathematically:

1. Variables 1-8

X_1 = Number of European based tanks (tk)
X_2 = Number of European based light antitank tracked vehicles (LATTV)
X_3 = Number of European based armored personnel carriers (APC)
X_4 = Number of European based antitank guided missiles (ATGM)
X_5 = Number of European based mortars (mort)

X_6 = Number of European based helicopters (hel)
X_7 = Number of European based artillery pieces (arty)
X_8 = Number of European based infantrymen (inf)

Equations for the goal program were written in terms of the individual weapons systems. For example the mathematical statement for Objective 13, the peacetime manpower restriction in Europe, would be the linear equation:

$$9.33500X_1 + 6.09512X_2 + 3.69000X_3 + 1.17500X_4 + 11.39200X_5$$

$$+ 23.07400X_6 + 40.13654X_7 + 1.33800X_8 \leq 81000$$

The number 81,000 represents the base case, slice-associated population in Europe in Active units and each coefficient represents the slice population. TRANSFORM equations, such as the above, were assumed to remain linear over a range of change. Equations representing each of the 31 objective functions were formulated to describe the respective objectives and their desired achievement levels.

2. Variables 9-16. The next 8 variables represented the same ordering of weapons systems but placed them in Europe prestock (POMCUS):

X_9 = number of POMCUS tk
X_{10} = number of POMCUS LATTV
X_{11} = number of POMCUS APC
X_{12} = number of POMCUS ATGM
X_{13} = number of POMCUS mort
X_{14} = number of POMCUS hel
X_{15} = number of POMCUS arty
X_{16} = number of POMCUS inf

Selection of a weapons system from POMCUS stocks contributed firepower potential (FPP) to the force on D-day and required no strategic lift of unit equipment from CONUS.

3. Variables 17-24. The next eight variables represented the choice of a stateside weapons system and again were arrayed in the same cyclical order. Selection of a stateside weapons system required strategic lift of the equipment and did not contribute FPP until D+19.

4. Variables 25-48. An identical set of equations were formulated with the exception that the coefficient representing the nonrecurring cost of the weapons system (Objective 22) represented the activation or higher cost. Once the base case level of a particular weapons system was exceeded, e.g., 3644 tanks, Priority 1 required that that particular weapons system be selected from variables X_{25} through X_{48} and that the higher nonrecurring cost be assessed for that system.

(b) Division Level Model. For the formulation of the division level model it was only necessary to know the density of weapons systems in a division.

Y_1 = number of European based infantry divisions
Y_2 = number of European based mechanized divisions
Y_3 = number of European based armored divisions
Y_4 = number of European based armored cavalry regiments
Y_5 = number of European based air cavalry combat brigades
Y_6 = number of European based artillery groups
Y_7 = number of European based Division Restructuring Study (DRS) heavy divisions
Y_8 = number of European based DRS + 2 attack helicopter companies

Table 6. Density of Weapons Systems for Division Level Units
Division Level Units

	Inf	Mech	Arm	ACR	ACCB	Arty Grp	DRS	DRS+2AHC
Tanks	54	306	360	51	--	--	360	360
LATTV	80	431	374	162	--	--	476	476
APC	48	220	217	105	--	--	232	232
ATGM	422	282	246	58	12	--	186	186
Hel	33	27	27	27	129	--	27	63
Arty	76	66	66	18	--	72	112	112
Infantry	2,529	1,588	1,345	310	120	--	882	882

Then the formulation of Objective 13 for the division level model would be:

$$10272.9Y_1 + 13242.4Y_2 + 12918.0Y_3 + 3933.4Y_4 + 3115.0Y_5 + 2889.8Y_6$$

$$+ 14873.4Y_7 + 15701.7Y_8 \leq 81000$$

Variables Y_9 through Y_{16} and Y_{17} through Y_{24} represented the same ordering of divisions but located them in POMCUS and CONUS respectively. The identical method for assessing the higher nonrecurring cost associated with the activation of new units was used at the division level, through the use of a separate set of variables (Y_{25} through Y_{48}) which contained the higher NRC coefficients.

(6) Combat Effectiveness Objectives. Imbedded in the goal program in the mathematical model are the design objectives which quantified the desired combat effectiveness of the force. These objectives were: Objectives 1-12 (hard, medium, soft, and artillery firepower on M, D and D+19) and Objectives 16 through 19 (reaction power on M-day).

(a) Firepower Potential Application.* This firepower potential methodology recognizes three types of targets: (1) soft, (2) medium, and (3) hard. A tank is an example of a hard target, an APC is an example of a medium target, and a ground-mounted antitank guided missile is an example of a soft target. The value of this methodology becomes apparent

*More explicit details of the firepower potential methodology are published in CAA Technical Papers TP-73-7 and TP-74-8.

when looking at a system such as the ground-mounted antitank weapon. Its firepower is best employed against hard targets, yet it is most vulnerable to area (soft) firepower such as that of mortars and artillery. The FPP value changes differentially by type of engagement.

(b) Influence of Tactical Posture. Since this study was designed for forward defense, the posture chosen was a Warsaw Pact attack against a US prepared defense. A US force optimized to perform well in this mode will have a defensive character.

(c) US as a Predator. US weapons systems were considered as the predator trying to match their hard, medium and soft firepower to the Pact hard, medium and soft targets (the prey). This was the first principle of design. Its goal was to match or exceed each type of Pact unsuppressed firepower potential with the suppressed firepower potential of the US weapons systems.* An example of such an objective is Objective 1 (match or exceed Pact hard FPP M-day) in the weapons system model.

$$.06047X_1 + .02011X_2 + 0X_3 + .00262X_4 + 0X_5 + .03920X_6 + .00414X_7 +$$

$$.00009X_8 \geq 154.5$$

where X_1 through X_8 are European based weapons system level variables as above. Again by knowing the density of weapons systems in a division, Objective 1 could be formulated in terms of division level units or:

$$7.81685Y_1 + 29.38441Y_2 + 31.38691Y_3 + 7.65095Y_4 + 5.29174Y_5 +$$

$$.29824Y_6 + 33.42960Y_7 + 34.83549Y_8 \geq 154.5$$

This formula represents a D-day hard firepower design objectives for the weapons system model and the division level model respectively. For example, in the fourth term of the first equation, $.00262X_4$, the .00262 is the hard firepower potential of a suppressed ATGM. The 154.5 is the amount of Pact firepower potential in hard targets. The principle of matching combat power in this fashion cannot be proven, but it was postulated that Pact cannot gain ground significantly without in turn exceeding significantly each type of US firepower potential.

(d) Counter Artillery with Artillery. This study exploited two characteristics of artillery. First, of the eight systems considered, US artillery is the only weapon that can hit Pact artillery. Second, artillery is often the first firepower which is available. Because of the range artillery has, it can change targets, in many cases, without the delay required in altering its position. This characteristic is an advantage during Pact penetrations. For these reasons, a further design postulate was that artillery firepower potential must equal or exceed Pact artillery firepower.

*Suppression of US weapons systems was considered as limiting the full FPP available and were selected judgmentally, e.g., the ground mounted TOW was suppressed 85%.

(7) Relationship of Objectives and Priorities. Table 7 reflects the 31 objectives of the goal program and their grouping into priorities.

Table 7. Objective Achievement for TRANSFORM Mathematical Models

Objective		Desired Achievement Level	Priority
1.	Match Pact hard FPP[a] M-day	154.5	10
2.	Match Pact medium FPP M-day	80.0	10
3.	Match Pact soft FPP M-day	102.3	10
4.	Match Pact arty FPP M-day	256.9	10
5.	Match Pact hard FPP D-day	175.2	5
6.	Match Pact medium FPP D-day	89.0	5
7.	Match Pact soft FPP D-day	116.7	5
8.	Match Pact arty FPP D-day	264.7	5
9.	Match Pact hard FPP D+19	433.9	5
10.	Match Pact medium FPP D+19	193.9	5
11.	Match Pact soft FPP D+19	266.2	5
12.	Match Pact arty FPP D+19	360.1	5
13.	Peacetime European manpower ceiling	81,000 pers	3
14.	Total wartime supported strength	370,000 pers	4
15.	Weight to be lifted	0 STON[b]	8
16.	Objective 1-Total static & mobile FPP	165	7
17.	Objective 2-Total mobile FPP	107	7
18.	Objective 3-Hard static & mobile FPP	50	7
19.	Objective 4-Hard mobile FPP	40	7
20.	Weapons system crew size	0 Pers[b]	9
21.	Recurring costs	$6,258.3 Mil	2
22.	Nonrecurring costs	$7,771.6 Mil	2
23.	Number of tanks in the active slice	3,644	1
24.	Number of light antitank tracked vehicles in the active slice	4,606	1
25.	Number of armored personnel carriers in the active slice	4,411	1
26.	Number of ground mounted antitank weapons in the active slice	4,543	1
27.	Number of mortars in the active slice	1,657	1
28.	Number of helicopters in the active slice	576	1
29.	Number of artillery tubes in the active slice	1,696	1
30.	Number of infantrymen in the active slice	25,929	1
31.	Nonrecurring Cost, $ value of currently authorized prepositioned materiel configured to unit sets (POMCUS)	$2,202 Mil	6

[a] Firepower potential

[b] Although objective values for lift and crew size were not listed, the minimization of these values was an objective of the goal program which attempted to limit the size of these two quantities.

D. Summary of Goal Programing for TRANSFORM. This study used goal programing as a system mix/force mix tool of discovery. It used goal programing to design mixes within a set of requirements which ranged from absolute (stay within cost constraints) to desirable (minimize strategic lift requirements). By investigating the impact of reordering priorities, useful insights were gained on potential force structure tradeoffs. By using goal programing with conventional units and restructured units as candidate variables, new perceptions of repackaging of divisional forces were gained. Goal programing could also have been used to design conceptual divisions.

III. Methodology Summary.

The TRANSFORM study adopted and extended the CONAF mathematical model and methodology. It used a systems approach to force design which accounts for individual weapon system potential performance and support and manpower required. A mathematical model which states desirable force objectives and resource constraints is translated into a problem statement in which the variables represent types of units - conventional, restructured or conceptual. Goal programing was used as an optimization technique to discover force and systems mixes which best met an ordered set of conditions. Designed mixes were verified by static analytical comparisons with the base case and by dynamic combat simulation.

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TITLE: Alternative Concepts for Evaluating Division Force Structures

AUTHORS: Dr. Philip H. Lowry and Henry J. Schroeder, Jr.
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ABSTRACT: As a base line to serious thought on methodology, and the application of current or new OR/SA tools in support of the Army of the 1980's, offered here is an overview of 30 years of associated activity in and with the Army.

In this overview authors concentrate on selected force structure analyses and the concepts, methodologies, and tools used in past key decision making relating to division restructuring since World War II. The authors then assess the state-of-the-art and utility of OR/SA methods for evaluation of doctrine and procedures, for the analysis and assessment of Army operations and of force planning and design. The interaction between OR/SA techniques, test and evaluation, staff analysis, and the decision making process will be projected and related issues discussed.

Authors refer to selected specific relevant events in which they participated to illustrate the interaction between methodology and division restructuring decisions.

SUBJECT: Alternative Concepts for Evaluating Division Force Structures

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Dr. Philip H. Lowry

ORGANIZATION: General Research Corporation

Since 1938 the US Army has fielded five new division structures, as shown on this slide.

NEW US ARMY DIVISION STRUCTURES

1938	TRIANGULAR DIVISION	(McNair Report)
1941	AIRBORNE DIVISION	
1956	PENTOMIC DIVISION	(PENTANA Study)
1961	ROAD DIVISION	
1964	AIRMOBILE DIVISION	(Howze Board)
1978	?	(DRS Study)

Two of these divisions were specialized--the airborne and airmobile—the remainder amounted to a major reorganization of the combat elements of the US Army.

Three of the new divisions were extensively studied and tested before being fielded, as shown, and two were not.

The studies and test reports have many elements in common. Nearly all emphasized tactical effectiveness and flexibility in combat.

Our emphasis this afternoon will not be on improvements in evaluating tactical effectiveness and flexibility. Rather, we will concentrate on some alternative concepts that we believe have been neglected.

The concepts we will discuss can be divided into three categories.

ALTERNATIVE CONCEPTS

STRATEGIC

TACTICAL

REAR AREA

The reason for discussing strategic concepts in evaluating divisional force structures is that the US is in a fundamentally different position from any other major power. Therefore we cannot off-hand adopt the structures of the German Army, the Soviet Army, or the Israeli Army.

The US Army is not structured to fight in the CONUS or in adjacent land areas. Canada and Mexico pose no threat. Our Army has been structured to fight thousands of miles across oceans. The Soviets say no nation, however powerful, can support in peacetime the Army required to fight and win against a major adversary. Similarly, we find it infeasible to maintain in Europe all the Army forces required to defeat a Warsaw Pact attack. Instead, we plan to reinforce the troops in Europe in an emergency. We have 40 percent of our active combat battalions deployed in Europe and about 50 percent of our battalion equipment there.

It follows that division structures should be evaluated in terms of the time, required effort, and feasibility of moving it from the CONUS to the theater and to sustain it once it gets there. This is in addition to its evaluation for tactical effectiveness once deployed.

In other words,

ALTERNATIVE CONCEPTS

STRATEGIC

- BALANCING STRATEGIC MOBILITY AND TACTICAL EFFECTIVENESS

TACTICAL

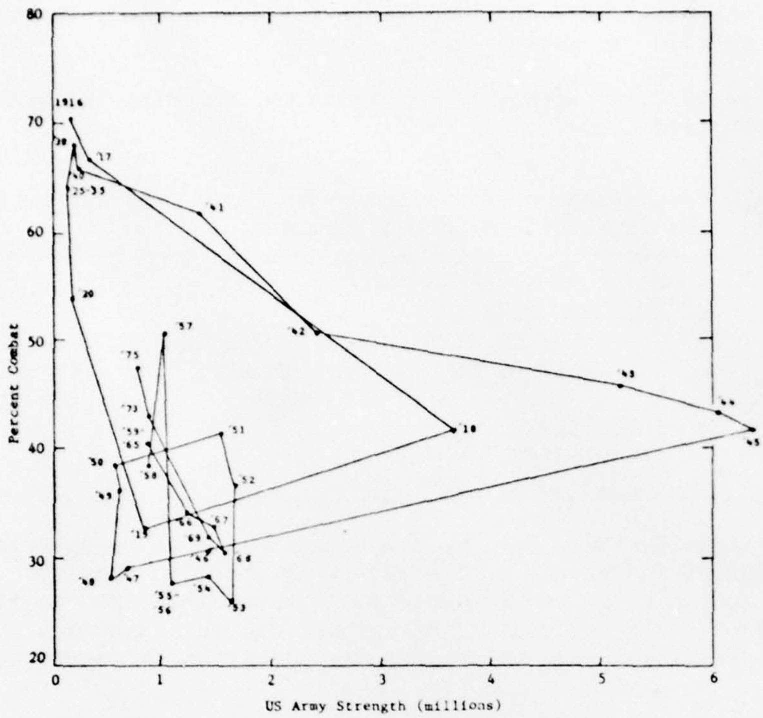
REAR AREA

It is of little value to have an effective force that cannot be deployed quickly overseas. It also is of little value to deploy quickly an ineffective force. Thus, a balance is needed between tactical effectiveness and speed of deployment.

* * * * *

Lets look at history from two points of view: our response in the past to the strategic requirements, and our response to tactical requirements.

First, our response to the strategic requirements.



This chart is a plot of the percent of manpower in combat divisions and non-divisional combat units, shown on the ordinate, against the total strength of the US Army over the last 60 years.

We start in 1916 at near 70 percent and drop during World War I to a little over 40 percent, all in Europe. Then we gradually increase back up to 70 percent in 1940. World War II follows almost the same slope as World War I down to about 40 percent. Then it starts back up.

Then came the Korean War.

This was the first war in the history of the US when we were in a hurry; when we needed fast deployment of combat forces.

The slope, shown between 1950 and 1951, was just opposite to the experience of World War I and World War II. We were converting service support units to combat units, both in Japan and in the US.

Then we converted them back again for two reasons. One was the development of a logistic infrastructure in Korea; the other was the deployment of forces to Europe and the development of an infrastructure there.

After the close of the Korean War, there was no perceptible trend. The spike in 1956 was an artifact of the Pentomic reorganization.

Then came Vietnam. Note that the slope followed closely the slope of World War I and II. We were not in a hurry.

Beginning in 1970, we appear to be gradually following the peacetime years of the 1920s and 1930s.

* * * * *

Now what are the lessons on divisional structures that can be drawn from this chart? What evaluations should be made?

EVALUATION NEEDED

- RESPONSE TIME
  - MARRY-UP WITH PREPOSITIONED EQUIPMENT
  - TRAINING READINESS vs TRANSIT TIME WITH EQUIPMENT
- FLEXIBILITY
  - PIECEMEAL MOVEMENT
  - NO INFRASTRUCTURE

First is response time. Can the new organization marry-up with prepositioned equipment faster and more effectively than the previous organization? What are the requirements for flexibility? No one likes piecemeal movement and commitment of divisional forces. But does the new organization suffer less degradation than the old if it is committed to this fashion?

Finally, how does a proposed division structure compare in strategic response time to a theater with no infrastructure? How long can it operate in an emergency, such as Korea, without a large-scale logistic base?

The next historical trend concerns our response to tactical requirements and one aspect of technology.

TRENDS 1945-1975		TECHNOLOGY 1945-1975 (lbs of fuel to move one ton one mile)	
ARMY DIVISION			
Weight	x 2	Cargo aircraft	} ~ 0.2
Fuel consumption(tons)	x 6	Wheeled vehicles	
Ammo consumption(tons)	x 4		
Total resupply(tons)	x 4	Tracked vehicles	} ~ 0.4
AIR FORCE		Combat aircraft	
Tons/Sortie	x 10	Helicopters	~ 1.0

Here we compare the current division with the World War II division, on the left.

The gross weight of an Army division has doubled; the fuel consumption to move the division a given distance has increased six times. The planned expenditure of ammunition has quadrupled—as has the total planned resupply.

The Army is not alone. The tonnage in theater required to fly one Air Force combat sortie has gone up 10 times.

Now one reason for this increase in fuel consumption has been the lack of any technological improvement in fuel economy. The World War II 2½ ton truck, the C-47, the current 5-ton truck, and the 747 aircraft all expend 0.2 lbs to move one ton one mile. (This is gross weight, not just payload.) Tracked vehicles and combat aircraft average about twice the fuel consumption. And helicopters more than double that figure. This explains the increase in fuel.

Not shown are other increases in fuel consumption: Air Force requirements, air defense and EW radar requirements, etc., which will compete with Army divisions for fuel and the resources to distribute fuel.

* * * * *

What are the implications of these trends on the division structure? And how can we evaluate the potential logistic vulnerability and tactical effectiveness of alternative division structures? And how should tactically desirable division structures guide R&D? Some of the needed evaluations are shown on this slide.

EVALUATION NEEDED

- WEIGHT vs COMBAT EFFECTIVENESS
- WEIGHT vs STRATEGIC MOBILITY
- WEIGHT vs TACTICAL MOBILITY
- WEIGHT vs SUPPORT MANPOWER

Weight here includes both the weight of major weapons systems and the weight of resupply.

The weight and resupply requirements affect tactical mobility, which we have separated from strategic mobility. This leads to our second major category, tactical concepts.

ALTERNATIVE CONCEPTS

STRATEGIC

TACTICAL

- IMPACT OF NO ASSURED AIR SUPERIORITY
- IMPACT OF NUCLEAR THREAT

REAR AREA

We have grouped the air superiority and the nuclear threat because we believe that many of the requirements to counter these threats are similar, if not identical.

General Kerwin, I believe, is the only officer on active duty with personal experience of the impact of a serious air threat on tactical army operations. We are not talking here about the management of air defense units,

which has been studied, but the impact on division operations as a whole, particularly the movement of brigade- or division-sized units.

We note, for example, that a major German concern in World War II was not the direct destruction of units and supplies by air attack. Rather, it was the uncertainty when units and supplies would arrive. The unpredictable delays produced by the response of troops to the threat of air attack and repair times of bridges, etc., were far more significant than the actual losses. But we note that no methodology currently exists to take account of these indirect but vital aspects of combat operations under air inferiority.

If limited nuclear operations occur, we believe that similar effects will occur. The effort to avoid unnecessary exposure to target acquisition by the enemy, the enormous gaps in our knowledge of human reaction to nuclear effects, and the difficulties and delays forecast in assessing the impact of limited nuclear strikes on army forces, if they occur, will produce uncertainties similar to those mentioned by the Germans.

Examples of needed evaluations are shown on this slide.

EVALUATION NEEDED

- MOVEMENT CONTROL vs SPEED OF RESPONSE
- DISPERSAL AND CONCEALMENT vs EFFICIENCY
- INTEGRATION OF G-3, G-4, AD, MP FUNCTIONS
- TECHNOLOGICAL AND TRAINING REQUIREMENTS

If the management of road traffic is centralized and must be flexible, what is its impact on speed of response of large units and resupply? How practical is it to change the routes used by a division-sized unit in mid-stream because of air and nuclear threats? What is the impact of night and bad-weather movement? What are the reporting and communication requirements of the units making the movement and of the traffic controllers (MPs)?

Dispersal and concealment of divisional units tend to increase response time, increase the use of radio, and dispersal of support units may decrease efficiency in the sense that productivity per man is less and the impact of casualties greater.

The DRS study makes an important point about the need for integrating and collocating G-3 and G-2 functions. Under conditions of air inferiority and high nuclear threat, there may be a need for much greater information transfer between the G-3, G-4, air defense, and military police functions.

Finally, we are not aware of an evaluation of the impact of new technology on the ability to move under air inferiority with an acceptable degree of concealment against enemy target acquisition means. What is the balance between night driving aids and position location systems and enemy airborne detection systems? What are the training requirements for drivers, MPs, and traffic managers? How do these requirements impact on the divisional structure?

* * * * *

Our last topic is rear area security. This is closely related to the impact of a serious air and nuclear threat, with additions.

ALTERNATIVE CONCEPTS

STRATEGIC

TACTICAL

REAR AREA
- SECURITY OF REAR AGAINST
  - OVERT THREAT
  - COVERT THREAT

The major dilemma here is that security against covert threats—enemy agents, infiltrators, sabotage teams—requires concentration. Security against an overt threat—nuclear strikes, conventional air attack, and airborne operations—requires dispersal.

REAR AREA SECURITY

- AIR THREAT
- AIRBORNE/HELIBORNE THREAT
- INFILTRATORS
- SABOTAGE TEAMS
- AGENTS
- REFUGEES

Enemy agents, line-crossers, sabotage teams, are said to pose a much larger threat to the US Army in Europe than any previous experience in Vietnam, Korea, or World War II. Presumably, the major objective of these elements will be nuclear units, communications centers, and headquarters. The wide frontages existing in US Army sectors in Germany tend to favor these enemy capabilities. The planned depths of division boundaries tend to increase the division's requirements for rear area security. Moreover, divisions will contain nuclear custodial units, and corps elements, such as LANCE, and even PERSHING units may be located within divisional areas.

The Soviet Union apparently does not agree with the conventional wisdom that major airborne operations are neither feasible nor desirable in the face of NATO's air defenses. They have increased rather than decreased the number of parachute troops and increased rather than decreased the combat equipment designed for airdrop or air-landed employment. They seem to be copying and modifying US heliborne operational concepts of Vietnam and adapting them to the requirements of a theater campaign in Europe.

Finally, refugees are an ever-present problem in any war fought in a densely populated area, such as Central Europe. The problems here relate to security—since enemy agents can be among the refugees; local control—which may require divisional resources; road management; and sanitation, which the Soviets discuss in detail. It is interesting to note that no recent thought has been given this problem, particularly in respect to the impact of refugees on combat and resupply operations.

To summarize, we believe that a number of important concepts in evaluating division force structures have not received the attention they deserve. We have discussed the five shown here.

ALTERNATIVE EVALUATION CONCEPTS

- STRATEGIC MOBILITY
- LACK OF ASSURED AIR SUPERIORITY
- SURVIVABILITY UNDER NUCLEAR/CHEMICAL ATTACK
- TACTICAL UNIT MOBILITY
- REAR AREA SECURITY

Strategic mobility is a dominating factor as long as the Army's mission is to fight overseas and as long as we cannot plan on a leisurely period of preparation and predeployment. The increased weight of weapons systems and the increased weight of resupply required for tactical effectiveness tend to reduce strategic mobility. Alternative division structures must be evaluated to achieve a balance between these conflicting demands. O.R. techniques exist but have not been applied to such an evaluation.

Air superiority cannot be assured in a future war in Central Europe. Evaluating alternative division structures in terms of their relative viability involves far more than the effectiveness of air defenses. O.R. techniques and much of the required data do not exist and must be developed.

The survivability under nuclear and chemical attack of alternative division structures has many elements in common with air inferiority. Evaluation involves more than modifications in the design of systems and training. O.R. techniques and data exist but have not been applied.

A key element in countering a numerical inferiority is superior tactical unit mobility. Evaluating the tactical mobility of alternative division structures requires O.R. techniques to account for the impact of C^3, dispersal, and the air, chemical, and nuclear threat on tactical mobility.

The covert threat and the overt threat from air, chemical, and nuclear threat and wide frontages increase the depth of the battlefield and the vulnerability of rear areas. O.R. techniques can be adapted to evaluate alternative division structures in terms of the often conflicting requirements of efficiency and survivability.

ABSTRACT

TITLE: Uncertainties and Inadquacies in Theater Level Combat Analyses

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ORGANIZATION: Organization of the Joint Chiefs of Staff, Studies, Analysis, and Gaming Agency, The Pentagon, Washington, D.C. 20301

ABSTRACT: This paper describes some of the uncertainties and inadequacies associated with the analysis of theater level wargames. Recent experience in a two year international study pointed out various problem areas in data inadequacy and variability in military judgement. Factors which appeared to affect the outcomes of the games are discussed and a proposed framework of investigation for future studies is proposed.

ESTIMATED TIME REQUIRED FOR PRESENTATION: 50 minutes

CATEGORY APPROPRIATE FOR THIS PAPER: Force Structure Analysis

Uncertainties and Inadequacies in
Theater Level Combat Analyses

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The theme of this year's symposium asks us to "Look Ahead. . . and explore both applications and methodologies appropriate for . . . OR support to the Army of the 80s." It is believed by some with the present state of the art using expanded techniques for simulating combat in conjunction with improvements in data manipulation, that the analyst is now provided with the potential for improved analyses of problems such as net assessment, disarmament options, as well as force planning and structuring analyses by using theater level models. This approach may be overly optimistic in light of uncertainties and inadequacies which have been shown to greatly affect the results of such models. These uncertainties and inadequacies affect basic assumptions as well as data inputs. However, theater level games offer us the only opportunity, short of actual conflict, to simulate the interactions of large complex general purpose forces over time in a theater of operations. This capability is important enough to warrant further investigation.

The purpose of this paper is to identify factors which have been shown to affect the results of theater level combat analyses. In so doing this paper may temper the optimism engendered by this year's theme because it addresses fundamental problems which will remain even as we progress toward the use of more sophisticated models and methodologies. However, it is not the purpose of this paper to degrade the usefulness of theater level models in addressing the topical areas indicated above, but it is hoped that at the conclusion of this paper reasonable judgments may be made by the reader concerning how the results of such analyses must be interpreted.

SAGA recently participated in a joint international study where force level analyses were conducted through the use of theater level models.1/ I intend to use this study as a means of highlighting uncertainties and inadequacies in theater level games. This work crystalized the problems of evaluating theater level results in such a manner that it was felt advisable to inform the analytic community of some

1/ "Study on Warsaw Pact and NATO Conventional Force Capabilities," Phase IV, IMSM-233-77, May 1977, (NS).

of the problem areas encountered. During the conduct of the study, it became clear that there were certain parameters, interactions, and data present which had a disproportionate effects on model results. These associated uncertainties were by no means readily apparent nor easily solvable. They were, to some extent, identified, and their impact outlined for the reader in the study. The joint study pointed out that three parameters in particular appeared to affect the intensity of combat in theater level models. These were rates of advance, casualty rates, and the density of forces in combat. In addition, I would add a fourth area, that of data inadequacies in external factors which affect theater games.

Each of these areas will be separately addressed during the remainder of this paper. The fourth area covers a broader perspective where uncertainty exists not only because of the range of values associated with individual data entries, but also in assessing the validity of measurement indices being used. In addition, some other external factors impacting upon the use of theater models are addressed in this section.

BACKGROUND

First of all, a definition of uncertainty as it will be used in this paper is necessary. Webster's dictionary states that it is a word meaning, ..."a condition of doubt; not sure." I intend to develop the areas affecting the results of theater level combat analyses listed above, with emphasis on the doubtfulness of each parameter. In the context of this definition, the joint study effort provided the following words on uncertainty in theater level gaming. "The spread of results among theater level games may be regarded as indicative of the uncertainty of any prediction of duration of the war produced by wargames or analytical simulations supported by military judgment. The rates that are used are based on the best military judgment available and the results provided nothing better than an indication of likely duration and recognition that it represents only one of a range of possibilities."1/

The joint effort mentioned earlier was conducted during the period April 1975 through July 1977, when the results were briefed to the NATO Military Committee. This international study was unusual, not because of its content,

1/ Working Papers, Study on Warsaw Pact and NATO Conventional Force Capabilities, March 1976 (S).

results, nor participants, but because of the methodological approach used. Multiple dynamic models were used to evaluate the capability of existing forces in a Central European scenario. The three countries used different deterministic models, which used a common set of starting conditions and data, similar or identical gaming rules to govern the conduct of game play, and a prescribed set of measures of capability to evaluate their results.

It was recognized early that the models resolved combat at different levels of aggregation, contained different attrition methodologies, and employed different approaches to game play. Recognition of these dissimilar approaches led the participants to accept the idea that a non-interested party was necessary to evaluate the results. Therefore, the SHAPE Technical Centre (STC) was tasked to act in that capacity. STC performed an excellent service for the study participants by evaluating all model results and pointing out certain uncertainties and inadequacies as potential problems which must be investigated.

RATES OF ADVANCE

Rates of advance refer to the speed at which formations advance or withdraw during the course of a battle period. For this study SAGA adopted the rates of advance used for the CEM and ATLAS models for 24 hour battle periods. Being been placed in a position of defending the use of such "historical" data, SAGA researched several studies purporting to define and validate these rates of advance.

The origin of these rates of advance can be traced to papers done in 1954 by Parson and Hulse, both of Operations Research Office (ORO). Parson and Hulse developed tables of infantry and armor battalion hourly rates of advance against specific types of resistance. These tables were then incorporated into wargaming handbooks prepared for the Army War College. In 1966, the Historical Research and Evaluation Organization (HERO) extracted considerable data on 37 Battles of WW II and Korea. HERO used this data to analyze material replacement policies. Subsequently, the data base was revised and expanded by HERO for a study on casualty rates. As the initial Parson and Hulse work showed a linkage between casualty rates, force ratios, and movement, the Research Analysis Corporation (RAC) used the HERO data base to conduct an in-house study in an attempt to derive insights and validate the existing rates of advance and force ratio tables used in wargames.1/

1/ Wainstein, L, "Progress Report on Historical Data for Simulations," WP-9, Institute for Defense Analyses, 1972. P. 1-4.

Although it appeared that adequate empirical evidence was available to develop the rates of advance tables, several recent studies have cast doubt upon their validity. Leonard Wainstein, in an Institute for Defense Analyses report, states that . . . "the two studies were rough, hasty efforts, inputs to a study, not careful pieces of research . . . numerous specified and unspecified qualifications surrounding the data have been overlooked . . . the variation among the sizes of the sample units examined in both papers makes questionable their generalized findings."1/ Secondly, HERO, once a proponent of a linkage between force ratios and rates of advance, no longer holds that opinion.2/ Further validation efforts on the force ratio linkage continued with a study done by VERTEX in 1974 using historical data extracted from the Lorraine and Ardennes offensives.3/ This same data was given to the University of London last year by the SHAPE Technical Centre. The University was charged to determine if any meaningful relationships exist in the data which links force ratios to the advance of forces, mechanized or infantry. The results: "No discernible relationship could be determined using statistical analysis--there was too much noise in the data."4/

Consequently, rates of advance tied to force ratios used in a whole generation of gaming models appear to be without logical or historical basis. However, there is some expected range of value within which these rates of advance exist. Questions concerning the validity and use of such factors represent an uncertainty in the evaluation of model outcomes. If the movement of the FEBA is the sole measure of effectiveness used, the determation of adequate rates to simulate the movement of formations is crucial for valid assessments of divisional and theater level conflicts.

CASUALTY RATES

The joint study also pointed out that differences in casualty rates were a second factor which affected the intensity of combat. Research in this area led us to investigate

1/ Wainstein, L, "An Examination of the Parson and Hulse Paper on Rates of Advance", IDA Report P-991, Dec 1973.
2/ Dupuy, T.N., Response to SAGA Opinion Survey, June 1976.
3/ Cockrell, J.K., "Research Study on Predictive War Game Factors," RMC-VERTEX Corporation, STC Contract C-72-03, March 1973.
4/ Goad, R., Comments made to author during meetings conducted on the Study on Warsaw Pact and NATO Conventional Force Capabilities, March 1977.

areas of uncertainty in more than just attrition. Attrition rates are developed in several ways depending upon the level of resolution of the model being used. Theater level models suffer from a requirement to aggregate the effects of high resolution simulations. J.A. Stockfisch, in a recent Rand Report, treats this problem in terms of levels of abstraction. A high resolution model, in order to be manageable, is abstracted from the larger context of an engagement. A hierarchy is established in some manner where some analytic and some judgemental outputs from one model are treated as inputs to another.1/ Hence, some means of assessing the results of high resolution attrition in the context of theater warfare is required.

The current operational theater level models generally employ several means of using aggregated data for assessing attrition to large units. First, the so-called "historical" approach places emphasis on measuring changes in opposing force ratios in order to provide an entry to personnel casualty tables. Secondly, entry values to attrition tables are determined by calculating a proxy variable or index. This technique requires a calibration to adjust the model output to a set of known conditions. Finally, a hierarchial chain may be established to pass the results of attrition progressively from lower to higher unit levels. This chain may be composed of a series of models or may be internally generated in one model.

Among these three possible methods, military judgement has most often been used to fill gaps in historical data; to decide what weapon will fire at another; to develop weighting schemes whereby heterogenous weapon systems are converted to a common additive base, and to judgementally determine what data is passed from one model to the next in a hierarchy. This is by no means an all encompassing list. Therefore, I feel that it is a fair statement that some form of military judgement can be found at every level where data is collected or manipulated to establish the basic aggregated attrition effects used in theater level models.

The first experimental results evaluated during the international study showed that variances between games could be traced to several factors. The first key to differences in casualty rates was indicated by apparently different impressions of what constitutes high intensity combat. Many other analytical agencies suffer in this regard by stating

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1/ Stockfisch, JA. "Models, Data, and War: A Critique on the Study of Conventional Forces", Rand Report R-1526-PR March 1975, Pg 9.

that their model or their studies used the Middle East conflict of 1973 as a basis for estimating battle intensity. This conflict emphasizes the greater attrition rates expected of modern weapon systems. It was our misfortune, initially, to discover that while all three study participants said they based their model calibration on Middle East data, each had selected a different conflict period as their base. Consequently, the term "calibrated to Middle East experience", itself connotes some measure of uncertainty.

SAGA attempted to resolve this issue by concentrating on improving the existing methodology for assessing attrition used in the IDAGAM model. The STC had earlier pointed out that each model used a different attrition methodology and, all used a Lanchester formulation in obtain final results. The SAGA methodology was based upon a technique which used ideal linear weights for expressing the value of weapons systems. This approach was developed by the Institute for Defense Analyses (IDA). Calibration of the model was necessary because its basic assumptions required that some form of scaling be made to convert potential casualties to actual casualties.1/ However, the initial calibration basis was not clearly prescribed causing some measure of confusion.

In the Army COMCAP Study (1973), the potential kills were related to actual kills by a scaling factor (i.e., the expected weapon kill probabilities at specified ranges are to be interpreted as potential kill probabilities, but that there exists a "fog of war" which may degrade this capability such that actual losses are somewhat less).2/ SAGA prepared an opinion survey keyed to estimating losses in a European scenario in order to establish a revised scaling factor to be used in IDAGAM for the joint study.

The results of this survey, conducted during the summer of 1976, formed the basis for calibration of the IDAGAM model. The survey showed that military judgment, per se, is a variable which should be treated as an uncertainty. The pure military judgment answers by military historians, senior commanders, War College students, analysts, and operations and staff officers were used to construct the scaling factor for losses. Model results could then be assessed using a confidence interval developed from the survey answers.

1/ Anderson, L.B., et al, IDA Ground-Air Model I (IDAGAM), Vol I, Comprehensive Description, Institute for Defense Analyses, Oct 1974, p. 47.

2/ Holter, W, Appendix D, NATO Combat Capabilities Study (COMCAP II) 1973 (S), Appendix D is unclassified.

The variability in military judgement was also noted by another study participant when they used an opinion survey to revise certain planning estimates affecting the tactical deployments of forces. This survey also allowed a confidence interval to be constructed. Results produced by its model were later evaluated using this confidence interval.1/

An interesting and thought provoking analysis of the use of models incorporating judgmental inputs, as well as those developed from analytic means, was incorporated into the Rand Report cited earlier. It should be required reading for all new theater level "modeleers". In it, Stockfisch states that the variability inherent in much of the judgemental data provided for use in theater level models produces results which become ". . . state histories as hypotheses in the scientific meaning of the word. At a minimum they are are creatures of both the model and the set of data fed into the model".2/

To summarize; different impressions of the intensity of combat produced variances in the results of these theater level games. Rates of advance and casualty rates both contributed to these different impressions. Military judgment has been shown to play a considerable part in the derivation and development of much of the attrition data which is used in theater level models.

DENSITY OF FORCES

The third area which affected the intensity of combat, was the density of forces in contact. This parameter has been observed to be affected by several interactions which have both judgemental and analytic foundations.

The density of forces is affected by the rate of tactical commitment of forces into battle. This was seen as a judgemental effect in the joint study. The early commitment of large numbers of forces creates massive theater resource allocation problems. The slower the rate of commitment, the easier it is for the theater and corps commanders to replace losses, resupply, and allocate reserve forces to critical areas. Density of forces is also linked to casualty rates. The more forces in contact the higher the force ratio, and higher attrition should be observed.

1/ Working papers, "Study on Warsaw Pact and NATO Conventional Force Capabilities," Sept 1976. (S)
2/ Stockfisch, J.A., Ibid., p 25.

To the extent that improved technology creates exchange ratios of two and three to one, for the defender, the forces in contact should remain small. However, there is a point where quantity overrides quality and the Lanchester effects of mass govern the losses incurred by the smaller force and allows a large number of units to be in contact.

Another military judgement decision specifies which forces may come in contact. In particular, the number of tactical positions planned to be occupied during a battle period can increase or decrease the density of forces in contact.

There is also an inherent problem in that most theater level models play only one set of tactics. Simulations of theater warfare using only one tactic incorrectly estimate the true capabilities of the forces involved if different concepts of execution of the conduct of the defense or offense exist. In addition, the employment of one set of tactics implies that standardization of forces exists. This assumption may be a driving factor in studies which address contrained supply levels.

I am also convinced that we too often employ or commit opposing forces based upon Western military judgments, rather than upon aggressor tactics. This is exacerbated by intelligence agencies which provide force laydowns which meet model input requirements, but are caveated to the extent that the force deployments may not be in accordance with aggressor intentions. The uncertainty which one must associate with intelligence inputs is also true of several external factors. These points are examined in the next section.

EXTERNAL INADQUACIES

This area encompasses problems of uncertainty and inadequacies in model inputs and the treatment of factors external to the model framework. Considerable effort and resources are now being expended in the development of large scale data bases for the newer theater models. J.N.D. Gupta in a recent article in INTERFACES states: ". . . the usual belief is that data requirements are dictated by the model and that the data exists some place. It is true that the usability of data is determined by the model, but the model does not provide data."1/ This is true of the current generation of theater level models and it will become a more acute problem as data requirements expand with newer models.

1/ Gupta, J.N.D., "Management Science Implementation: Experiences of a Practicing Manager," INTERFACES Vol 7, Number 3, May 1977, pg. 88.

The US has relied heavily upon historical and empirical data in order to develop inputs for theater level games. Wainstein pointed out that there is a paucity of continuing historical research to determine planning factors for large unit operations. The limited efforts at data generation using actual records on WW II are complicated since, for the majority of battles, data is spotty or inadequate and its extraction has been time consuming and expensive. Equal treatment of opposing forces is complicated because all the German records were returned to Bonn a few years ago. As a result of the effort required to extract relevant model oriented data, many studies have relied upon the same set of historical data. Hence, relationships may exist, such as in the tenuously empirical basis of rates of advance factors, which have limited foundation in empirical evidence.1/ I would point out that the above conditions still exist. Furthermore, the condition exists without definition of the bounds of uncertainty surrounding the data used.

Stockfisch points out that . . . "any number when confronted by an analyst, decision maker, or other interested party, should be probed. . . ."2/ It is in the best interests of all concerned if it can be determined whether the number was based upon empirical evidence or military judgment, but it begs further analysis whether judgmental or analytic derivations provide the best source of data for games. This problem is compounded because there are conflicting theories among analysts concerning the applicability of historical and empirical analysis to simulations of modern warfare. It is not the purpose of this paper to degrade the use of one or the other approaches, only to point out that both have associated uncertainties and must be treated accordingly. The manipulation of numbers into measures of capabilities (MOC) allows connotations of military effectiveness to be drawn by analysts which are based on single point estimates. These indices, normally, are drawn from changes in proxy variables. Such indices are only surrogates for reality and are abstractions of combat used for convenience and manageability. The uncertainty in the numbers raise questions on the validity of well known indices as being truly reflective of reality. The uncertainty associated with the development of most deterministic data used in theater models should be clear at this point. Nonetheless,

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1/ Wainstein, L, WP-9, "Progress Report on Historical Data for Simulation," Institute for Defense Analysis, January 1972, pg. 1-4.
2/ Ibid, pg. 6.

the use of indices allows conclusions and observations to be made. It is important that this use of indices be restricted to the identification of trends rather than as an absolute measure of capability.

There are also numerous external influences upon theater level models. In particular, it was extremely difficult to determine the basic organizational structure of certain allied forces. Consequently, assumptions concerning their organization, tactics and requirements were made. Considering the amount of detail required to drive the current theater models, this area needs more emphasis. A point in fact has been the recent reorganizations of the German, UK, and French armies. The revised organizations, when projected into the 1980s for use in game simulations, represent a source of uncertainty which must be addressed.

Next, theater level models incorporate air power in remarkably different ways. This presents some problems. Several efforts have supposedly developed methodology which represents the air ground interface. To date no one study has provided an agreed methodology usable by all Services. Consequently, in the current theater level models, air is often treated as just another parameter and most often in the absence of theater air constraints or allocations. Additionally, air weapons data is often based upon test range and/or contractor estimates, particularly those associated with precision guided munitions. The relatively high estimates of probabilities of kill for both ground and air PGMs beg further study on the synergistic affects of their joint use on the battlefield. This is compounded by the fact that there is no clear definition of which targets will be struck by ground and which by air. The linkage of theater level air and theater level ground combat is difficult, to say the least. The inadequacies of ground attrition data in the presence of perhaps overly optimistic air estimates present the user with a problem in weighing the contribution of both realistically.

And finally, the a revised US Army doctrine for offense and defense, outlined in the recently published FM 100-5 has come under fire from several sources. It is known that wargames were used to test this new doctrine. It is possible that the uncertainties associated with the fixed inputs used in those games raise questions concerning evaluation of the results. The risk is high, in the face of the opposing threat, that areas of uncertainty which impact upon tactics and doctrine may not have been fully explored.

CONCLUSIONS

The uncertainties identified in this paper should temper some of the optimism engendered by the potential usage of theater level models. It should also be remembered that no matter how detailed, a model can not yet simulate the most critical factors recognized by senior commanders, namely training, morale, and leadership. I would venture to say that until such time that sufficient effort is placed upon resolving the uncertainty factors identified, the results of theater level games used to evaluate these topical areas will need to be qualified.

However, the prudent use of theater models in examining the various aspects of net assessment, disarmament options and force planning can provide a guide to the decision maker. The recognition that uncertainty exists in several key areas can be addressed during the conceptual phase of any study. It is sufficient to identify excursions or sensitivity analyses which bound the key areas affecting the intensity of combat. In the future, uncertainties will not go away simply by building better mousetraps. Since the projected models require significantly more data than the current model, the risks associated with not investigating the sensitivities of parameters causing uncertainty may become unacceptable.

In addition to providing bounds upon uncertainty in areas of investigation, a study to determine appropriate rates at which formations move in combat is absolutely essential to improve theater level models. In the near future two major field tests are to begin. Instrumentation can be incorporated in these tests which may capture the necessary required data. While only be a beginning, it would direct attention to critical areas of uncertainty which are important to the analytic community.

In my opinion, the best use of theater models will be to provide a means of assessing relative changes in well defined areas of investigation. It must be emphasized again that the current and projected models should not be used to provide single predictions of conflict outcome. On the other hand, relative changes can be evaluated by the use of trend analysis techniques using multiple, rather than single, measures of capbility.

It is hoped that the reader, in light of the above discussions will feel more comfortable in interpreting the results of theater combat analyses. SAGA has initiated a new study to support the development of the Joint Strategic Objective Plan (JSOP). It is a developmental effort where some of the above

uncertainties in force planning may be quantified and the decision makers made aware of their impact upon the development of force requirements for Central Europe. The methodology and model to be used for this study will capitalize on the observations and efforts of the recent international study. Its successful conclusion will, hopefully, provide a better appreciation of uncertainty and risk in selected areas of force planning. It may provide an excellent basis from which we can return again to this forum in the future.

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A Proposed Probabilistic Monte Carlo Analogue Concept

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1. BACKGROUND. The purpose of this study is to develop a credible analytic model that would handle combat between opposing Red and Blue divisions involving many kinds of weapon systems. Typically, these involved hundreds of tanks and light armored vehicles. Vehicle and ground mounted anti tank weapons, helicopters and artillery were also included in the mix of weapons. It was considered desirable that the model be capable of standing alone and not have to rely on other engagement or battle models. Another specification was that it be fast running. Since Monte Carlo methodologies did not satisfy these requirements, an analogue of a Monte Carlo concept was developed for handling fairly complex probability problems involving time dependent events in a rather straight forward routine manner. The analytic form of the attrition equations allowed them to be programed on a small hand held electronic calculator. It was suggested that the probability methodology needed to be presented in terms of a challenging real problem if it were to be meaningful and convincing. The methodology will therefore be applied within the context of the analytical combat model referred to earlier. Model development to date has been restricted to weapons firing at other weapons, rather than being fired at area targets in an indirect fire mode.

2. DISCUSSION. Important effects from reaction times, flight times, time between shots, single shot kill and survival probabilities of the targets and the firers, the number of shots, the probabilities of terrain line of sight, detection and acquisition, the combinations of weapon types in each sub-engagement, and the numbers of sub-engagements (which are obtained from using targeting rules to be described later) have been incorporated into the probability equations.(Symbols are described in Figures 13-16).

a. Attrition Results. Figure 1 titled, "Examples of Attrition With Reinforcement," shows the fraction of surviving weapons with time for Red and Blue weapon types and varying percentages of reinforcement based upon the initial quantities. Fighting is assumed to continue during the reinforcement period. As would be expected, the fraction surviving (based upon the initial quantity) shows an increase during the time that weapons are being added.

b. Basic Attrition Equations With Reinforcement. The equations in Figure 2 present the rationale of the model. The formulation is simply expressed by the first equation which basically states that the differential decrease in the number of surviving weapons of type N is equal to the attrition loss minus the number gain by reinforcement. The attrition loss is the loss per engagement multiplied by the number of engagements per battle. The latter term is the number of weapons per battle divided by the numbers per engagement. Dividing by the time per battle TPB gives the loss per unit time and multiplying by dt results in the differential loss in dt time. The replacement fraction RF_n is multiplied by the initial number $NU_n(o)$ giving the absolute number replaced. Dividing by TFR, the time for replacement results in the number replaced per unit

time. Multiplying by dt results in the number replaced in dt time. L and R are the loss and replacement rates used over the integration interval. If L and R change appreciably, new values may be used in a subsequent integration interval, starting with a new value of time, ti. The equation is a modified version of the Lanchester equation. The modifications have allowed us to obtain the coefficients from the targeting and probability methodology described below for combined arms combat.

c. Targeting Scheme. To determine attrition by means of the above equation, it must first be determined who is shooting at whom, for all the weapon systems. It was assumed that the defense targeted and fired first. The target priority was based on those weapons which were the greatest threat to the defense. The defense then used those weapons which had the highest probability of kill against the target priority list previously selected by the defense. The offense fired back at those weapons that were targeting it. The offense targeted its remaining weapons using the same rules.

d. Targeting Results. Figure 3 titled, "Engagement Targeting Results," shows the results of the above targeting rules. (Double digit numbers ending in 1 or 2 represent different weapon types on the defense and offense side respectively.) The rules were applied over a light and heavy combat sector. Several variations of the rules were also tried. The figure shows that the majority of sub-engagements were 1 vs 1, with some 1 vs 2 and 2 vs 2 types. These and more complicated types of sub-engagements will be discussed below.

e. Application of Probability Methodology. Figure 4 titled, "Probability Application," illustrates the concept when it is applied to determining the probability of a given round (the third round, for example) from weapon 3 killing weapon 6 when both weapon 6 and weapon 5 are firing at weapon 3, thus affecting its probability of killing weapon 6. The single shot survival and kill probabilities of each round is allowed to vary from shot to shot. If the firing times for each weapon and the time of flight of each round have already been determined, then an event sequence can be set up as shown in the figure. The figure lists the four basic requirements that must be satisfied to solve the problem. The survivability and kill probability terms that must be included and those that are not pertinent follow directly from the basic requirements which are shown in parenthesis next to the terms in the solution. The solution for the kill probability based upon the three participants is not the conventional binomial expression. It involves specific contributions from all three participants; and is independent of the kill probability value of the second and all subsequent shots fired by weapon 5 at weapon 3. This is because these shots arrive at weapon 3 after the killing shot from weapon 3 is fired. Probability of line of sight, detection and acquisition, are handled as shown in the figure. It is assumed that previous specific random events do not influence subsequent random events. The frequency of specific subsequent random events is influenced only by the probability associated with the event at the specific time in the event sequence. These probabilities may however vary from shot to shot. Figure 5 is an additional example of the method for a 4 by 4 sub-engagement. Much lengthier and complex sequences would require more steps and possibly computer solution to obtain final analytic formulaes. Since the

actual problem we were solving involved developing an analytic model of combat and putting it into a hand held calculator it was decided to make certain simplifying but acceptable assumptions. Once this was done, it was possible to develop equations which satisfied the basic survivability, kill and calculation requirements discussed above.

f. Mechanization of Probability Computation. The firing times and flight times are still allowed to be different for each weapon. The first equation in Figure 6 expresses analytically the result obtained previously where a sequence of events chain had to be set up for each problem. The latter is tedious and time consuming unless it were done on a computer. To make the problem more manageable, it was assumed that the time between shots and the flight times did not vary from shot to shot. In addition the kill probability of each arriving shot was assumed to be constant. These assumptions reduced the amount of input data that had to be handled by a very significant amount. It was now possible to automatically calculate which probability terms needed to be included for each participant in the sub-engagement. The required value of the exponent is now calculated by the last equation shown in the figure. For any desired value of the killing shot fired from weapon 3 or M, the equation calculates the number of shots from N or N' that would arrive at M prior to the killing shot being fired. The attritor must survive these numbers of shots to kill with its next shot. The lowest integer term, (not allowed to be negative) insures that we use only integer numbers of arriving shots. Another constraint, not shown in the figure, is that $LA_{N'M}$ not exceed the number of shots available to N' for firing at M. Figure 7 generalizes the previous case for 2 vs 2 type combat. It shows how the loss coefficient L for the basic attrition equation may be calculated from input and previously derived values. It is assumed that the probability of weapon N surviving shots from M is independent of the probability of weapon N surviving shots from weapon M'.

g. Sensitivity. Figure 8 shows the sensitivity of the Probability of Kill per Target, PKPTT(N) to some of the 28 independent variables involved in a 2 x 2 sub-engagement. The first column showing nominal values of some of the major variables results in a probability of kill of target N equal to 0.1. This is because the 4 rounds from N and N' arrive at M before M can fire its round at N. (If there is no entry in a subsequent column, the value reverts to the nominal value in the first column.) Increasing the time to fire from N to M from 0 to 20 seconds increases the probability of killing N to only .11 since the rounds from N' are still able to arrive in time to prevent M from firing at N. When the time of flight for N' is also increased to 20 seconds, the probability of killing N increases to .99 as would be expected. Decreasing the probability of kill by N and N' of M to .01 (from .7) increases the probability of killing N from .1 to .92. If the time to fire from N and N' to M is made 1 second compared to 0 seconds from M to N, then M can fire its round before it is killed and the probability of kill of N is increased from .1 to .99. Similarly, decreasing the probability of line of sight from N to M and from N' to M to .5 increases the probability of kill of N from .1 to .52. Figures 9 and 10 present additional sensitivities, with all the nominal values specified in the first column. Notes at the bottom of these two figures give a short physical explanation of each effect. The input values used in these figures and other parts of

the report are chosen to show responsiveness of the model and do not necessarily bear any resemblance to actual weapon system values. Each column represents another 30 second run on the hand calculator. The results show that the model performs as would be expected for each of the input variables. Figure 11 relates PKPTT(n) to the fraction of weapon systems surviving, based upon the initial number when combat started. It may be noted that the fraction that survives is quite sensitive to the total kill probability, thus, illustrating the importance of the effects of the many offense/defense parameters encountered in combined arms combat situations.

h. Comparison Between Results from the Analytic and a Simulation Model. A comparison was made between the results of the analytic model and a firepower simulation model which involved engagements between many divisions on three fronts. There was a large variation in battle activity from front to front and division to division in the simulation model. As a result, divisions were selected which had the same losses as the analytic model in an initial 12 hour period. Figure 12 shows that the analytic attrition curve has approximately the same shape as the attrition curve from the simulation model for the first 5 to 6 days after the initial 12 hour interval.

3. OBSERVATIONS AND SUMMARY. A system of analytic equations is presented and applied to division combat between opposing forces. Sensitivities to many input variables are presented and shown to conform to physical expectations. The attrition curves are shown to have approximately the same shape as those obtained from a simulation model.

4. ACKNOWLEDGEMENTS. The author wishes to express his appreciation to numerous members of the War Gaming Directorate of the Concepts Analysis Agency whose ideas and work contributed substantially to this paper. In particular, I would like to acknowledge the contributions of COL K. H. Gates, Jr., Mr. H. K Graves, Ms. Pat Grossman, Ms. P. T. Shields, and the Word Processing Center. Responsibility for the paper itself, of course, is the author's.

FIGURE 1. EXAMPLES OF ATTRITION WITH REINFORCEMENT

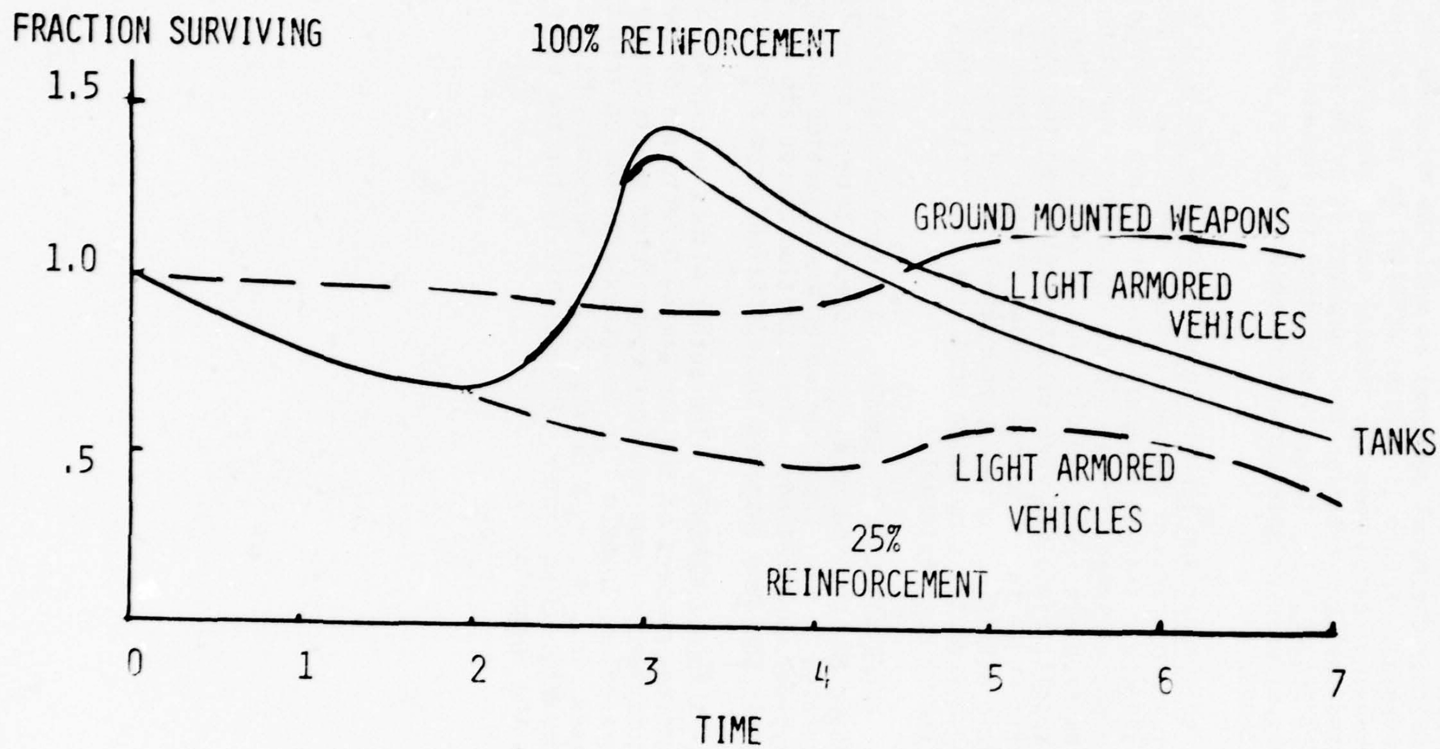

FIGURE 2. <u>BASIC ATTRITION EQUATIONS, WITH REPLACEMENT</u>

$$-dNU_N = LPE_N \frac{NU_N}{NPE_N\ TPB}\, dT - \frac{RF_N\ NU_N(0)}{TFR}\, dT$$

OR

$$\frac{dNU_N}{\underbrace{\left(\frac{LPE_N}{NPE_N\ TPB}\right)}_{\equiv L} NU_N - \underbrace{\left(\frac{RF_N}{TFR}\right)}_{\equiv R} NU_N(0)} = -dT$$

INTEGRATING FROM TI TO T AND REARRANGING

$$\frac{NU_N(T)}{NU_N(0)} = e^{-L(T-TI)} \left[\frac{NU_N(TI)}{NU_N(0)} - \frac{R}{L}\right] + \frac{R}{L}$$

FIGURE 3. ENGAGEMENT TARGETING RESULTS

	LIGHT COMBAT SECTOR			HEAVY COMBAT SECTOR		
NUMBER OF SUB-ENGAGEMENT	NUMBER OF SUB-ENGAGEMENTS*	WEAPON TYPES M	N	NUMBER OF SUB-ENGAGEMENTS*	WEAPON TYPES M	N
		M′	N′		M′	N′
1	8	22	21	16	22	21
		32			32	61
2	25	22	21	13	22	21
		12				61
3	5	22	21	1	22	51
						61
4	21	22	31	23	22	51
5	24	22	51	16	22	31
6	22	22	61	33	12	11
7	11	22	11	19	12	41
8	38	12	11	1	12	61
9	25	12	41	5	42	61
10	1	12	71			

*ALSO, TGTSPE, TGTS. PER ENG.

FIGURE 4. PROBABILITY APPLICATION

$\overset{\downarrow}{N} = 6 \quad N' = 5$

$\overset{\uparrow}{N} = 3$

DETERMINE PROBABILITY OF THIRD ROUND FROM WEAPON 3 KILLING WEAPON 6. ($PK_{3,3;6,5}$)

SEQUENCE OF EVENTS.

F	F	F	(K,S)	(K,S)	F	(K,S)	F	F	(K,S)	(K,S)	F	(K,S)	(K,S)
136	163	153	136	163	236	153	263	253	236	263	$\underline{336}$	253	$\underline{336}$
	$PK_{3,3;6,5}$ =		S(3)	S(2)		S(2)			S(3)	S(2)		1(4)	K(1)
			$13\underline{6}$	$16\underline{3}$		$15\underline{3}$			$23\underline{6}$	$26\underline{3}$		253	$\underline{336}$

REQUIREMENT (1) THIRD SHOT FROM WEAPON 3 MUST KILL WEAPON 6, $\underset{336}{K}$.

(2) THE ATTRITOR, 3, MUST NOT BE KILLED PRIOR TO FIRING ITS KILLING ROUND, $\underset{336}{F}$.

(3) THE ATTRITEE, 6, MUST NOT BE KILLED PRIOR TO $\underset{336}{K}$.

(4) NOTE ABSENCE OF $\underset{253}{S}$ TERM. NO REQUIREMENT FOR THIS TERM.

TO INCLUDE EFFECTS OF PROBABILITY OF LINE OF SIGHT, PLOS, ETC., LET $\underset{LMN}{S}$ IN ABOVE $= 1 - (\text{PLOS} \times \text{PDET} \times \text{PACQ} \times \text{K})_{LMN}$ AND $\underset{LMN}{K} = (\text{PLOS} \times \text{PDET} \times \text{PACQ} \times \text{K})_{LMN}$

FIGURE 5.

DETERMINE PROBABILITY OF FIRST ROUND FROM WEAPON 2 KILLING WEAPON 7 (PK_{127}).

SEQUENCE OF EVENTS:

$$\overset{F}{116}\ \overset{(K,S)}{116}\ \overset{F}{153}\ \overset{F}{162}\ \overset{F}{147}\ \overset{(K,S)}{162}\ \overset{(K,S)}{153}\ \overset{F}{172}\ \overset{(K,S)}{147}\ \overset{F}{\underline{127}}\ \overset{F}{181}\ \overset{F}{138}\ \overset{(K,S)}{172}\ \overset{(K,S)}{\underline{127}}\ \overset{(K,S)}{181}\ \overset{K,S}{138}$$

$$PK_{127} = \overset{S}{116}\ \overset{F}{162}\ \overset{S(2)}{162}\ \overset{I}{153}\ \overset{S(3)}{147}\ \overset{F}{127}\ \overset{K(1)}{127} + \overset{K}{116}\ \overset{OF}{162}\ \overset{I(4)}{162}\ \overset{I}{153}\ \overset{S(3)}{147}\ \overset{F}{127}\ \overset{K(1)}{127}$$

$$= \overset{K}{127}\ \overset{S}{147}\left(\overset{S}{116}\ \overset{S}{162} + \overset{K}{116}\right) =$$

REQUIREMENT:

(1) FIRST SHOT FROM WEAPON 2 MUST KILL WEAPON 7.

(2) THE ATTRITOR 2 MUST NOT BE KILLED PRIOR TO FIRING ITS KILLING ROUND, $\overset{F}{127}$.

(3) THE ATTRITEE 7 MUST NOT BE KILLED PRIOR TO $\overset{K}{127}$.

(4) WEAPON 6 HAS ALREADY BEEN KILLED AND CAN HAVE NO SUBSEQUENT EFFECT.

FIGURE 6. <u>MECHANIZATION OF PROBABILITY COMPUTATION</u>

$N = 6 \quad N' = 5$

$M = 3$

$$PK_{3,3;6,5} = S_{63}^{LA_{63}} \qquad S_{53}^{LA_{53}} \qquad S_{36}^{3-1} \qquad K_{36}$$

* REQMT 2 REQMT 2 REQMT 3 REQMT 1

$$PK_{LKF,M;N,N'} = S_{NM}^{LA_{NM}} \qquad S_{N'M}^{LA_{N'M}} \qquad S_{MN}^{LKF_{MN}-1} \qquad K_{MN}$$

$$TF_{N'M}+TFT_{N'M}+TBF_{N'M}(LA_{N'M}-1) \leq TF_{MN}+TBF_{MN}(LKF_{MN}-1)$$

OR

$$LA_{N'M} = \left[\frac{TF_{MN}-TF_{N'M}-TFT_{N'M}-TBF_{MN}+TBF_{N'M}}{TBF_{N'M}} + \frac{TBF_{MN}}{TBF_{N'M}} LKF_{MN}\right] \quad \text{L.I.}$$

APPLICABLE FOR N' = N, ALSO.

$N = 6 \quad N' = 5$

$M = 3 \quad M'$

FIGURE 7.

DETERMINATION OF LOSS COEFFICIENT FOR ATTRITION EQUATION

$$PKPTT_N = 1 - (1-PKPT_{M;N,N'})\ (1-PKPT_{M'N})$$

WHERE

$$PKPT_{M;N,N'} = \sum_{LKF_{M,N} = 1,\ LKFMX_{M,N}} PK_{LKF,M;N,N'}$$

AND,

$$PKPT_{M'N} = 1 - (1-K_{M'N})^{LKFMX_{M'N}}$$

$$LPE_N = \sum_{NSE} TGTSPE_{NSE,N}\ PKPTT_N$$

THE LOSS COEFFICIENT L FOR THE BASIC ATTRITION EQUATION IS THEREFORE:

$$L = \frac{LPE_N}{NPE_N\ TPB} = \frac{\sum_{NSE} TGTSPE_{NSE,N}\ PKPTT_N}{NPE_N\ TPE}$$

FIGURE 8. TYPES M&M' VS N&N'

TIME TO FIRE (NM)	0	20	20		1.0	
TIME TO FIRE (N'M)	0		20		1.0	
TIME TO FIRE (MN)	10				.0	
SS PROB OF KILL (NM&N'M)	.7			.01		
PROB. LOS, DET, ACQ (NM&N'M)	1.0					.5
MAX. NO. OF RDS (NM&N'M)	4.					
MAX. NO. OF RDS (MN&M'N)	1					
SS PROB OF KILL (MN)	.99					
SS PROB OF KILL (M'N)	.1					
PROB OF KILL PER TGT, TOTAL, (N) PKPTT (N)	.1	.11	.99	.92	.99	.52

TF (NM)	0		TIME		TO	FIRE			1.
TFT (NM)	0		20	FLIGHT			TIME		
TBF (NM)	.001	TIME		20	BETWEEN			FIRES	
LKFMX (NM)	4.				1	MAX NO		OF	RDS
K (NM)	.7	SINGLE	SHOT				KILL		PROB.
K (MN)	.99					.1			
K (M'N)	.1					.01			
LKFMX (M'N)	1.						4		
TF (N'M)	0								1.
TFT (N'M)	0		20						
TBF (N'M)	.001			20					
LKFMX (N'M)	4.				1				
K (N'M)	.7								
LKFMX (MN)	1.							4	4.
TF (MN)	10.								0
TBF (MN)	10.								.1
PKPTT (N)	PROB.	OF	KILL	PER		TGT, TOTAL			
PKPTT (N)	.10		.99	.18	.18	.01	.34	.10	1.0
	(1)		(4)	(5)	(6)	(8)	(9)	(10)	(12)

FIGURE 9.

TYPES M&M'
VS. N&N'

(1) NOM.;

(2) N&N' ARRIVE LATE

(3) & (4) 1st RDS FROM N&N' ARRIVE EARLY

(5) DUE TO K (M'N); M KILLED BY N&N'

(6) M' FIRES MORE RDS.

(7) M KILLED BEFORE FIRING

(8) M'S 4 RDS. FIRED BEFORE KILLED

FIGURE 10.

EFFECT OF LINE OF SIGHT, DETECTION AND ACQUISITION PROBABILITIES ON ATTRITION

K (NM)	.7								SINGLE SHOT KILL PROBABILITY
K (N'M)	.7								
PLOS (NM)	1.	.5	.5	.5	.5				PROB. OF LINE OF SIGHT
PLOS (N'M)	1.	.5	.5	.5					
PDET (NM)	1.		.5	.5	.5				PROB. OF DETECTION
PDET (N'M)	1.		.5	.5					
PACQ (NM)	1			.5	.5				PROB. OF ACQUISITION
PACQ (N'M)	1			.5					
K (M,N)	.99								
K (M'N)	.1								
PLOS (MN)	1.					.5	.5	.5	
PLOS (M'N)	1.					.5	.5	.5	
PDET (MN)	1						.5	.5	
PDET (M'N)	1						.5	.5	
PACQ (MN)	1							.5	
PACQ (M'N)	1							.5	
PKPTT (N)	.10	.13	.28	.52	.10	.05	.03	.01	PROB OF KILL PER TARGET, TOTAL
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	

(1) NOM. (2) EFFECT ON N OF DEGRADATION IN LINE OF SIGHT FOR N AND N' (3) PLUS DEGRADATION IN DETECTION (4) PLUS DEGRADATION IN ACQUISITION (5) ONLY WPN N DEGRADED NOW (6) LINE OF SIGHT FOR M & M' NOW DEGRADED (7) PLUS DEGRADATION IN DETECTION (8) PLUS DEGRADATION IN ACQUIS. PROB.

FIGURE 11.

FRACTION SURVIVING VS. PROBABILITY OF KILL

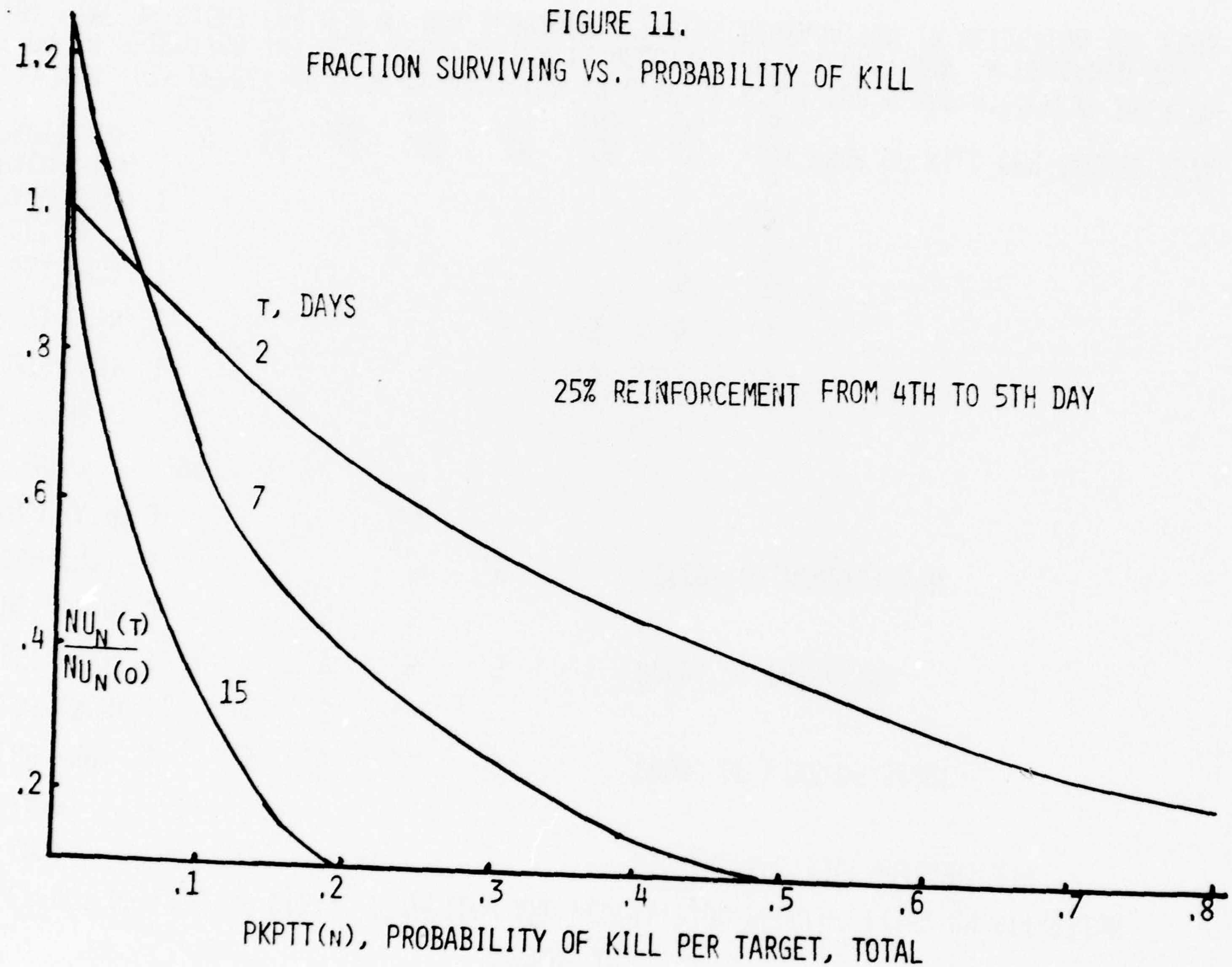

FIGURE 12. COMPARISON BETWEEN ANALYTICAL AND SIMULATION ATTRITION CURVES

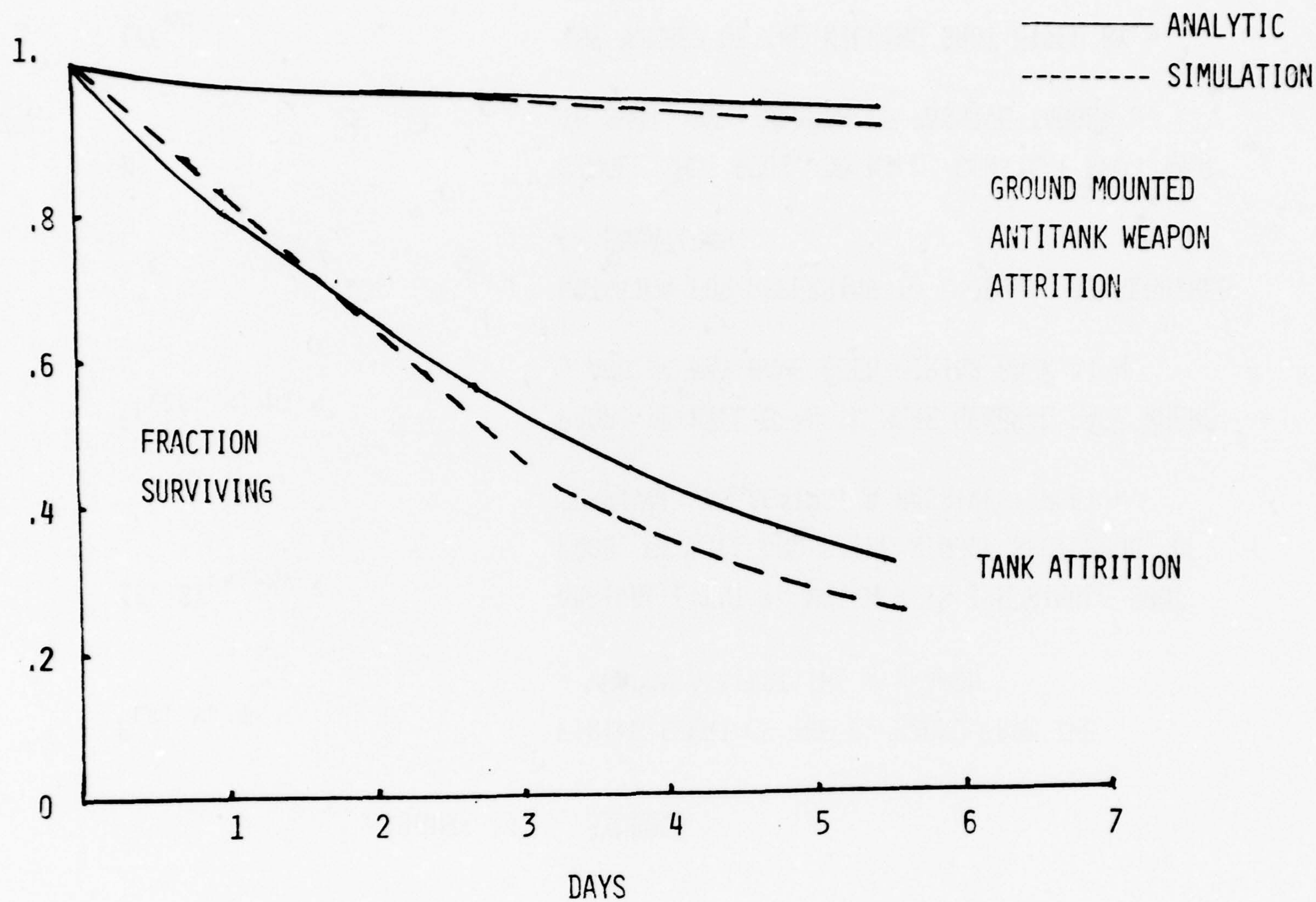

FIGURE 13. SYMBOLS

$F_{LK, M, N}$	FIRING EVENT OF THE LK ROUND FROM THE M WEAPON AGAINST THE N WEAPON.
$(K, S)_{LK, M, N}$	ARRIVAL EVENT IN WHICH K IS THE SINGLE SHOT PROB. OF KILL AND S THE SINGLE SHOT PROB. OF SURVIVAL, AS AGAINST A PASSIVE TARGET N.
$PK_{LKF, M; N, N'}$	PROB. OF KILL OF N IN MORE GENERAL CASE WHERE N AND N' MAY HAVE BEEN FIRING BACK AT M.
N''	NOTATION FOR REFERRING TO N OR N', AS REQUIRED BY EQUATIONS.
K_{IJ}	SINGLE SHOT PROB. OF KILL, CONSTANT FROM SHOT TO SHOT, BY I WEAPON OF PASSIVE TARGET J.
LKF_{MN}	THE NUMBER OF THE KILLING SHOT FIRED BY M AGAINST N.

FIGURE 14.

$LA_{N''M}(LKF_{MN})$, OR $LA_{N''M}$	THE NUMBER OF SHOTS THAT ARRIVE FROM N″ TO M, PRIOR TO LKF_{MN}.
TF_{IJ}	TIME TO START FIRING FROM I TO J WEAPON.
TFT_{IJ}	TIME OF FLIGHT FROM I TO J.
L.I.	NEXT LOWEST INTEGER. MUST BE ≥ 0.
$PKPT_{M;\,N,\,N'}$	PROB. OF KILL PER TARGET, TOTAL (DUE TO ALL THE SHOTS THAT M CAN FIRE AT N); M IS FIRING AT N; N AND N′ ARE FIRING BACK AT M.
$PKPT_{M'N}$	PROB. OF KILL PER TARGET, TOTAL, FROM M′ AGAINST N.
$PKPTT_{N}$	PROB. OF KILL PER TARGET N, FROM M AND M′.
$LKFMX_{IJ}$	MAXIMUM NUMBER OF SHOTS THAT I IS ALLOWED TO FIRE AT J.
NSE	NUMBER OF SUB-ENGAGEMENT TYPE CONSIDERED.

FIGURE 15.

$TGTSPE_{NSE,\ N}$	TARGETS PER SUB-ENGAGEMENT TYPE. TARGETS ARE OF TYPE N, THE TYPE FOR WHICH ATTRITION IS BEING CALCULATED.
LPE_N	LOSS (OF TYPE N) PER ENGAGEMENT. LOSS IS FROM ALL TYPES OF WEAPONS FIRING AT N.
NU_N OR $NU_N(T)$	NUMBER OF WEAPONS OF TYPE N, IN THE ENTIRE BATTLE, THAT HAVE SURVIVED UP TO TIME T.
NPE_N	NUMBER PER ENGAGEMENT OF WEAPONS OF TYPE N.
TPB	TIME PER BATTLE. (1 DAY)
DT	DIFFERENTIAL TIME CONSIDERED.
DNU_N	DIFFERENTIAL CHANGE IN NU_N IN TIME DT.
OF_{LMN}	DENOTES THAT M WEAPON HAS PREVIOUSLY BEEN KILLED AND CAN'T FIRE.

FIGURE 16.

1_{LMN}	UNITY - NO EFFECT ON PROBABILITY, SINCE THE M WEAPON WAS ALREADY KILLED BEFORE IT WAS SCHEDULED TO FIRE OR THE N TARGETED WEAPON HAS ALREADY BEEN KILLED.
$PLOS_{LMN}$	PROBABILITY OF MAINTAINING SUFFICIENT LINE OF SIGHT FOR FIRING L M N.
$PDET_{LMN}$	PROBABILITY OF SATISFYING DETECTION CRITERIA REQUIRED FOR FIRING L M N.
$PACQ_{LMN}$	PROBABILITY OF SATISFYING ACQUISITION CRITERIA REQUIRED FOR FIRING L M N.
I J K	I ROUND FROM J WEAPON AGAINST K WEAPON (FIRING OR ARRIVAL OF ROUND).
RF_N	REPLACEMENT FRACTION ADDED FOR TYPE N.
TFR	TIME MEASURED FROM TI DURING WHICH REPLACEMENT IS OCCURRING.

US ARMY OPERATIONS RESEARCH SYMPOSIUM XVI

ABSTRACTS SUBMITTED BUT NOT PRESENTED

Title: An Analysis of Military Traffic Management Command (MTMC) Participation in the REFORGER 76 Exercise

Authors: MAJ John M. Burbidge and CPT Raymond A. Schaible, Military Traffic Management Command

Abstract: The annual exercise, Return of Forces to Germany (REFORGER), has been for the last decade the focal point of the United States' reiteration of its commitment to the defense of Europe. In past years, the emphasis has been on the airlifting of troops to prepositioned equipment in Germany for participation in the annual NATO exercises. In 1976, however, the REFORGER Army division, the 101st Airborne Division (Air Assault), was deployed with its organic equipment. The exercise involved the simultaneous movement of personnel by air and equipment by sea. The Military Traffic Management Command developed and tested the conceptual plans for the deployment as well as coordinating the surface movement of unit equipment from CONUS home stations to Europe and return. Transportation planners employed a total systems approach in bringing together the transportation requirements for the exercise and optimizing strategic air and sealift assets in the deployment of the REFORGER division. The proposed paper analyzes MTMC participation in the REFORGER 76 Exercise and evaluates the Defense Transportation System's capability to support the transportation of the equipment of an airmobile division, including over 300 helicopters, from a CONUS origin to a potential combat employment destination overseas.

Title: The Effect of the Operational Readiness of the Float on the Operational Readiness of the Entire Fleet

Author: Dr. Frank Fox, US Army Aviation Systems Command

Abstract: The operational readiness (OR) rate of a fleet of aircraft is calculated by dividing the hours in commission for the fleet by the hours on hand for the fleet and multiplying by 100 to convert to a percentage. The OR calculation includes both the aircraft assigned to the unit and the float aircraft which serve the unit. When an aircraft assigned to a unit needs extended maintenance, it is replaced in the fleet by exchanging it for a float aircraft which is operationally ready. Hence the float has given up an operationally ready aircraft for one which may be out of service for a lengthy period. This process of continually giving up operationally ready aircraft for non-operationally ready aircraft causes the OR rate of the float to be much lower than that of the fleet as a whole. Therefore, the inclusion of the float aircraft in the calculation of the OR rate for the entire fleet decreases the OR rate from what it would otherwise be if the float aircraft were not included. In this paper some useful relationships between the OR rates for the fleet as a whole and its float are developed and illustrated with equations, graphs, and tables. For example, one

equation shows the decrease in the OR rate caused by including the float aircraft in the calculation. The magnitudes of the effects illustrated by these results depend, of course, on the values of the parameters involved. However it is clear from the range of possible values for the parameters that the OR of the float can decrease the OR of the whole fleet by a few percentage points. This is significant amount, since when the OR rate is below its goal it is often by only a few percentage points.

Title: Logistics Enhancement Handbook
Author: COL Aaron L. Lilley, Jr., Headquarters, First US Army
Abstract: The handbook entitled Logistics Enhancement is intended to aid the Active and Reserve Components in obtaining an increased level of logistical efficiency. Included in the handbook are chapters on Property Book, Supply, Maintenance, and Personnel and Training. Since the Property Book Officer is the key to a successful unit supply operation, Chapter 2 outlines the necessary steps to be taken in order to achieve the elusive goal of having on hand 100% of all authorized property. The attainment of a responsive supply system is crucial to the proper functioning of the entire organization. Accordingly, nearly half of the handbook is devoted to the supply operation. Maintenance is inexorably linked to supply. A good maintenance system will result in the achievement of fully combat ready units. Personnel and training actions should be well-planned and prudently executed to insure that the path to optimum logistical posture is a smooth one. The Logistics Enhancement handbook is currently being reviewed by MUSARC DCSLOGs. Early indications are that it is being well-received.

Title: PERT as a Data Base
Author: Mr. Michael E. Neyer, US Army Tank-Automotive Materiel Readiness Command
Abstract: PERT has seemingly come full cycle. Managers are extremely interested in actual schedules and projects as well as in probabilities of accomplishment. Previous drawbacks of PERT were a lack of memory and slow updating. This discussion will show how to overcome these drawbacks by use of PERT as a management data base. The data base design offers the advantages of real time computer applications, the ability of a manager to include various sub-networks in the entire PERT network, and the ability to monitor any sub-networks separately.

Title: Role of the Operations Research Analyst in a Product/Project Manager's Office (PMO)
Author: Mr. Michael E. Neyer, US Army Tank-Automotive Materiel Readiness Command
Abstract: This discussion will cover four topics. The first topic covered will be how and when a PMO should determine that the addition of an OR analyst to the staff would be beneficial to the office. Secondly, the discussion will cover the historical projects of an OR analyst in the office of the PMO M880. Next, will be the general advantages that accrue to the PMO by having

an OR analyst on his staff, and the last topic will cover the use of a PMO assignment for career development of OR analysts.

Title: Research Analysis or Cost Analysis: An Army Viewpoint
Author: Dr. Theodore Trybul, US Army Materiel Development and Readiness Command
Abstract: This paper presents a corporate look at Army Resource Analysis problems including the DARCOM Resource Management System. New concepts, principles, and trends are presented from a top management overview. Strategic approaches to resource analysis problems are explained. Methodologies such as review and command assessment of programs, command management review and analysis, measures of research effectiveness, overall performance indicator review and analysis, productivity trend evaluation, and milestone techniques are discussed with emphasis on innovative management approaches and applications. The resource management system includes the set of disciplines and procedures encompassing the process involved in planning; establishing goals, estimating required resources; assigning objectives and time-phased tasks to operating agencies; issuing program planning and budgeting directives; evaluating and analyzing performance and resource utilization; and rendering managerial and operational decisions. This paper discusses how resource analysis results in cost reductions and cost effective weapon systems, and how new resource management techniques improve the management processes of planning, evaluation and control.

Title: Cost of Terminating Contracts Study (COTCOS I)
Author: Mr. J. S. Sutterfield, US Army Aviation Systems Command
Abstract: The Cost of Terminating Contracts Study (COTCOS) was initiated because of concern as to whether the progress payments being made by foreign military purchasers of Army type aircraft were adequate to defray the cost of contract termination. A set of termination liability tables had been developed by the Air Force and were being recommended for use by DARCOM. There was doubt, however, regarding the adequacy of these tables for foreign military transactions involving Army aircraft. Thus, the objectives of the study were:

1. Determining whether the Air Force information would have been adequate for several representative airframe and spares contracts.
2. In the event the Air Force information was found to be inadequate, developing a curve that would provide adequate progress payments.

The assumptions forming the basis for the study were as follows:
1. That government furnished equipment has a negligible effect upon the incurrence of cost.
2. That the reporting of cumulative costs "lags" the actual incurrence of costs; this reporting lag is assumed to increase linearly until it reaches 90 days midway through the contract and to remain constant, thereafter, for the remaining half of the contract.

3. That a "normal" contract is closed out 90 days after the last delivery is made.

4. That cost incurred is a continuous function, when in reality it is discrete and discontinuous.

5. That the small number of contracts available for analysis is sufficient to provide a basis for sound generalizations about future cases.

The following conditions constrained the study:

1. A relatively small number of "clean contracts" on which to perform the analysis.
2. Lack of uniformity in the incurrence and reporting of costs.

The study resulted in the average or "equally likely" cost incurred curve for Army Aviation.

The conclusions from the study were that:

1. The AVSCOM average or "equally likely" curve of the cost incurred fulfills the current DARCOM definition for a termination liability curve.
2. A greater number of cases must be analyzed in order to render the analysis more statistically sound.

The foregoing conclusions gave rise to the following recommendations:

1. That the AVSCOM average or "equally likely" curve be adopted as the basis for reckoning the payments to be made by FMS customers who purchase Army aircraft.
2. That a computer system be developed for using future contractual reporting information to modify the AVSCOM curve as may be required to make it more descriptive.

Title: Technology Forecasting at the Bench Level

Author: Dr. Norman Slagg, US Army Armament Research and Development Command

Abstract: It has been pointed out that technology forecasting is a tool that can lead us from the logical future based on current knowledge to a "willed" future. This paper is concerned with the application, through forecasting, of our most important resource, the accumulated scientific engineering knowledge, to the resolution of future military problems. Prior to 1900 science and technology developed independent of each other. The steam engine was developed prior to the formulation of thermodynamics. Now there is much closer coupling and this situation must be recognized as we look ahead. Technology forecasting has been used by the three Services in broad planning. Unfortunately it's carried out at a level somewhat distant from the research and development laboratories and divisions where the major technical competence exists. In this paper a general discussion is presented of the potential benefits from an aggressively pursued technology program at the bench level to programs on bombs, shaped charges, and

electro-optical devices. This discussion will include the effect of forecasting on the coupling between fundamental science and military programs, on innovations, and the time between innovation and realization. Methods are suggested on how forecasting can be simply included in current R&D organizations.

Title: Reducing Unsought Consequences in Automatic Data Processing Equipment Procurement
Author: 1LT Andrew C. Rucks, US Army Concepts Analysis Agency
Abstract: The interplay of competing intra-agency and inter-agency objectives for automatic data processing equipment (ADPE) procurement create an apparent conundrum. An Army agency attempting to acquire new or replacement ADPE is primarily concerned with optimizing system performance relative to its workload. Competing with this objective are inter-agency objectives (originating in various Army and non-Army governmental agencies that regulate ADPE procurement) which include: allocating scarce fiscal resources, assuring coordination among all interested governmental bodies, assuring technical quality in a proposed system, and assuring competition among potential suppliers of ADPE or ADP services. The stimuli for these inter-agency objectives are trends in the government's overall procurement policy. Failure to respond to changes in policy diminishes an agency's contribution to the achievement of inter-agency objectives and can lead to unsought consequences in the form of delays, degraded system performance and negative budget variances. The apparent conundrum created by the aforementioned competing objectives can be transformed into a soluble problem by careful and purposeful planning and plan execution. This paper will explore plan formulation and execution by the US Army Concepts Analysis Agency in its efforts to acquire replacement ADPE. The paper will cover the following topics:

1. Steps in the planning process.
2. Elements of the ADPE acquisition process.
3. The application of activity/event network analysis to ADPE acquisition planning.
4. Workload analysis.
5. Preparation of a comparative economic analysis.

Title: Cost Effectiveness of the Armor Force
Author: Mr. Hung Wang, US Army Concepts Analysis Agency
Abstract: This paper presents a method of developing the combat effectiveness per dollar relationships of armor forces. The work was performed in support of the Tank Force Management Group. Combat effectiveness was estimated by use of a methodology which quantifies the contribution of firepower, mobility, survivability, etc, and then expresses their composite value as a numerical score. The cost associated with the armor forces was determined through a multi-step process: a) assuming the Army is comprised of economic sectors; b) determining what percentage of each sector's budget supports the armor

force; and c) calculating, through input-output analysis, the total budget required to sustain a given armor force. The costs and combat effectiveness scores were input into a series of computer programs which yielded combat effectiveness-per-dollar ratios. In essence, the programs distributed the effectiveness measure among the sectors according to a series of rules; and then divided the scores by their associated budget. Several alternatives were analyzed and showed that increased funding of the armor forces result in increased Army combat capability.

Title: Supporting the Procurement Process Through the Use of An Analytical Cost Model
Author: Mr. Harvey J. Slovin, US Army Electronics Command
Abstract: The Army Electronics Command has been using an analytical cost model to estimate costs for development and production programs, in setting design to unit cost bogeys, and for evaluating the cost impact of design tradeoffs. In the interest of better utilizing our dwindling resources, and to take full advantage of the capabilities of the model, application of the parametric model has been extended to include supporting the procurement process. New applications of the model are in management planning for short - and long - range programs, for developing independent estimates for development and production contracts, in pinpointing areas of concern during "should cost" evaluations, in supporting procurement and production planning in areas of lead time and delivery schedules, and in assisting in the source selection process. While the model is not intended to completely replace traditional methods of performing these functions, cost model assistance in the procurement process allows the better utilization of resources, completing the tasks faster, and with more confidence.

Title: Evasive Targets for Tanks
Author: Mr. Edward C. Christman, US Army Materiel Systems Analysis Activity
Abstract: The Army's requirements documents for tanks have included a hitting probability requirement for constant velocity targets but not for accelerating targets. This omission was at least partly due to lack of methodology. It was found that lead computers designed to meet the requirement frequently performed poorly against varying velocity targets which undoubtedly occur more frequently than constant velocity targets. In response to this void, AMSAA is doing a study to do the following:

a. Determine the nature of the motion of tank targets (how to characterize and categorize target motion), determine the spectrum of the categories to be encountered, and determine the frequency of occurrence to the various categories of target motion.

b. Develop a new model for estimating probability of hitting accelerating targets.

c. Use a. and b. to study design and performance tradeoffs and develop a draft requirement statement concerning accelerating targets.

AMSAA will present the content, results and conclusions of this study.

Title: Long Range Artillery Opportunities and Challenges
Author: Mr. Charles T. Odom, US Army Materiel Systems Analysis Activity
Abstract: Measurement of the benefits to be gained from the use of long range artillery is dependent upon a number of factors. The method of propulsion for the projectile will have an effect upon the delivery accuracy and the weapon's reliability. Any projectile which flies an unguided ballistic trajectory will generally have large errors at the terminal end of long range trajectories which will degrade the effectiveness of the projectiles. Any increase in muzzle velocity of a cannon weapon required to increase its range will generally increase the forces into the weapon thereby degrading its reliability. The weight of ammunition which can be supplied to an artillery unit is a constraint which must be considered. The amount of ammunition required to meet a given level of effectiveness at long range is much greater than for the same target at shorter ranges. Thus, either the attack criteria for long range targets must be lowered to prevent inordinate expenditures which would tend to degrade the units shorter range capability, or the military value of the longer range target must be sufficiently large to compensate for the values of shorter range targets which will not be fired because of the lowered amount of ammunition available to the unit. The question of a long range artillery capability is one of both system design and operational trade-offs which must be carefully weighed and considered by both weapon designers and by the tacticians.

Title: Fire Control Against Evasive Ground Targets
Author: Mr. S. S. Wolff, US Army Ballistic Research Laboratory
Abstract: Upper bounds are derived on the performance of an arbitrarily sophisticated - but non-clairvoyant - "lead" algorithm working from manual tracking inputs against a deliberately evasive target. Such bounds are useful in assessing the utility of contemplated modifications to current systems.

Title: Engineer Estimate--V (US) Corps
Author: Mr. Lyle G. Suprise, Engineer Studies Group (ESG)
Abstract: This paper will present the methodology and unclassified results of an effort sponsored by HQ V Corps. The requirement for the study grew out of the revision of the V Corps General Defense Plan to reflect FM 100-5 doctrine, the target servicing technique, and the active defense. An engineer estimate is the result of solving a specially defined resource allocation problem. Limited engineer and related resources are allocated among the possible Corps area engineer tasks to promote assured target servicing by the active defense subject to the requirements for overall economy of manpower and materiel. The study focuses on the feasibility and effectiveness of the obstacle/barrier plans and ways that they can be improved as well as Corps-wide engineer requirements. The relation of engineer work to combined arms effectiveness is addressed. Engineer tasks are identified in the functional areas of mobility, survivability,

countermobility, and general engineer support for the covering force area, main battle area, division/brigade rear, Corps area, and the Corps area of interest in the forward COMMZ. Important considerations in the analysis are the trade-offs that can/must be made within the engineer system and trade-offs between engineer and non-engineer tasks. In sum, the paper will provide a case example of applied combat engineering in today's Army.

Title: The Value of Information in Combat Decision Making
Authors: COL Allen F. Grum, MAJ David R. E. Hale and CPT Terry A. Bresnick, US Military Academy
Abstract: The Lanchester model of combat has generally been used with deterministic values of initial force strengths. In reality these are probabilistic parameters that can be described by probability distributions. Techniques of evaluating the worth of perfect information on force strengths have been investigated in an earlier study. This paper discusses extensions of this methodology using concepts developed in Decision Analysis.

Title: SQT: A System for Individual Training Accountability
Authors: Dr. Ward Harris and MAJ Benjamin Whitehouse, US Army Training Support Center
Abstract: With the implementation of the Skill Qualification Test (SQT), the Army has launched a new system for evaluating the job competence of soldiers. The new system will provide training managers at all levels information regarding specific tasks that individual soldiers can and cannot do. Training managers can then focus training efforts and resources on tasks and soldiers that need it the most. There are many players in the SQT, and each has specific responsibilities in achieving the purpose of SQT: to improve the combat effectiveness of the Army by getting soldiers to do their jobs better. This presentation deals with the SQT participants, their responsibilities and the individual training accountability system that provides feedback for better individual training.

Individuals. Every member of the Army has a responsibility related to individual training. Specifically, each soldier is personally responsible for attaining and maintaining proficiency in the critical tasks that define his job. To insure that a soldier knows what is expected of him, the Army issues a Soldier's Manual to every enlisted soldier for his current and next higher skill level. This manual lists critical tasks by number and describes what the soldier must do under what conditions and how well he must do it. In addition, relevant references are given which amplify the task, its conditions, or standards. These related training materials include TEC, correspondence courses, literature and special training packages. To focus the soldiers efforts prior to taking the SQT, a notice is sent two months prior to the test period indicating precisely which tasks will be tested and how they will be tested to include sample questions. Whether the soldier meets his responsibility for individual proficiency or not is measured to a large

degree by the SQT. The Individual Soldiers Report (ISR) provides an evaluation of the soldiers job proficiency indicating tasks failed. Ultimately the burden is on the individual to maintain job proficiency using existing training materials.

Training Managers. The training manager has responsibility for scheduling of time, facilities, equipment and other resources in support of individual training. If the soldier is to meet the job standards of his soldiers manual, then the commander must provide time, assistance and access to the referenced training materials. To assist the training manager, the Commander's Manual provides task identification and where the task should be taught and reinforced. The first line trainer (sqd ldr, etc.) uses a job book, which gives the status of each soldier by job task, to conduct concurrent, on the job evaluation of soldier proficiency. The effectiveness of commanders and supervisors, in context with other relevant factors, as trainers is accounted for in the SQT summary reports which are given to commanders at all levels.

Training Developers. The base for individual proficiency is the job and task analysis which builds the Soldier's Manual and the task related training materials. The training developer integrates doctrine, information, procedures and all aspects of job performance into supporting training materials for use by the individual in achieving and maintaining his job proficiency. The quality of these training products, in the context of other relevant factors, are measured in the performance of the soldiers in the tasks being trained in these products. The school commandant is accountable for the efficiency and effectiveness of the training systems his school creates for the specialities and units for which he is proponent.

Training Program Managers. For those delivery systems which are centrally managed such as TEC, ACCP, Armywide Training Literature, and Training Devices, there is a responsibility to the individual training system for timely and effective delivery of the training materials developed in the schools. The precision of the SQT will allow for examination of each of these products and their contribution to the soldiers ability to do his job. Each program manager is accountable, in the context of other relevant factors, as to whether his product achieves its objectives.

SQT Accountability. Lastly, the SQT must be a fair and relevant performance oriented, criterion referenced test. The SQT uses formative and summative methods of test design and review. However, the final judges are the soldiers and commanders who use the SQT to achieve and maintain unit readiness.

The SQT Training Accountability System is completely interactive in that each variable must be well defined and specifically documented in determining where the system is flawed when a soldier cannot meet the standards of his job. The SQT is an evolutionary evaluation tool to achieve and maintain unit readiness.

Title: WSTEA - An Analytical Approach
Authors: Major Albert J. Erickson and Mr. James P. Hamill, US Army Armor School
Abstract: The paper will describe an approach developed by the USAARMS for conducting a Weapons System Training Effectiveness Analysis (WSTEA). Included will be the methodology and analytical techniques used to:

1. Conduct a complete front end analysis to include equipment, personnel and intended use.
2. Obtain data on the performance of individuals and units using the current training program.
3. Analyze the data to determine the effectiveness of the program.
4. Determine whether there are modifications which should be made to the program and determine whether the modifications should be cost-effective.
5. Develop the cost-effective modifications in detail.
6. Test the modifications to determine whether they accomplish their objectives.
7. Integrate the proven modifications into the existing program.
8. Implement the modified program.

Title: A Performance Risk Analysis
Authors: Mr. Jose Antunes, Mr. Gerald Klopp, Mr. James O'Brien, and MAJ K. Mark Woodbury, US Army Logistics Management Center
Abstract: Risk analysis is a systematic process to evaluate alternative research and development projects. The areas analyzed in a risk analysis include the triad of cost, schedule, and performance. Cost and Schedule were first addressed in the 1950's by CPM and PERT; since the late 1960's more sophisticated computer simulations like RISCA, SOLVNET, and VERT provide ample means to focus on the problem areas of cost and schedule risk. This paper discusses a computer simulation process to analyze risk associated with performance. The risks and uncertainties in various performance parameters are evaluated as probabilities and probability distributions. Via Monte Carlo simulation, the interrelationships among the various performances parameters are quantified. Their effect on achieving an assigned level of performance can thus be measured. The objective of this article is to demonstrate a technique to analyze performance risk or uncertainty. To achieve this objective, parametric equations which describe the relationship among design, operational, and environmental variables of a helicopter system will be presented. Certain of these variables will be described in terms of subjective distributions to indicate the measure of uncertainty in particular variables. The conclusions of the analysis demonstrate an approach to address performance risk analysis. Finally, the performance risk analysis is related and analyzed in conjunction with schedule and cost risk analysis for the helicopter system.

Title: The T-COR Model and its Applications
Author: Mr. Michal Mruz, The BDM Corporation
Abstract: T-COR is a theater-level combat model being developed for the Defense Nuclear Agency. The model features a Simulation Control Software (SCS) package, dynamic storage allocation, a mixed event/time-stepping capability, and a multi-granular hexagonal coordinate system, all of which serve as tools to be used by the modeler/analyst to develop very powerful and flexible simulation models. The modeling routines in T-COR use a "player-centered," as opposed to "process-centered" approach, and contain routines which treat perceptions of reality by the various units played in the game. This allows T-COR to model the value of information to the commander, and the effects of distorted perceptions on the outcome of the conflict. T-COR has undergone a preliminary development phase, and at present the modeling routines contain only a low level of detail. During the second phase, which continues through this summer, the modeling will be developed to a level of detail sufficient to use T-COR in a study effort later this year. Input and output of data for the study will be accomplished with the User-Oriented Input Language (UOIL), which allows the analyst to put data into the model using English-like sentences and phrases, and to get results of the simulation out of the model in the same manner.

Title: Helicopter Cargo Systems Effectiveness Analysis
Authors: Mr. Timothy D. Evans and Mr. Jules A. Vichness, US Army Aviation Systems Command
Abstract: This paper presents the results of a project to evaluate the effectiveness of proposed alternative helicopter cargo handling systems. Mathematical modeling and simulation techniques were used to develop the analysis method presented here. Factors considered in the evaluation include combat threat, terrain types, and weather conditions. Also considered were the physical constraints of the cargo system itself, such as: loading and unloading time, number of sorties required per ton load, reliability, maintainability, and availability, and effect on aircraft vulnerability. The analysis provides measures of life cycle cost, operational effectiveness, productivity and performance in both combat and noncombat situations. The results of this evaluation are used to assist in establishing research priorities in terms of benefits achieved, costs incurred, and risks assumed.

Title: PERT as a Management Tool for Implementing a Realignment Action
Authors: Mr. A. J. Rymiszewski, Mr. Donald S. Case, Mr. Phillip Powers and Mr. Kurt Wussow, US Army Tank-Automotive Materiel Readiness Command
Abstract: This paper will present how USATARCOM responded to the DA request to implement a realignment action for the TARCOM Support Activity, Selfridge Air National Guard Base, Michigan. Emphasis will be placed on the multi-disciplined task team composition and their use of OR/SA techniques to develop an implementation milestone schedule. The use of PERT as a management flow diagram to define as well as control the required interrelationships between various TARCOM functional directorates with DARCOM, DA, AAA, DCAA and SBA will be highlighted.

Title: Application of Dynamic Mean to Demand Forecasting
Author: Mr. Daniel Meng, US Army Tank-Automotive Materiel Readiness Command
Abstract: The objective of this study is to find practical forecast techniques other than the moving average currently employed in forecasting secondary item demand. New parameters were derived through the dynamic mean process. The resultant forecast was compared with both moving average and Kalman filter techniques based on six years of tank-automotive repair parts demand histories. Better forecast parameters are recommended.

Title: New Management Coordination Tool: Definition/Prioritization/ Management (DPM) Technique
Author: Mr. V. Berger, US Army Aviation Systems Command
Abstract: This paper presents a process whereby project and weapon systems managers, and their supporting network of functional managers recognize and prioritize topics of mutual interest. The process is designed to accommodate project dynamics, to provide a procedure for management by exception, and to generate an audit trail of management activities in complex projects. The DPM technique is suitable for use in conjunction with DRA-guided program management. The paper provides examples of DPM technique applications in weapon systems management.

Title: MOEs for Division Level Models
Author: MAJ John H. Shuford and CPT Fredrick H. Knack, US Army TRADOC Systems Analysis Activity
Abstract: High level excursions, using the Division Battle Model (DBM) or a similar game, are expected to become more important in the performance of future Cost and Operational Effectiveness Analyses (COEAs). It is therefore necessary that a good Measure of Effectiveness (MOE) for use with these games be developed. Certain MOE, such as the force exchange ratio or other ratios, have become accepted as providing good estimates of the results of high resolution, company/battalion level combat simulations. Efforts have also been made to develop analytical weighting systems for the different weapons in order to compute weighted MOE. Both of these methods have been used to analyze the outcome of DBM, a low resolution, division level war game, but neither has been entirely satisfactory. It is hoped that this paper will stimulate interest and further investigation into the analysis and interpretation of combat simulation results.

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